

# Fair Allocation

**3<sup>rd</sup> International Workshop on  
“Game Theory - Economics and Mathematics”**

OCTOBER 2-3, 2014

UNIVERSITY OF SOUTHERN DENMARK

HOSTED BY

DEPARTMENT OF BUSINESS AND ECONOMICS

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# Programme

Thursday sessions take place in room Ø95.

Friday sessions take place in the IVØ seminar room.

## Thursday, October 2nd

- 9:45-10:00 Arrival
- 10:00-11:30 Session 1 chaired by Lars Peter Østerdal  
Michel Le Breton (Toulouse School of Economics, France)  
*Fairness versus bargaining in French regional elections:  
a cooperative game theory perspective*  
José Manuel Zarzuelo (Universidad del País Vasco)  
*Highway Games and Airport Games*
- 11:30-11:40 Coffee break
- 11:40-12:30 Session 2 chaired by Karol Szwagrzak  
Karol Szwagrzak (University of Southern Denmark)  
*Allocating divisible and indivisible resources according to conflicting claims:  
collectively rational solutions*  
Tina Gottschalk (University of Southern Denmark)  
*A characterization of Atkinson-Bourguignon dominance for welfare comparisons*
- 12:30-13:30 Lunch
- 13:30-15:00 Session 3 chaired by Peter Sudhölter  
Tommy Andersson (Lund University, Sweden)  
*Transferring ownership of public housing to existing tenants: a mechanism design approach*  
Norovsambuu Tumennasan (Aarhus University)  
*The deferred acceptance mechanism in dynamic markets*
- 15:00-15:15 Coffee break

15:15-17:15 Session 4 chaired by Lone Grønbæk Kronbak

Marko Lindroos (University of Helsinki, Finland)

*International fisheries management: choice of management instruments under coalition formation*

Pedro Pintassilgo (University of Algarve, Portugal)

*Application of an almost ideal sharing scheme to the formation of international climate agreements under uncertainty*

Lone Grønbæk Kronbak (University of Southern Denmark)

*On international fisheries agreement, entry deterrence and ecological uncertainty*

18:30- Dinner at Den Gamle Kro

### **Friday, October 3rd**

9:15-10:45 Session 5 chaired by Rafael Treibich

Paolo Piacquadio (University of Oslo, Norway)

*Fair intergenerational utilitarianism: risk, its resolution over time, and discounting*

Rafael Treibich (University of Southern Denmark)

*Welfare egalitarianism with other-regarding preferences*

10:45-11:00 Coffee break

11:00-12:40 Session 6 chaired by Trine Tornøe Platz

Peter Sudhölter (University of Southern Denmark)

*Variants of the core for games with precedence constraints*

Jesper Normann Breinbjerg (University of Southern Denmark)

*Strategic behavior and social outcomes in a bottleneck queue: experimental evidence*

Trine Tornøe Platz (University of Southern Denmark)

*On games arising from multi-depot traveling salesman problems*

12:40-13:30 Lunch and goodbye

# Abstracts

1. Tommy Andersson (Lund University, Sweden)

*Transferring ownership of public housing to existing tenants: a Mechanism design approach*

(joint work with Lars Ehlers and Lars-Gunnar Svensson)

This paper explores the situation when tenants in public houses, in a specific neighborhood, are given the legislated right to buy the houses they live in but can choose to remain in their houses and pay the regulated rent. This type of legislation has been passed in many European countries in the last 30-35 years (U.K. Housing Act 1980 is a leading example). The main objective with this type of legislation is to transfer the ownership of the houses from the public authority to the tenants. To achieve this goal, the selling prices of the public houses are typically heavily subsidized. The legislating body then faces a trade-off between achieving the goals of the legislation and allocating the houses efficiently. This paper investigates this specific trade-off and identifies an allocation rule that is individual rational, equilibrium selecting, and group non-manipulable in a restricted preference domain that contains "almost all" preference profiles. In this restricted domain, the identified rule is the equilibrium selecting rule that transfers the maximum number of ownerships from the public authority to the tenants. This rule is also weakly preferred to the current U.K. system by both the existing tenants and the public authority. Finally, a dynamic process that finds the outcome of the identified rule, in a finite number of steps, is provided.

2. Jesper Normann Breinbjerg (University of Southern Denmark)

*Strategic behavior and social outcomes in a bottleneck queue: experimental evidence*

(joint work with Alexander Sebald and Lars Peter Østerdal)

We consider a class of three-player queuing games where players independently choose when to arrive at a bottleneck facility that serves only one at a time. Players are impatient for service but cannot arrive before the facility opens and they dislike time spent in queue. We derive the equilibrium arrivals under the first-in-first-out (FIFO), last-in-first-out (LIFO), and service-in-random-order (SIRO) queue disciplines and compare these equilibrium predictions to outcomes from a laboratory experiment. LIFO provides higher equilibrium welfare than FIFO and SIRO since the players arrive such that lower congestion is induced. Experimental evidence

confirms that employing different queue disciplines indeed affects the strategic behavior of players and thereby the level of congestion. The experimental participants do not, however, behave as prescribed by the equilibrium predictions. They obtain significantly higher welfare than prescribed by equilibrium under all queue disciplines. Our results moreover suggest that people perceive LIFO as the most unfair of the three disciplines although the theoretical results suggest that it is welfare optimal.

3. Michel Le Breton (Toulouse School of Economics, France)

*Fairness versus bargaining in french regional elections: a cooperative game theory perspective*

The main objective of this paper is to construct a cooperative game with transferable utility that describes the spectrum of alliance strategies arising in a specific two round electoral system in a multimember district ( $K$  seats have to be filled) where the competing parties present closed lists of  $K$  candidates. This system is used in France to elect the members of the regional and municipal councils. The mechanism works as follows: If at the end of the first round a list obtains more than a majority of the votes, the election is over : a fixed fraction of the seats go to that list (as a bonus) and the remaining fraction is allocated through a proportional formula to the lists (including the winning list) whose score is above some lower threshold. Otherwise, a second round is organized. In that case, parties whose score belong to the interval defined by the lower threshold and an upper threshold (in the case of the French regional elections, the interval is  $[5\%,10\%]$ ), cannot participate to the second round unless a party with a score larger than the upper threshold accepts to form a list merging some of its candidates with some candidates of that partner. There is no limit on the number of partners that a party can consider in this merging operation. After these operations, a second round with the new closed lists takes place : a fraction of the seats is allocated through a proportional formula while the remaining fraction goes (again as a bonus) to the list with the highest score. The paper offers a structural framework to understand how the positions in the lists resulting from a merging operation were allocated among the partners in the coalition. Is it the case that some bargaining logic is at works or should we instead consider that the parties have followed some exogenous norm without taking advantage of their bargaining power. To do so, we write down a characteristic function which captures the strength of each coalition in such a strategic setting and examine the implications in terms of seats allocations of three solutions: two of them (the nucleolus and the Shapley value) are game theoretical and take into account the bargaining strengths of the players (here the parties) in the negotia-

tions surrounding the merging operation. The third one (called Gamson) suggests, as a fairness principle, to use a proportionality norm using the scores at the end of the first round. We contrast the predictive power of these three solutions on real electoral data (the 2010 French regional elections).

4. Tina Gottschalk (University of Southern Denmark)

(joint work with Troels Martin Range, Peter Sudhölter, and Lars Peter Østerdal)

We look at stochastic dominance techniques used for welfare comparisons. A seminal contribution in the field is made by Atkinson and Bourguignon. However we find the existing equivalent definitions of this stochastic dominance concept rather theoretical and we miss some intuitive understanding of when this concept determines that one population is unambiguously better off than another. We will examine their definition of first order dominance in order to find an equivalent definition. We will make it clear how we can characterize Atkinson and Bourguignon dominance in terms of underlying elementary operations.

5. Lone Grønbæk Kronbak (University of Southern Denmark)

*On international fisheries agreement, entry deterrence and ecological uncertainty*

(joint work with Hans Ellefsen and Lars Ravn-Jonsen)

From the game theoretical perspective, a prerequisite for an international fishery agreement (IFA) to be stable is that parties expect their benefits from joining the agreement to exceed benefits from free riding on the agreement, and parties only comply with the agreement as long as this is true. The agreement, therefore, implicitly builds on an expectation of the ecological condition of the natural resource. Typically, the game theoretical models assume that all parties have the same (often perfect) information of the resource and the exploitation is an equilibrium use of the stock. As known from experts from natural science the fish ecology still has many open questions, e.g. predicting population dynamics, migration patterns, food availability, etc. In some cases parties disagree about the state, abundance and migration of a stock, which can significantly disturb the possibilities for reaching an agreement for exploitation of the stock. The paper develops a model, applied to the North East Atlantic mackerel fishery, analyzing IFA under different ecological scenarios combining it with the economic theory of entry deterrence. The model is empirically used to determine whether the original parties to the agreement have an advantage in that the results from fishing the stock down to a smaller size to prevent another party from entering into the fishery. The paper presents a novel method for

illustrating the obstacles in why an agreement for the North East Atlantic mackerel has been so hard to achieve.

6. Marko Lindroos (University of Helsinki, Finland)

*International fisheries management: choice of management instruments under coalition formation*

The purpose of the current paper is to study how different management instruments work when we allow for coalition formation. We will modify the Ruseski (1998) model to include coalition formation at the country and/or fisher level. We use the same two management instruments, subsidies and fleet licenses in the present study. However, later studies could also incorporate other management options including TACs and/or ITQs. As an extension of our model we study a case where one country uses subsidies and the other fleet licenses. This allows us to take one step towards the question: What is the optimal management for each country. From the social/global point of view it is also interesting to study which combination of management instruments yields the highest economic returns, or largest stable coalitions.

7. Paolo Piacquadio (University of Oslo, Norway)

*Fair intergenerational utilitarianism: risk, its resolution over time, and discounting*

The paper examines the welfare economics of intergenerational risk. Risk and its resolution are modeled as a decision tree: in each period, the outcome of the current one-period living generation is to be traded-off against uncertain benefits of future generations; as time passes, the planner observes the realized shocks and becomes more informed about the true state of the world. The axiomatic approach singles out the family of fair intergenerational utilitarian criteria, avoiding serious drawbacks of expected utilitarianism. According to such criteria, each generation's welfare is measured by a CES aggregation of the outcome at each history relative to an endogenously determined reference, called fair prospect; total welfare is the risk-adjusted discounted sum of a CRRA transform of each generation's welfare. Depending on the magnitude of risk, on the timing of its resolution, and on the planner's risk attitude, specific discounting formulas obtain, including exponential, quasi-hyperbolic, and hyperbolic as special cases.

8. Pedro Pintassilgo (University of Algarve, Portugal)

*Application of an almost ideal sharing scheme to the formation of international climate agreements under uncertainty*

This presentation explores the potential of the Almost Ideal Sharing Scheme (AISS), proposed by Eyckmans and Finus (2009), to stabilize international climate agreements under uncertainty and is based on the research undertaken by Finus and Pintassilgo (2013). The AISS has some interesting features. First, every coalition that can be potentially internally stabilized will be internally stable with the AISS. Second, if a coalition is stable without transfers, then with the AISS that coalition will either be stable or a larger coalition with additional players will be stable. Third, among those coalitions which can be potentially internally stabilized, the AISS stabilizes the coalition with the highest global payoff. This explains the word "Ideal" in the name of the transfer scheme. Because this may not be the grand coalition if free-rider incentives are too strong, the word "Almost" is used. From these three features it can be concluded that, with the AISS, stable coalitions will be at least as large and successful (in terms of aggregate payoffs) than without transfers.

9. Peter Sudhölter (University of Southern Denmark)

*Variants of the core for games with precedence constraints*

(joint work with Michel Grabisch)

In a series of papers we investigate cooperative games with restricted cooperation induced by a partial order on the set of players. An element of the possibly unbounded core of such a cooperative game with precedence constraints belongs to its bounded core if any transfer to a player from any of her subordinates results in payoffs outside the core. It is shown that for transferable utility (TU) games the bounded core is the union of all bounded faces of the core. Moreover, we prove that both for TU games and for NTU games certain variants of the reduced game property, its converse, and the reconfirmation property, all with respect to the Davis-Maschler reduced game, together with some variants of other well-accepted properties, still characterize the core and its bounded variant. For a TU game with precedence constraints the positive core and the positive prekernel and its bounded variants are also axiomatized similarly as, even simpler than, in the classical case. Hence, the axiomatic approach indicates that these core extensions are excellent nonempty substitutes of the prenucleolus and the prekernel that both may not exist when precedence constraints are present.

10. Karol Szwagrzak (University of Southern Denmark)

*Allocating divisible and indivisible resources according to conflicting claims: collectively rational solutions*

(joint work with Albin Erlanson)



We consider the problem of allocating multiple divisible and indivisible resources according to conflicting claims on these resources. We prove that choosing allocations maximizing a separable social welfare function is a consequence of three basic principles: consistency, resource-monotonicity, and the independence of irrelevant alternatives.

11. Trine Tornøe Platz (University of Southern Denmark)

*On games arising from multi-depot traveling salesman problems*

A Steiner traveling salesman problem (STSP) can be represented by a connected (di)graph in which a fixed node is denoted the depot, and a weight function is defined on the edges of the graph. A solution to the STSP is a minimum weight tour that starts and ends at the depot and visits every node of the graph. If different players are associated with the nodes of the graph that are to be visited, a cooperative traveling salesman game is induced. The paper generalizes this notion of a traveling salesman (TS) game to allow for multiple depots in the underlying STSP and analyzes the submodularity of such multi-depot TS games by characterising globally (and locally)  $k$ -TS submodular graphs and digraphs. A graph  $G$  is called globally (locally)  $k$ -TS submodular, if the TS game induced by a multi-depot TSP on  $G$  is submodular for any (some) choice of  $k$  depots and every non-negative weight function.

12. Rafael Treibich (University of Southern Denmark)

*Welfare egalitarianism with other-regarding preferences*

We study the fair allocation of a one dimensional and infinitely divisible good (money) when individuals have other-regarding preferences. Using only ordinal and non-comparable information about such preferences, how should society measure the social welfare associated to any conceivable allocation? In both a model of average and positional externalities, we characterize a class of social preferences which satisfy appealing efficiency and equity properties. These rankings require giving full priority to the worst off individual in the economy, where utility is measured by the quantity which would leave the individual indifferent to the current allocation if it was consumed equally by everyone.

13. Norovsambuu Tumennasan (Aarhus University)

*The deferred acceptance mechanism in dynamic markets*

Many matching markets are intrinsically dynamic. In such markets, it is common that the priorities of one side of a market depend on previous allocations, which creates opportunities for manipulations. Indeed, in the dynamic version of the school-choice problem, Kennes et. al (2014) show that no mechanism exists that is both stable and strategy-proof. In this paper, we show that under a suitable restriction on the schools' priorities, the fraction of players with incentives to manipulate the deferred acceptance mechanism approaches zero as the number of participants increases. When this restriction is not satisfied, the mechanism remains manipulable even in large markets.

14. José Manuel Zarzuelo (Universidad del País Vasco)

*Highway Games and Airport Games*

A particular class of cost allocation problems in which some agents jointly use a common resource or facility is studied. The peculiarity is that this resource is an ordered set of several indivisible sections, where each agent requires some consecutive sections, and each section has an associated a fixed cost. The issue is how to share the total cost of the sections among the users in an efficient and fair way. A simple example that illustrates these situations is the case of a linear highway, where the sections are delimited by the entry and exit points, and each car or truck only needs the highway sections between his entry and exit point.

It turns out that cooperative games are especially appropriate to model these situations. And, moreover, the solution concepts of these games embody a criterion of fairness in their definition and satisfy certain properties or axioms which make them particularly suitable for solving these problems. In this presentation the attention mainly focus on two solution concepts: the Shapley value and the nucleolus, and their application to some instances of these special cost allocation problems.

Firstly, it is considered the case of airport problems: These classical problems refer to the particular case of a highway problem in which all the agents use the same entry. We recall the formula for the Shapley value of Littlechild and Owen (1973), and also Littlechild's (1974) algorithm to find the nucleolus of airport problems whose calculation is considerably more complex. Next we consider a richer model by considering the potential revenues or benefits by the agents for using the facility, and we provide an algorithm to find the nucleolus in this special case (Brânzei, Iñarra, Tijs and Zarzuelo, 2006). Next it is analogously a formula for the Shapley value and an algorithm to find the nucleolus for the general situation of highway problems (Mosquera, Kuipers and Zarzuelo, 2013). Finally it is addressed the initial question of how to share the total cost from a non-cooperative perspective,

seeking to complement cooperative approach. Namely, they are proposed two non-cooperative bargaining games with a unique subgame perfect equilibrium outcome whose payoffs  $s$  are respectively the Shapley value and the nucleolus of an airport problem (Albizuri and Zarzuelo, 2014). Furthermore, it is shown that all the subgame perfect equilibria of these games are also coalition-proof.