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A Leverage-Based Measure of Financial Instability

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Abstract: We employ a model of leverage-induced explosive behavior in financial markets to develop a measure of financial market instability. Specifically, we derive a quantitative condition for how large levered investors can become relative to the whole market before the demand curve for securities suddenly becomes upward-sloping and small price declines cascade as levered investors are forced to liquidate. The size and leverage of all levered investors and the elasticity of demand of unlevered investors define the minimum market size for stability (or MinMaSS), the smallest market size that can support a given group of levered investors. The ratio of actual market size to MinMaSS is termed the instability ratio, and can give regulators and policymakers advance warning of financial crises. We apply the instability ratio in an investigation of the 1998 demise of the hedge fund Long-Term Capital Management. We find that a forced liquidation of the fund threatened to destabilize some financial markets, particularly for bank funding and equity volatility.

Keywords: Leverage; financial crisis; financial stability; minimum market size for stability (MinMaSS); instability ratio; Long-Term Capital Management (LTCM).
JEL Classification Numbers: E0; E58; G01; G10; G20; G21.

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1 Introduction

Financial crises are difficult to understand using the neoclassical benchmark assumptions of financial economics. These assumptions permit neither incorrect valuation of assets nor large price changes without changes in expectations, nor do they even countenance a role for liquidity, money or banks. As a result, economists tend to view financial crises as arising from some "market imperfection"—language that suggests that the phenomenon is not a result of behavior or conditions that are deep or fundamental. In this paper, we recognize that leverage and credit constraints are basic and universal features of financial markets, and show that these virtually alone are sufficient to produce financial crises under a range of assumptions about behavior. We develop a generalized measure of leverage and use it to derive a quantitative condition for instability. The condition can be evaluated from observable characteristics and can give policymakers advance warning of financial crises. We define a financial market equilibrium as unstable when the process of *tatonnement* pushes the system away from, rather than towards, equilibrium. In the context of a limit order book, this corresponds to a situation where demand rises with price, and does so faster than supply. It is well-known that levered investors may have upward-sloping demand curves, and that this causes shocks to asset prices to be amplified via what is known in the literature as a "financial accelerator" mechanism.[1] It is therefore clear that too large a proportion of levered investors makes it likely that the aggregate demand curve for assets will become upward sloping, leading equilibria to be unstable. Prices can thus undergo large and rapid changes even without any change in expectations, news, the real economy, or underlying market architecture.

We then show how the model can be applied to a complex case with sophisticated strategies and financial instruments. Using the model, we reinterpret the 1998 col-

[1] See, for example, Adrian and Shin (2010) who admit the possibility of upward-sloping demand leading to a feedback effect when markets are hit by a shock, even if they do not construct a model.

lapse of hedge fund Long-Term Capital Management (LTCM). The instability coefficients emerging from our framework imply that LTCM was not large enough to destabilize equity or Treasury markets in general. However, it could, in an inelastic demand environment, have been large enough to destabilize the markets for bank funding and equity volatility. The consequences of this potential instability eventually prompted the Federal Reserve to step in and coordinate a private-sector bailout.

Section 2 relates briefly the approach we take to the previous literature. Section 3 develops a simple, intuitive version of the model. Section 4 fully generalizes the model to a degree that permits it to be used in the real world. As part of the generalization, we show that the use of financial derivatives is equivalent to leverage and is usually more likely to create the conditions for instability. We will also see that when levered investors play in multiple markets, a crash in one market can lead to contagion into other markets. Section 5 applies our measure of financial instability to the 1998 collapse of hedge fund Long-Term Capital Management. Section 6 concludes.

2 Previous Literature

Broadly speaking, models where limitations on leverage and collateral play an important role have been referred to in the literature as models with "net worth effects." These models fall into three main categories.

The first, generally known as financial accelerator models, focus on the impact of credit constraints tied to net worth on the real economy. The two seminal contributions in this area are Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who show that negative net worth shocks force the most productive entrepreneurs to liquidate and therefore have persistent effects on the economy.

A second strand of the literature evaluates the effect of collateral requirements specifically on financial markets, showing how forced liquidations from negative net worth shocks can lead to large fluctuations in asset prices. These include Geanakoplos (2003), Fostel and Geanakoplos (2008), Yuan (2005), Xiong (2001), Adrian and Brunnermeier (2011), Chowdhry and Nanda (1998), Acharya and Viswanathan (2011), and Gromb and Vayanos (2010). These stylized models tend to have only two or three periods and sometimes permit only a discrete number of possible asset prices, which causes challenges in quantitative empirical application or in use as a policy tool.

A third strand of the literature has examined feedback loops where small shocks cause cascading liquidations through channels other than the collateral constraint. These include fund redemptions (Shleifer and Vishny, 1997), price movements being interpreted as fundamental signals (Brunnermeier and Pedersen, 2009; Diamond and Verrecchia, 1980; Gennotte and Leland, 1990), tightening margin requirements (e.g., Brunnermeier and Pedersen, 2009; Fostel and Geanakoplos, 2008), or uncertainty about bank solvency (Gorton and Metrick, 2012). We produce similar dynamics from features that are more universally present in financial markets, suggesting that some of the mechanisms in these earlier models may be amplifications of the undefrlying mechanism of the crisis rather than the essence of the mechanism itself.

There is also a large literature on banking crises. The seminal contribution is by Diamond and Dybvig (1983) and the literature is reviewed by Claessens and Kose (2013). In these models, the financial crisis arises on the funding liquidity side while in the present model it is on the market liquidity side.

In addition to the theoretical literature on financial crises, we are aware of a few authors who have attempted to create measures of financial instability. These include CoVAR (Adrian and Brunnermeier, 2011), an Asset Quality Index (Kamada and Nasu, March 2010) and the credit-to-GDP ratio (Borio and Lowe, 2002). Finally, there exists a modest literature on the near-collapse of LTCM. Two full-length books (Dunbar, 2000; Lowenstein, 2000) offer accounts in a journalistic style.

Two academic studies examine the failure of LTCM through a business framework (Perrold, 1999) and a sociological framework (MacKenzie, 2003). Jorion (2000) conducts a calibration exercise and deducts risk management lessons from the case of LTCM's failure. Schnabel and Shin (2004) compare the near collapse of LTCM and the historical northern European crisis of 1763, and point at the timelessness of certain determinants (e.g., the potentially devastating effects of liquidity risk). Dungey et al. (2006) quantify the contribution of contagion to the spread of increased volatility in international bond markets after LTCM's recapitalization announcement.

3 The Basic Model

In our model there are four types of agents: *levered investors*, who are sufficiently confident in their views that they lever their positions to the maximum level permitted by their lenders; *fully funded investors*, who have a downward-sloping demand curve for the asset, and who deposit any excess funds in a bank account; *banks*, who provide credit to the levered investors at the market interest rate and a fixed margin requirement (the reasons for these assumptions are discussed later); and a *central bank*, whose sole function is to hold interest rates fixed in the near term by providing credit to the market against sound collateral.

3.1 Levered Investors

Suppose an investor is extremely enthusiastic about an asset, so that she wishes to purchase as much of it as she is able with as much leverage as her lenders permit.

Such seemingly simplistic rule of thumb behavior can in fact be supported by appropriate microfoundations; the literature investigating optimal portfolio choice in the presence of net worth constraints and credit constraints finds that sufficiently optimistic rational agents do indeed employ leverage to the maximum degree permitted by their lenders (e.g., Grossman and Vila, 1992; Liu and Longstaff, 2004).[2] The net worth of the levered investor is then given by:

$$\text{Net Worth} = (\text{Margin Percentage}) \cdot (\text{Assets}) \tag{1}$$

$$\equiv \lambda \cdot p_t m_t^{\text{lv}} \tag{2}$$

where p_t is the price of the asset at time t, m_t^{lv} is the quantity of the asset held by the levered investor at time t and λ is the margin requirement imposed by lenders or by regulators (the minimum proportion of the investor's assets that have to be covered by equity). Each period, the investor will reap the benefit of all price appreciation and dividends d_t from the assets and pay interest rate r_t charged on margin loans, so the change in net worth will be given by:

$$\Delta NW = \text{Appreciation} + \text{Dividends} - (\text{Margin Interest}) \tag{3}$$

$$= (p_t - p_{t-1})m_{t-1}^{\text{lv}} + d_t \Delta t \cdot m_{t-1}^{\text{lv}} - (1-\lambda) r_t \Delta t \cdot p_{t-1} m_{t-1}^{\text{lv}} \tag{4}$$

Adding (2) and (4), and simplifying, we have:

$$NW_t = m_{t-1}^{\text{lv}} \cdot [p_t + d_t \Delta t - (1-\lambda)(1 + r_t \Delta t) p_{t-1}] \tag{5}$$

This equation says simply that net worth is given by the current value of last period's assets, plus any dividends received on those assets, minus the value of debt (with

[2] Even if real-world levered investors have some slack and cushion built in for the short term, investors still tend to target a certain leverage ratio over the medium term: they voluntarily liquidate when their net worth declines in order to avoid forced liquidations later, so they face what is effectively a "soft" margin requirement.

interest) funding those assets.

The enthusiastic investor invests her profits back into the asset with leverage. A combination of (2) and (5) simplifies to:

$$m_t^{\text{lv}} = \frac{m_{t-1}^{\text{lv}}}{\lambda} \cdot \left[1 + \frac{d_t \Delta t - (1-\lambda)(1 + r_t \Delta t) p_{t-1}}{p_t}\right] \tag{6}$$

This gives the levered investors' demand for assets m_t^{lv} as a function of the price p_t; it is their demand curve for assets, and it is almost certainly upward-sloping.[3] Investors targeting a specific leverage ratio demand more of an asset as its price increases. Finally, leverage introduces path-dependent demand; levered investors' demand depends on both yesterday's holdings and yesterday's price.

While we express levered investors' behavior as a rule of thumb, it is one that can be the result of optimizing behavior as discussed above. There is much to be gained from this approach. No empirically ungrounded assumptions are required about the functional forms of preferences or processes. We are able to incorporate heterogeneity without having to worry about strategic or game theoretic effects. Relying on previous findings of optimizing behavior allows us to avoid the mathematical complexities of nesting optimization within our model. This means that the model is far more tractable, and therefore more transparent about what is driving the results.

[3] The only way it is not is when both the time step between margin checks is large and the dividend is very large compared to the margin interest rate. To see how unlikely this is, consider the case where $\Delta t = 1$, that is, portfolio reallocations and margin calls take place only once a year. Consider a high capital ratio of 90% (nine dollars of equity for every dollar of debt), and a margin interest rate of just 1.5%. The dividend yield would then have to be greater than 10.15% in order for the demand curve to be downward-sloping.

3.2 Fully Funded Investors

The remaining investors who invest without leverage are termed fully funded investors. These investors are not modeled in detail, but are assumed to have a downward-sloping demand curve for assets. This downward-sloping (rather than horizontal) demand may be for a variety of reasons, including heterogeneity of opinion about the value of the asset, relative value considerations, and the desire for portfolio diversification.[4] More simply, investors who eschew the use of leverage are limited in their asset purchases by their equity, and so the maximum number of shares they are able to purchase is a declining function of the share price. Demand by fully funded investors is then given by:

$$\text{Demand for Assets} = (\text{Proportion of Fully Funded Investors}) \times (\text{Population of Investors}) \times (\text{Demand per Fully Funded Investor}) \tag{7}$$

or:

$$m^{\text{ff}} = (1-\mu)N \cdot D(p) \tag{8}$$

where m_t^{ff} is the total demand for assets by fully funded investors, μ is the proportion of investors that are levered, N is the total number of investors in the economy and $D(p)$ is the number of assets that the average fully funded investor demands as a function of price. We shall assume that $D'(p) < 0$ so that demand is downward-sloping and demand does not depend upon the investor's net worth.[5]

[4] Relative value is a method of determining an asset's value by considering the value of similar assets. The value investor does not invest into the more overvalued asset, which puts downward pressure on its price. Portfolio diversification is when an investor wishes to keep a fixed proportion of her portfolio in different assets, such as the orthodox portfolio split of 60% stocks and 40% bonds. As the price of stocks rises, she needs to hold fewer shares to account for 60% of her portfolio.

[5] This is an abstraction which is unlikely to be correct in the real world. However, the dependence on price can capture this effect at each moment in time, since we have not specified a functional form. At first glance, it might appear that wealth effects could be large enough that they make fully funded investors' demand upward-sloping. Closer examination reveals that this is not the case. Suppose that unlevered investors hold a proportion of their wealth $\beta(p)$ in an asset, and that they have net worth y. Then the number of units of the asset each fully funded investor demands is given by:

$$D(p) = \frac{y\beta(p)}{p} \tag{9}$$

3.3 Banks

Banks are assumed to be conduits that lend to all comers against collateral at the prevailing interest rate, which is fixed by a central bank, and with a fixed margin requirement. The existence of banks links the money supply to credit growth and hence links monetary policy to credit provided to speculative endeavors. While the model could work just as well without banks, we include them in order to ensure we meet adding-up constraints.

3.4 The Central Bank

The central bank simply supplies liquidity to the banking system in order to keep the interest rate fixed at r, which is given exogenously but which we may vary in order to examine the impact of central bank policy.

3.5 Model Dynamics

The model's dynamics are governed largely by the behavior of the levered and fully funded investors. We shall investigate how the model behaves over short periods of time, where interest and dividend payments can be neglected.

The total demand for the asset, m_t, is the sum of the demand by levered and fully

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Differentiating in logs gives:

$$\frac{d\log D(p)}{dp} = \frac{1}{y}\frac{dy}{dp} - \frac{1}{p} + \frac{\beta'(p)}{\beta(p)} \tag{10}$$

The last term in this equation is negative. After examining the first term, we shall see that it is always outweighed by the second term, so that demand remains downward-sloping. Suppose that the unlevered investor owns m assets at price p, and has additional assets y_0. Then

$$y = pm + y_0$$

and

$$\frac{1}{y}\frac{dy}{dp} = \frac{m}{pm + y_0} < \frac{1}{p}$$

for $y_0 > 0$. So the demand curve remains downward-sloping.

funded investors:

$$m_t = m_t^{\text{ff}} + m_t^{\text{lv}} \tag{11}$$
$$= (1-\mu)N \cdot D(p_t) + \frac{m_{t-1}^{\text{lv}}}{\lambda} \cdot \left[1 - \frac{(1-\lambda)p_{t-1}}{p_t}\right] \tag{12}$$

It is clear that if $(1-\mu)N \cdot D(p)$ is large compared to m_{t-1}^{lv}, the fully funded investors will dominate and the demand curve will be downward-sloping, with the supply-demand diagram for securities appearing like panel A of Figure 1. However, if levered investors begin to do well, reinvest their proceeds and accumulate the asset, m_{t-1}^{lv} will begin to grow large relative to $(1-\mu)N \cdot D(p)$.

Eventually, the market reaches a tipping point. If levered investors get infinitesimally richer, the demand curve deforms itself from a curve like D into a backward-bending curve like D' in panel B of Figure 1. This infinitesimal change in the levered investors' fortunes changes the character of the system. Point E becomes an unstable equilibrium: any slight positive demand shock (to curve D'' in panel C) would result in excess demand but a lower equilibrium price at point E''. The demand shock would thus result initially in a price that was above equilibrium but with excess demand. The excess of buyers over sellers would push the price up further, resulting in still greater excess demand. The price would explode upward until it reached a price so high that even levered investors were not willing to buy, and this high price would be the new equilibrium.

Similarly, a slight negative demand shock when the market reaches its tipping point (for example from curve D'' inward to D') will result in excess supply but a higher equilibrium price, with no mechanism to pull the price back up to equilibrium. Supply exceeds demand, resulting in price declines, leading to even greater excess supply. A slight increase in demand therefore results in the price exploding to a very high level; a slight decrease in demand results in the price crashing to the point where fully funded investors can absorb all the assets.

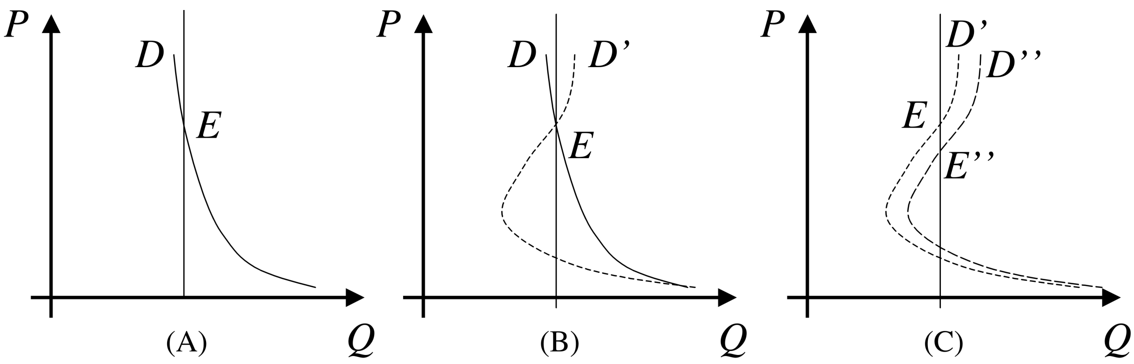

Figure 1: Supply and Demand with Levered Buyers

Appendix A explains how this process would unfold in a market where, like many financial markets, the price-discovery mechanism is a limit order book. If the demand curve is upward-sloping a negative shock to the price of an asset would result in mass liquidations. This event is usually referred to in the literature and the popular press as "panic" selling, which seems to imply that it is somehow irrational or incomprehensible. It is neither; in this model the mass liquidations are a required response to a decline in asset values.

Equation (12) determines when the system will exhibit explosive behavior. If its derivative is positive, at the previous period's equilibrium demand slopes upward. The system is unstable and a crash or price spike will result. Otherwise it is stable. Let us examine this formally. Differentiating (12), we have:

$$\frac{dm_t}{dp_t} = (1-\mu)N \cdot D'(p_t) + \frac{(1-\lambda)}{\lambda} \frac{p_{t-1} m_{t-1}^{\mathrm{lv}}}{p_t^2} \tag{13}$$

If we know $D'(p)$, the demand response of the fully funded investors to small changes in price, we thus can determine whether the system is stable.

Define $A \equiv (1-\mu)NpD(p)$ to be the total dollar amount fully funded investors in the aggregate wish to hold of the asset. Substituting into equation (13) and rearranging, the condition for stability ($\frac{dm_t}{dp_t} < 0$) then becomes:

$$\frac{NW_{t-1}}{\lambda^2} + (1 - \eta_D)A < p_{t-1} m^{\text{lv}}_{t-1} + A \tag{14}$$

where η_D (a positive number) is the fully funded investors' price elasticity of demand $-pD'(p)/D(p)$. The right hand side of the equation is the market size: the total assets held by levered investors, plus the total assets held by fully funded investors.

The left-hand side of equation (14) is the net worth of levered investors divided by the square of the margin requirement, plus the amount unlevered investors hold of the asset adjusted for their elasticity of demand. This quantity defines the *minimum market size for stability* of the market, which we shall term *MinMaSS*. It is the smallest market size that is consistent with stability; if $\eta_D = 1$ then MinMaSS is just the net worth of levered investors divided by the squared leverage ratio. We can form a ratio of MinMaSS to the actual market size; we call this the *instability ratio*. If the instability ratio is greater than one, the market is unstable; if it is less than one, the market is stable. We can use the instability ratio in a straightforward way to determine how close the market is to an instability point. The closer is the instability ratio to one, the closer is the market to becoming unstable.

Higher margin requirements support stability with a higher share of levered investors, while a relatively small number of levered investors can create instability if the leverage is high. For example, suppose levered investors have a 10% margin requirement ($\lambda = 0.1$) and the elasticity of demand η_D is 1. Then levered investors will only need net worth of 1% of the total demand for the asset to create an unstable situation. This suggests that instability in markets may not be a particularly rare state of affairs. On the other hand, if the capital ratio is moderate, say 1:1, levered investors will need net worth of 25% of the total demand for the asset to create an unstable situation. A lower capital buffer is so dangerous because, as equation (14) shows, MinMaSS goes as the *square* of the margin requirement λ.

Once the instability ratio exceeds one and the market becomes unstable, one of two events occurs. Either there is an immediate crisis where the price crashes and levered investors go bankrupt, or the price explodes up to the maximum levered investors will pay (call this v_π). The speed of this explosion prevents levered investors from fully levering their net worth to buy the assets without driving the price up past v_π. After such an explosion, a small shock to fully funded investors' demand does not change the price at all; levered investors simply adjust their holdings to absorb the shock. However, if a downward shock to fully funded investors'demand is large enough, the levered investors may be forced to fully lever to absorb the shock. At this point, any additional shock will reduce the price of the asset below v_π and will force liquidations. The levered investors will be constrained again and the demand curve below v_π will be given by equation 12. This means we can still use equation 14 to assess the stability of the market. However, it is important to note that the relevant margin requirement λ is the minimum leverage ratio required by financial regulators (or targeted by investors), rather than the actual margin requirement observed in the market.[6]

This section has developed the basic concept of MinMaSS and the instability ratio, showing that there is a minimum market size required for asset markets to be stable. MinMaSS is determined by the characteristics and holdings of levered investors and by the demand curve of fully funded investors. Instability results from insufficient capital: If levered investors grow in the market, their perverse demand curves overwhelm the downward-sloping demand from more prudent investors, eventually causing total demand to become upward-sloping. This makes the typical equilibrium between supply and demand an unstable "knife edge" with no mechanism to force a

[6]This minimum capital ratio may be more difficult to observe than actual leverage. In general, using actual rather than minimum margin requirement would bias the estimate of MinMaSS downward and thus lead to insufficiently conservative policy, because higher minimum capital leads to lower MinMaSS, *ceteris paribus*.

convergence to that equilibrium. The lower capital ratio that is required, the more fragile the market in the sense that it takes a smaller share of levered investors to cause an unstable situation.

4 An Operationalizable Version of the Model

We have so far restricted ourselves to a market with only one asset, no short-selling, and only one class of levered investors, a choice made for expositional clarity. In this section, we expand the MinMaSS framework to incorporate markets where different investors lever to differing degrees, markets where investors sell short and take positions using derivatives, and markets with multiple assets, which will give rise to contagion. The resulting measure accounts for sufficient diversity that it might be used by macroprudential regulators as an early warning sign against financial crises.

Let us suppose that there are many assets, indexed by j, and that each asset has some derivative contracts associated with it, indexed by δ. Levered investors are indexed by i and have net worth NW_i. Each asset has a *collateral value*, the maximum amount that can be borrowed against it, which may vary by investor. If the price of asset j at time t is p_t^j, then its collateral value for investor i is defined to be $(1-\lambda_i^j)p_t^j$. Each derivative contract δ on asset j must be also collateralized. At each time t, investor i allocates a proportion of her net worth $\pi_{it}^{j\delta}$ (which may be a function of the price vector $\mathbf{p_t}$) to collateralize each asset and derivative contract in which she invests. As above, we assume that levered investors are leverage-constrained, meaning they use all their capital: $\sum_{j\delta} \pi_{it}^{j\delta} = 1$.

Each investor's direct demand for the asset is given by the quantity she can buy with the share π_{it}^{j} of her net worth she devotes to that asset:

$$\text{Direct Demand} = \frac{\text{Capital Devoted to Asset}}{\text{Margin Requirement per Unit}} = \frac{\pi_{it}^{j} NW_{it}}{\lambda_i^j p_t^j} \equiv m_{it}^{jd} \tag{15}$$

With regard to derivatives, we consider contracts with single-period margining, meaning that any changes in fair value of the contract are paid or received each trading period. The most common examples of such contracts are exchange-traded futures and options contracts, although most credit default swaps and interest rate swaps have similar features.

The value of each derivative contract δ on asset j is a function of the price of the asset, which we shall denote $f_{j\delta}(p_t^j)$. We treat a short sale as the special derivative contract where $f(p) = -p$. For each investor i, asset j, derivative contract δ, and time t, we shall say that she holds $C_{it}^{j\delta}$ derivative contracts. For each of these contracts, she must post a fixed dollar amount of collateral $\chi_{it}^{j\delta}$ as *initial margin* with the exchange or counterparty.

We assume that levered investors may be on either the long or short side of the derivative contracts, so that C may be positive or negative. Of course, every derivative contract has two sides, so the net supply of derivative contracts must be identically equal to zero. Therefore, we introduce *market makers* who are assumed to absorb any disparity in demand between long and short speculative positions and hedge these positions in the cash market. For each derivative contract, the position $C_{ht}^{j\delta}$ held by market makers is just the inverse of the net position of the levered investors:

$$C_{ht}^{j\delta} = -\sum_i C_{it}^{j\delta} \tag{16}$$

In order to be hedged, a market maker wishes to be indifferent to price changes in the underlying asset. She therefore demands assets m_t^h according to the condition:

$$\frac{d}{dp_t}[C_{ht}^{\delta} f(p_t) + m_t^h p_t] = 0 \tag{17}$$

or

$$m_t^h = -C_{ht}^{\delta} f'(p_t) = \sum_i C_{it}^{\delta} f'(p_t) \tag{18}$$

The market maker is thus "delta hedging" with $\Delta \equiv C_{ht}^{\delta} f'(p_t)$. Each investor i makes a contribution to the market maker's delta hedging activities for derivative contract δ in proportion to the investor's holdings. It therefore makes sense to refer to this contribution as the investor's indirect demand for the asset:

$$\text{Indirect Demand} = C_{it}^{j\delta} f'_{j\delta}(p_t^j) = \sum_{\delta} \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta}} \cdot f'_{j\delta}(p_t^j) \tag{19}$$

Investor i's total effective demand for asset j is the sum of direct and indirect demand:

$$m_{it}^j = \frac{\pi_{it}^j NW_{it}}{\lambda_i^j p_t^j} + \sum_{\delta} \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta}} \cdot f'_{j\delta}(p_t^j) \tag{20}$$

At this point it is helpful to bring in two concepts from the options pricing literature and practice, delta (Δ) and gamma (Γ). Delta and gamma will be the building blocks of our stability analysis. For each investor i and asset j, her Δ_{it}^j is the change in her net worth for every dollar increase in the price of asset j. That is:

$$\Delta_{it}^j \equiv \frac{\partial NW_{it}}{\partial p_t^j} \tag{21}$$

To get an explicit expression for Δ, we differentiate each investor's net worth this period with respect to the price of a specific asset j' to obtain $m_{i,t-1}^{j'}$. The investor's Δ is thus her total net effective demand for the asset.

We will also be interested in gamma (Γ). For each investor i and asset j, Γ_{it}^j measures the change in the investor's exposure to the asset as its price changes, assuming she does not actively adjust her positioning. It represents the convexity of her net worth relative to the price of asset j. By definition gamma is the price derivative of delta:

$$\Gamma_{it}^{j} \equiv \frac{\partial \Delta_{it}^{j}}{\partial p_t^j} \tag{22}$$

$$= \sum_{\delta} \frac{\pi_{i,t-1}^{j\delta} NW_{i,t-1}}{\chi_{i,t-1}^{j\delta}} \cdot f_{j\delta}''(p_t^j) \tag{23}$$

These definitions of delta and gamma are analogous to those in the options literature (e.g., Hull, 2006).

4.1 Stability Analysis

As with our previous analyses, the market for each asset j will be stable if demand is downward-sloping. As before, we add fully funded investors with a downward-sloping demand curve:

$$m_t^{j,\text{ff}} = (1-\mu) N D_j(p_t^j) \tag{24}$$

The total demand curve is just the sum of the demand of all the investors:

$$m_t^{j,TOT} = \sum_i m_{it}^j + m_t^{j,\text{ff}} \tag{25}$$

The slope of the demand curve is:

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \frac{dm_{it}^j}{dp_t^j} + (1-\mu) N D_j'(p_t^j) \tag{26}$$

$$= \sum_i \frac{dm_{it}^j}{dp_t^j} - \frac{\eta_j A}{p_t^{j2}} \tag{27}$$

where η_j (a positive number) is the elasticity of demand of fully funded investors and A is the total value of the assets they hold, as before.

Expanding the total derivative in terms of partial derivatives, and conducting a number of transformations (which are described in detail in Appendix B), gives the slope of the demand curve in terms of Δ and Γ as:

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[-\frac{m_{it}^j}{p_t^j} + \Gamma_{i,t+1}^j \cdot \frac{f''(p_t^j)}{f''(p_{t+1}^j)} + \frac{\Delta_{i,t+1}^j \Delta_{it}^j}{NW_{it}} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}} \frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j} \right] - \frac{\eta_j A}{p_t^{j2}} \tag{28}$$

Note that while some of the subscripts in this equation have the value $t+1$, these values are nonetheless all known at time t.[7]

To find MinMaSS and evaluate the stability of an equilibrium, we will be interested in the sign of this derivative in steady state, *i.e.* when $p_{t+1}^j = p_t^j$. In other words, if p_t^j is an equilibrium, is it stable?

The condition for stability in asset market j is that $dm^{TOT}/dp < 0$. Imposing this condition, dropping the now superfluous subscripts j and t, and rearranging terms gives another form of the stability condition:

$$\sum_i NW_i \cdot (p\Delta_i/NW_i)^2 + p^2 \left\{ \sum_i \Gamma_i + \sum_{\delta i} \frac{\partial m_{it}^j}{\partial \pi_i^\delta} \frac{\partial \pi_i^\delta}{\partial p} \right\} + (1-\eta)A < \sum_i pm_i + A \tag{29}$$

We define the left-hand side of equation (29) as MinMaSS; the right side is the actual market size, given by the total assets of the levered investors, plus total assets of the fully funded investors.[8] The four terms on the left side of equation (29) determine

[7] Mathematically, this can be seen in Appendix B in the transformation from equation (34) to (35).

[8] Equation (29) gives the stability condition where the independent variable is the price of an asset. However, many fundamentals-based investors consider relative value in their asset allocation decision, so that their demand for an asset depends not only on the price p^j of asset j but also on the price level of assets generally. It is easy to incorporate this into equation 29 via a change of variable. For example, we might let P be the general level of asset prices and $q^j = p^j - P$ be the idiosyncratic component of the price of asset j. Then $\partial/\partial q^j = \partial/\partial p^j$, so the equation does not change, but on the right side the elasticity η has a slight definitional change, and becomes:

$$\eta = \frac{(q^j + P) \cdot [\partial D^j(P, q^j)/\partial q^j]}{D^j(P, q^j)}$$

the minimum market size for stability.

In the first term, each investor makes a contribution to MinMaSS that is proportional to the square of her Δ relative to her net worth. Recall that Δ tells us the dollar amount that an investor's net worth changes for each dollar change in the price of an asset. The expression $p\Delta/NW$ is therefore a measure of leverage. Indeed, in the simple case where there are no derivative contracts and an investor is invested in only one asset, $p\Delta/NW$ is precisely equal to the conventionally defined leverage ratio.

Equation 29 therefore shows that each levered investor makes a contribution to MinMaSS in proportion to her net worth and the *square* of her leverage. This non-linearity means that a single investor can have a large impact on the market. The fact that contribution to MinMaSS is proportional to delta squared also means that a levered investor always makes a destabilizing contribution, whether she is long or short. If two levered investors enter into a futures contract, taking offsetting positions, both investors increase their squared delta and thus both contribute positively to MinMaSS. MinMaSS and instability increase with the total absolute value of levered investors' positions, not their aggregated net position, meaning that derivatives used for speculative purposes increase instability in proportion to the *square* of the net open interest in the contract.

This term also contains a contagion effect. If the price of another asset falls, so that the net worth of an investor decreases while Δ^j stays constant, the first term in equation (29) will increase, MinMaSS will rise, and the market will move closer to instability.

The second term of equation (29) is more subtle, but it will be intuitive to derivatives market participants as representing purchases or sales by market markers as a part of their delta-hedging activities. This term arises because the value of a derivative in general will not be linear in the price of the underlying asset (long options positions have positive convexity, for example). As a result, if the price of the underlying asset changes, $f'(p)$ will change, and the market maker will no longer have a neutral stance with respect to the price of the asset (that is, equation (18) will no longer be satisfied). The market maker will therefore have to adjust her position in the underlying asset to remain hedged. This change in hedging demand in response to price is what is described by the second term, and it shows that levered investors contribute to instability and MinMaSS in proportion to their net gamma position.

Because gamma only arises in relation to open derivative contracts, the total gamma in the market is zero.[9] However, we need to exclude from our calculation the gamma position of market makers, so this term will not in general be zero, although it is likely to be quite small compared to the first term. This leads to the interesting prediction that when levered investors sell volatility, stability in the underlying market is enhanced.[10]

The third term in equation (29) captures the contribution to MinMaSS from levered investors rebalancing their portfolios in response to price movements. In practice, this term is likely to be negative, though this is not certain; sufficiently inelastic substitution away from appreciating assets can cause levered investors actually to increase the proportion of net worth held in an asset as its price rises.

The fourth term in equation (29) is the elasticity-adjusted amount unlevered investors hold of the asset. This term is negative for $\eta > 1$. In this case, the greater share of assets held by fully funded investors, the lower is MinMaSS and the more stable is the market, all else equal.

[9] Recall that if the value of an investors' portfolio is a function $v(p)$ of the underlying price of the asset, then $\Gamma = v''(p)$. If the investor holds only outright long or short positions and has not entered into derivatives contracts, then the value of her portfolio is simply proportional to the price p, and $\Gamma = v''(p) = 0$.

[10] This phenomenon was confirmed in a conversation with a market participant who managed a large portion of the derivatives portfolio: he told us that when a large investor writes options, market makers delta-hedging their positions are forced to buy when the market falls and sell when it rises, reducing volatility.

In this section, we have seen how to generalize the model to accommodate a wide variety of investors with different strategies and risk parameters. Each levered investor contributes to instability in proportion to the square of her leverage (Δ) and in proportion to her net volatility position (Γ). In theory, all the information necessary to evaluate the stability condition defined by equation (29) could be collected by a systemic regulator and straightforwardly aggregated to evaluate the stability of the market. While such an undertaking may sound daunting, in fact the information is simply a standard set of summary statistics of the portfolios of investors and is already compiled daily (or even more frequently) by all sophisticated investors in their risk reports. In the next section we show how the model can be applied to the 1998 collapse of the hedge fund Long-Term Capital Management.

5 The Collapse of Long-Term Capital Management

We now apply the model to the 1998 collapse of hedge fund Long-Term Capital Management and show that it risked destabilising those markets where it was both highly levered and relatively large.

LTCM was a large relative value hedge fund that was in business from 1994 until 1999.[11] The hedge fund employed a strategy of *relative value arbitrage*, in which it bought some assets it considered to be relatively cheap while selling short other, very similar assets it considered to be relatively expensive.[12] Relative value hedge funds

[11] The historical facts in this narrative are taken largely from Lowenstein (2000) and MacKenzie (2003), as well as conversations with former LTCM principals who wished to remain anonymous.

[12] An example of a typical trade would be for LTCM to buy a 29 1/2-year Treasury bond and sell short a 30-year Treasury bond. LTCM made money because the 30-year bond was more liquid, so it traded with a slightly lower yield (i.e., higher price). After six months, the Treasury would issue a new 30-year bond, and the 30-year bond LTCM had sold short would become a 29 1/2-year bond while the 29 1/2-year bond it owned would become a 29-year bond. Because the 29 1/2-year bond has similar liquidity characteristics to the 29-year bond, the yields would converge, and LTCM could liquidate the trade at a profit. More examples of LTCM's trades can be found in Perrold

are typically highly levered institutions, and LTCM was perhaps the archetype of such a fund. LTCM was also very large: at its peak in April 1998 it had $4.87 billion in capital, $125 billion in assets, and another $115-125 billion of net notional value in off-balance sheet derivatives.[13]

For the first several years of its existence, Long-Term's results were spectacular, and the fund grew as it succeeded. However, as it grew it attracted imitators both in the hedge fund community and among the trading desks of the Wall Street banks, leading returns and opportunities to dwindle (see Table 1). LTCM's troubles began in late spring of 1998 and continued into the summer 1998, especially when Salomon Brothers, whose trading behavior was close to that of LTCM, began to close down its arbitrage desk, both to reduce risk and in response to poor results. In

Date	Beginning Assets under Management (Net Capital)	Annualized Return	End of Period Leverage (Excluding Derivatives)
3/94-2/95	$1.1 billion	25%	16.7
3/95-2/96	$1.8	50%	27.9
3/96-2/97	$4.1	34%	27.9
3/97-2/98	$5.8	11.5%	26.8
3/98-7/98	$4.7	-35%	31.0

Table 1: LTCM Performance and Leverage Ratio, Excluding Derivatives.
Source: Perrold (1999)

August and September, LTCM began to lose money in a dramatic fashion. However, LTCM's principals found themselves unable to liquidate to reduce risk at anything close to what they viewed as a reasonable price. Other market participants moved to liquidate ahead of LTCM, pushing prices against it and causing even deeper distress.[14]

(1999).

13 Other sources generally report a figure of $1.25 trillion in off-balance sheet derivatives. However, this figure fails to take into account that many of these positions were not just negatively correlated but in fact perfectly offsetting, that is, long and short exactly the same instrument. This is because LTCM, like other market participants, typically engaged in an offsetting trade when it wanted to close out a swap contract, rather than ending or selling the original contract. The so-called "replacement value" of these swaps was $80-90 billion (Interview with LTCM Principal, 2010), and equity derivatives accounted for an additional $35 billion (Dunbar, 2000).

14 Partner Eric Rosenfeld compared LTCM to a large ship in a small harbor in a storm—it was too large to maneuver, and all the other boats were just trying to get out of its way (Rosenfeld,

By the end of September, LTCM had barely been able to reduce its risk at all and its capital had been severely depleted. The Federal Reserve, cognizant that a default could result in a sudden liquidation of a portfolio that included $125 billion in assets and $1.25 trillion gross notional value of derivatives, and that this could destabilize markets, stepped in to orchestrate a bailout by LTCM's counterparties.[15]

We will never know for certain whether fears of a severe financial disruption would have been realized had LTCM been allowed to fail. We can, however, examine how the MinMaSS framework could have been used to assess whether financial markets would have suffered from an episode of instability had LTCM been forced to liquidate. We shall find that the instability might have occurred in at least a few of the markets in which Long-Term played.

5.1 Stability of LTCM's Markets

We now examine LTCM's impact on the stability of the global equity markets, global equity volatility markets, US bank funding markets and US Treasury markets in the late summer of 1998.[16] The bank funding markets and the equity markets are two of the most economically significant and transparent markets in which LTCM operated, and accounted for a significant portion of the fund's risk. Furthermore, the bank funding market is of particular systemic importance since a dysfunctional bank funding market may cause contagious bank failures.[17]

2009).

[15]As then-Chairman Alan Greenspan put it, "our sense was that the consequences of a fire sale...should LTCM fail on some of its obligations, risked a severe drying up of market liquidity." (Greenspan, 1998) New York Fed President McDonough said, "there was a likelihood that a number of credit and interest rate markets would experience extreme price moves and possibly cease to function for a period of one or more days and maybe longer." (McDonough, 1998)

[16]While other funds and banks had made similar trades, these funds are considered here as behaving like fully funded investors since they were generally not facing distress and forced liquidations.

[17]The choice of these markets has been informed by the positions LTCM held, and in some sense seems obvious. We can use the MinMaSS framework to examine the stability of any market segment we can define. However, the narrower the definition of the market, the more elastic is the demand of unconstrained investors, and thus the smaller is the minimum market size for stability. When

Hard portfolio data on LTCM and its competitors are very difficult to come by because there were no public reporting requirements and the funds were very secretive while they were trading. Because of the media scrutiny to which LTCM was subject after the crash, some of the partners in the fund were more forthcoming than they had been previously, and some information is available on its portfolio. This information has been compiled from a number of media and academic sources, as well as a discussion with former LTCM principals. Information on the size of markets has been compiled from public sources such as the flow of funds accounts from the Federal Reserve.[18]

LTCM's largest equity trades were sales of volatility on broad stock indexes in the US and Europe.[19] According to Dunbar (2000), by January 1998 LTCM's 5-year equity option position was about \$100 million per percentage point of volatility. Using the Black-Scholes option pricing formula, this implies that LTCM had written options with a notional value of about \$11.5 billion (see Appendix C.1 for the calculation). The consequences of this position for stability in the equity market can be examined utilizing equation (29). For stability of the equity market as a whole, the consequences of LTCM's distress were small. LTCM's hedges meant that it had no exposure to outright stock price movements—it had a Δ of zero. Its Γ, however, (or $p^2\Gamma$) was about \$10 billion (see Appendix C.2 for the calculation).

Plugging these figures into equation (29), assuming no changes in LTCM's portfolio

market size is larger, the levered investor such as LTCM must control a larger overall proportion of the market to destabilize it.

[18]Data for LTCM's competitors and copycats, in particular large investment banks and hedge funds, is even more scarce. The investment banks that were competing with LTCM were required to publicly report losses at the time of the collapse of LTCM, but these were generally only on an aggregate basis. Their arbitrage portfolios were only a part of the proprietary trading businesses. Similarly, LTCM's hedge fund competitors were under no obligation to disclose their holdings and took care not to do so. However, these funds tended to be both significantly smaller and less levered than LTCM (Anonymous, 1998), meaning that for our theory they contribute far less to MinMaSS. Note that contributions to MinMaSS are proportional to fund capital and to the square of leverage. If our calculation was extended by data for firms that had similar trades to LTCM, the estimated MinMaSS would have been higher and the market closer to instability. Set against this, however, is that the other firms were not in financial distress and hence forced to liquidate.

[19]More background on these trades, and the reasons behind them, are described in Dunbar (2000); Lowenstein (2000); Perrold (1999).

distribution and assuming the elasticity of fully funded demand to be equal to one, gives a MinMaSS in the equity markets of \$10 billion. The US and European equity markets at the time were larger than this by a factor of over a thousand, implying a negligible instability ratio. Clearly, a forced liquidation of LTCM was nowhere near enough to destabilize the equity markets. Indeed, while equity markets declined along with most risk assets during the summer of 1998, they never ceased to function in an orderly manner.

The market for equity volatility was affected, however. LTCM's sales of volatility were one of the trades that also hurt it the most, with losses of \$1.3 billion. We can use again equation (29) to determine the stability of the market for volatility by calculating first MinMasSS. We assume no changes in LTCM's asset allocation shares during a forced liquidation, volatility priced around 20% and \$100 million exposure per point of volatility.[20] This translates into LTCM having sold short \$2 billion of volatility.[21]

We now calculate the total market size. According to Dunbar (2000) and Lowenstein (2000), LTCM was responsible for about a quarter of the long-term volatility sales, while the investment banks were responsible for the rest, meaning that the demand for volatility on the part of unlevered investors, A, was about \$8 billion. This volatility was mainly sold to pension funds and unit trusts that had promised their owners a minimum rate of return.

Since pm is LTCM's position (negative \$2 billion), the actual market size is therefore \$6 billion, and the instability ratio, given by the ratio of MinMaSS and actual market size, simplifies to:

$$\text{Instability ratio} = \frac{1}{3}(5 - 4\eta) \tag{30}$$

Assuming the elasticity of unconstrained demand to be equal to one (i.e., $\eta = 1$),

[20] Lowenstein (2000) states that volatility was priced at 19%, while Perrold (1999) cites a figure of 20%.

[21] See to Appendix C.3 for detailed calculations.

the instability ratio is 0.33 (Table 2). If the elasticity of demand of unlevered investors was less than 0.5, then MinMaSS would have been higher than $2 billion and the market for volatility would have been unstable. This is not entirely implausible, because the volatility was sold to insurance companies and pension funds that were using it to hedge guaranteed returns on their policies and likely would not have been inclined to sell their options to take advantage of short-term price movements. This shows that LTCM was of the right order of magnitude to destabilize this market.

	Equity Volatility	Bank Funding	US Treasury
LTCM Net Notional Exposure ($p\Delta$)	-$2 billion	$20 billion	$20 billion
LTCM Net Worth	$2.1 billion	$2.1 billion	$2.1 billion
Notional Position of Unconstrained Investors	$8 billion	$618 billion	$5.5 trillion
MinMaSS (Assumes $\eta = 1$)	$2 billion	$200 billion	$200 billion
Actual Market Size	$6 billion	$638 billion	$5.5 trillion
Instability Ratio	0.33	0.31	0.04

Table 2: Stability Analysis for Selected LTCM Markets

A similar analysis is possible for another of LTCM's trades, a bet on swap spreads, a measure of bank funding costs. Comparison across sources suggests that LTCM's exposure to US swap spreads in the late summer of 1998 was about $16 million per basis point of swap spread.[22] This corresponds to an exposure of about $200 million per point of the price of a 10-year bond (MacKenzie, 2003), and therefore a notional exposure of $20 billion both to Treasury bonds and to bank credit.

For the bank funding market we have LTCM's position ($pm =$ $20 billion) and assets held by other investors ($A =$ $618 billion).[23] Equation (29) implies a MinMaSS

[22] Lowenstein (2000, p. 187) implies that LTCM's exposure to a 15 basis point adverse move in swap spreads was $240 million, which implies an exposure of $16 million per basis point. This would imply that the trade lost $160 million for LTCM on August 21, 1998, a day when swap spreads moved 10 basis points (although, as many authors note, they moved up to 20 basis points intraday). Overall, LTCM lost $550 million on August 21, of which $160 million was due to a merger arbitrage trade gone wrong, and perhaps a bit more was due to increases in equity volatility. This would mean that losses on US 10-year swap spreads were a substantial portion of the remaining losses, which is likely given that this was by all accounts one of LTCM's core trades (see, for example Perrold, 1999). Additionally, a former LTCM principal told us that $10 million per basis point was a *plausible* estimate, which we take to mean that it is within a factor of two.

[23] Assets held by other investors is the sum of $188.6 billion owed by commercial banks, $193.5

of \$200 billion, if the elasticity of unconstrained demand was equal to one. The associated instability ratio is about 0.3. However, if the demand for bank credit was inelastic ($\eta <$ about 1/3), then MinMaSS would have moved up towards the market size and the instability ratio would have approached one. In this case, LTCM alone could have been enough to destabilize the market for bank credit.

By contrast, there were around \$5.5 trillion of Treasury securities outstanding, which is around nine times the size of the bank funding markets. This leads to an instability ratio of less than 0.05, so LTCM likely was nowhere near big enough to destabilize the Treasury markets.

Real-world behavior is always more complex than economic models, but the narrative of the rise and fall of LTCM generally corroborates the key behavioral assumptions and predictions of the model. The next section provides an extended discussion of these real-world factors in light of the insights stemming from our model.

5.2 Corroborating Evidence

A more qualitative investigation shows that LTCM's behavior and financial market dynamics more closely match what is proposed in this paper or earlier models of capital-constrained arbitrage and investing behavior such as Grossman and Vila (1992), Shleifer and Vishny (1997), or Liu and Longstaff (2004), than an unconstrained neoclassical agent. This is true in at least three important respects. First, the size of LTCM's positions during most of its existence was determined much more by its capital than by its assessment of available opportunities. Second, when the market tottered in the summer of 1998, the savviest investors were certain that prices were divorced from fundamental value, a view that was later proved correct. Third, prices in the destabilized markets became ill-defined as liquidity dried up.

billion owed by bank holding companies, \$212.4 billion owed by savings institutions, \$1.1 billion owed by credit unions, and \$42.5 billion owed by broker-dealers (Federal Reserve Board, 2013), minus the \$20 billion held by LTCM.

It is clear from even a casual glance at LTCM's assets and equity (which Perrold (1999) has obtained directly from LTCM, Figure 2) that the fund's equity is an important determinant of its assets. As LTCM ramped up its operations and began both to raise and earn capital, it was able to increase its assets as well, although, as Perrold points out, the growth in assets outpaced the growth in capital for a time as LTCM built its operations. In fact, LTCM's balance sheet size was, in the early stages of its existence, more governed by right-sizing its assets to meet its capital base than by its assessments of the available arbitrage opportunities in the marketplace.

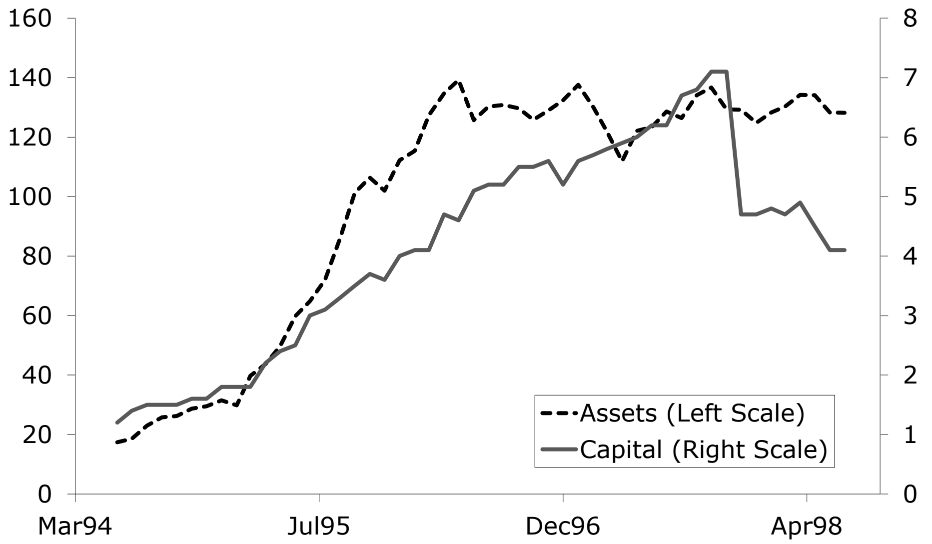

Figure 2: LTCM Assets and Equity Capital
Source: Perrold (1999)

LTCM was not completely price-insensitive in its buying, contrary to what the model assumes about levered investors. Instead, as it grew and was imitated, LTCM noticed that favorable opportunities were "drying up big," as Eric Rosenfeld put it (quoted in MacKenzie, 2003). LTCM reacted by returning capital to investors, putting a ceiling on its willingness to continue its strategy at less and less attractive pricing. This decision to return capital to investors kept LTCM's capital at a level where margin constraints had to be considered. Instability was thus still very much a possibility because, as all the narrative histories of LTCM that we are aware of point out, capital cannot be easily raised by a fund once it begins to lose money and approaches its credit or net worth constraint.

This instability was realized in the summer of 1998 when prices began to move against LTCM. Even at the time, sophisticated market participants understood and commented that asset prices were moving away from fundamental value. As William Winters, head of J.P. Morgan's European Fixed Income business, put it at the height of the crisis, "any concept of long-term or fundamental value disappeared" (Coy and Woolley, 1998).

Other bankers agreed that prices were not being driven by fundamental value but by tactical considerations. As one Goldman Sachs trader put it, "If you think a gorilla has to sell, then you sure want to sell first." Goldman CEO Jon Corzine did not deny that the firm "did things in markets that might have ended up hurting LTCM. We had to protect our positions. That part I'm not apologetic about." (Lowenstein, 2000, p. 175) Lowenstein cites similar sentiments from executives at other banks, in particular Salomon Brothers, which had a portfolio of a similar size to LTCM (Dunbar, 2000).

In this situation, a value-oriented, unconstrained investor would be betting on prices to converge. So, too, would a levered investor who saw prices moving against her long-term view, and indeed, this is precisely what LTCM *wanted* to do. LTCM principals continued to have confidence in their trades even as the market moved further and further against them. As Meriwether put it in his August 1998 letter to investors (reproduced in Perrold, 1999):

> With the large and rapid fall in our capital, steps have been taken to reduce risks now ... On the other hand, we see great opportunities ... The opportunity set in these trades at this time is believed to be among the best that LTCM has seen ... LTCM thus believes it is prudent

> and opportunistic to increase the level of the Fund's capital to take full advantage of this unusually attractive environment.

Rosenfeld put it more directly: "We dreamed of the day when we'd have opportunities like this" (Lowenstein, 2000, p. 166).

This was not just talk. No source disputes that Meriwether was actively trying to raise additional capital. As Lowenstein (2000), Dunbar (2000), and others note, the partners' faith in their trades was ultimately proven correct. The consortium of investment banks that took over the fund was left with double-digit returns one year later. Yet as markets were moving against it, creating more attractive opportunities, LTCM was liquidating some trades, adding to the price pressure. According to Dunbar (2000, p. 194), LTCM decided at the end of June to reduce its daily value-at-risk (VAR) from $45 million to $35 million.[24]

In the markets that LTCM destabilized, prices were not only divorced from fundamental value but in some cases were not even well-defined as liquidity evaporated. The instability ratios presented in Table 2 indicate that LTCM was of the right order of magnitude to destabilize the equity volatility market, but not the equity market. Liquidity conditions during the late summer of 1998 corroborate this finding. Despite significant declines, cash equity markets continued to function normally with reasonable liquidity. The market for long-dated volatility in equities, however, was thrown into disarray and became almost completely illiquid. Trading became very sparse and price quotes spiked and became divorced from fundamentals, according to market participants quoted in Dunbar (2000) and MacKenzie (2003).

As one banker said:

> When it became apparent that they [LTCM] were having difficulties, we thought that if they are going to default, we're going to be short

[24] VAR is a measure of risk, typically quoted as the 95% confidence interval of daily profit and loss.

> a hell of a lot of volatility. So we'd rather be short at 40 than 30, right? So it was clearly in our interest to mark at as high a volatility as possible. That's why everybody pushed the volatility against them, which contributed to their demise in the end. (MacKenzie, 2003)

This quotation demonstrates that in the long-dated volatility markets, prices were not clearly defined. LTCM's counterparties had considerable discretion in what prices to place on the options that LTCM was short. This is only possible in a market that is not liquid. If the market had been active with many participants ready to buy and sell at their estimate of fundamental value, such discretion would not be possible because prices would be determined by the intersection of supply and demand. LTCM was such a big player that the possibility of it being forced to liquidate was enough to prevent prices from being well-defined. This is a hallmark of instability.

Given that the market moves were driven by fear of instability, it is no surprise that LTCM's losses were concentrated in the areas where it had most destabilized markets. Lowenstein (2000, p. 234) provides a breakdown, showing that of the $4 billion lost by Long-Term in 1998, $1.6 billion was in swaps and another $1.3 billion in equity volatility. No other category of losses even tops $500 million.

6 Conclusion

This paper has drawn a quantitative link between leverage, market size, and financial instability. Specifically, markets tend to become unstable when levered investors accumulate too large a share of the assets. The total net worth and the distribution of net worth held by levered investors together determine a minimum market size for the market to be stable. The ratio of this minimum market size to actual market size defines an instability ratio which determines how close the market is to an instability-induced crisis.

We applied the model to study the collapse of Long-Term Capital Management in 1998 and how it affected the markets. Most accounts of the demise of LTCM argue that the fund's fundamental failure was that it was too highly levered and took too much risk. Lowenstein, Dunbar, the President's Working Group on Financial Markets, and the Basel Committee all accept this hypothesis. Our analysis makes clear that LTCM's leverage was, however, only part of the story. The true problem was that it was *both* highly levered *and* large relative to the markets in which it invested. Had it been smaller it might have survived. LTCM's very existence destabilized the markets, creating the potential for much larger price moves than would have been possible in the absence of the fund's existence. Instead of the risk being mitigated by LTCM's hedges, as it would have been had the fund been smaller, once the crisis hit risk became governed by the theoretical notional exposure. All of a sudden, the correlations between the assets LTCM was betting on changed, precisely because LTCM was betting on them.

Our model has a number of advantages relative to other models of financial crises. The simpler framework of our model means that it can accommodate a richness of financial instruments and heterogeneous beliefs, making it easier to apply operationally. The chosen methodological design implies that the results are not sensitive to small changes in model specification. Finally, the key variables in the model are *observable* and *measurable*: leverage, margin requirements, the interest rate, and the net worth of levered investors versus unlevered investors. There is a minimum of reliance on unobservable, psychological variables such as utility, expectations, and subjective probability distributions. The measure we have presented is sufficiently general and simple that it could be calculated and applied by macroprudential regulators to provide advance warning of a crisis, warnings that might prevent or mitigate future crises.

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7 Appendix

A Limit Order Book Price-discovery Mechanism

In this part we explain how the price-discovery mechanism works in a typical financial market affected by a shock to the asset price when the demand curve is upward-sloping. For illustrative purposes, consider the closest situation to the Walrasian case: a market where (unrealistically) the full retail demand curve is expressed in the form of open buy and sell orders in the limit order book.[25] The levered investors' demand curve below the market price is expressed in terms of *stop-loss* orders: If the market price falls, these stops convert automatically into sell-at-market orders that are matched against the highest bid. Their demand curve above the market price is expressed as a series of stop-buy orders, which convert into buy orders as the price rises.[26]

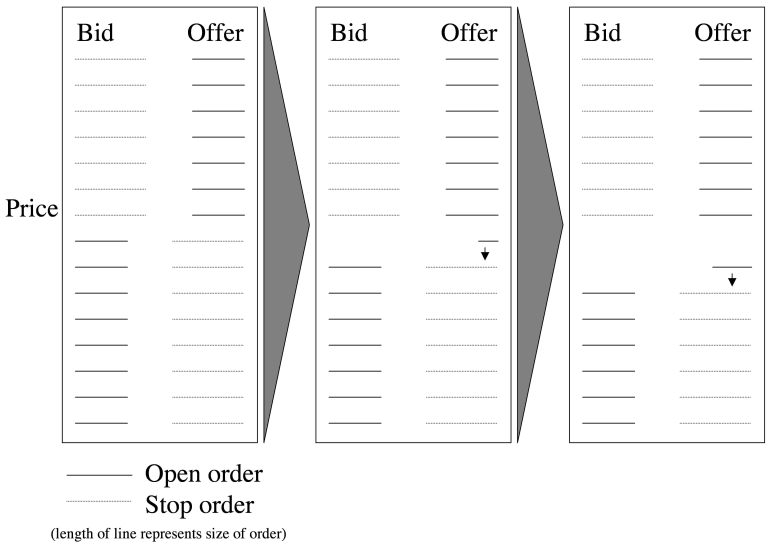

Figure 3: The limit order book and a negative price shock

Figure 3 visualizes how the limit-order book will change after a negative price shock and upward-sloping demand. The left panel shows the initial limit-order book, where the first column lists the bids, the second the offers, the solid line indicates open orders, the dotted line marks stop orders and the size of each line represents the size of order. Once the tape prints a downtick, the first stop-loss order is activated. It is matched against the highest bid, but because the demand curve is upward-sloping

[25] To be clear, these orders are tied to the derivative of the demand curve; they are the *changes* in quantity that would be required for each change in price level.

[26] We assume the levered investors already have finance lined up to purchase additional units when their equity increases.

the highest bid is not for a large enough size to fully fill the sell order (middle panel). The part of the sell order that could not be filled jumps to the next lowest price and executes against the next highest bid (right panel). When these orders are matched, the tape prints another downtick, the next stop-loss order is activated, and the process cascades as all the stops are ultimately activated and quickly execute against progressively lower bids as the market crashes to a level where retail investors are able to fully absorb the liquidations by the levered speculators.

Once the liquidations begin, rational non-levered investors, if they understand the process underway, will respond by withdrawing their bids and waiting for the dust to settle at the new equilibrium, rather than supporting the price all the way down. Some of the more sophisticated of these investors will even engage in predatory selling, temporarily worsening the collapse, along the lines of what is modeled by Brunnermeier and Pedersen (2005). The price will gap downwards rather than fall smoothly. After the fall, the model produces a condition of depressed asset prices. Because the levered investors lose their entire net worth, the stock of assets must be held by a smaller group of unlevered investors. These investors have downward-sloping demand, so the asset price must be less than it was before demand became concentrated in a small group of levered investors, as it might in a bubble.

If the initial shock is upward, the result is similar. Theoretically, the upward cascade continues to infinity; in practice it continues until the levered investors have fully exhausted their credit lines and must raise more finance in order to continue to buy assets.

These adjustments are not simply an ordinary process of market adjustment to a new, efficient equilibrium. Rather, as in much of the literature on currency crises (for example, Krugman, 1999), multiple equilibria are possible and prices can move discontinuously in response to small shocks.

B Calculating the Stability Condition

Expanding the total derivative from equation (27) in terms of partial derivatives, we obtain:

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[\frac{\partial m_{it}^j}{\partial p_t^j} + \frac{\partial m_{it}^j}{\partial NW_{it}} \frac{\partial NW_{it}}{\partial p_t^j} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}} \frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j} \right] - \frac{\eta_j A}{p_t^{j2}} \tag{31}$$

We will aim to rewrite this equation in terms of Δ and Γ. Working just with the first term, we have:

$$\frac{\partial m_{it}^j}{\partial p_t^j} = \frac{\partial}{\partial p_t^j} \left[\frac{\pi_{it}^j NW_{it}}{(1-\lambda_i^j) p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta}} \cdot f'_{j\delta}(p_t^j) \right] \tag{32}$$

$$= -\frac{\pi_{it}^j NW_{it}}{(1-\lambda_i^j) p_t^{j2}} - \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta 2}} \cdot f'_{j\delta}(p_t^j) \frac{\partial \chi_{it}^{j\delta}}{\partial p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta}} \cdot f''_{j\delta}(p_t^j) \tag{33}$$

If the initial margin on derivative contracts χ is proportional to the price, as is usual, then we can simplify further:

$$\frac{\partial m_{it}^j}{\partial p_t^j} = -\frac{1}{p_t^j}\left[\frac{\pi_{it}^j NW_{it}}{(1-\lambda_i^j)p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta}} \cdot f'_{j\delta}(p_t^j)\right] + \sum_\delta \frac{\pi_{it}^{j\delta} NW_{it}}{\chi_{it}^{j\delta}} \cdot f''_{j\delta}(p_t^j) \tag{34}$$

Substituting equation (20) in the first term and equation (23) in the second term gives:

$$\frac{\partial m_{it}^j}{\partial p_t^j} = -\frac{1}{p_t^j}\left[m_{it}^j\right] + \Gamma_{i,t+1}^j \cdot \frac{f''(p_t^j)}{f''(p_{t+1}^j)} \tag{35}$$

We can now substitute equation (35) into the first term in the demand curve (31):

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[-\frac{m_{it}^j}{p_t^j} + \Gamma_{i,t+1}^j \cdot \frac{f''(p_t^j)}{f''(p_{t+1}^j)} + \frac{\partial m_{it}^j}{\partial NW_{it}}\frac{\partial NW_{it}}{\partial p_t^j} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}}\frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j}\right] - \frac{\eta_j A}{p_t^{j2}} \tag{36}$$

Working now with the third term, we can substitute equation (21) to give:

$$\frac{\partial m_{it}^j}{\partial NW_{it}}\frac{\partial NW_{it}}{\partial p_t^j} = \left[\frac{\pi_{it}^j}{(1-\lambda_i^j)p_t^j} + \sum_\delta \frac{\pi_{it}^{j\delta}}{\chi_i^{j\delta}} \cdot f'_{j\delta}(p_t^j)\right] \cdot \Delta_{it}^j \tag{37}$$

$$= \left[\frac{m_{it}^j}{NW_{it}}\right] \cdot \Delta_{it}^j \qquad \text{by (20)} \tag{38}$$

$$= \left[\frac{\Delta_{i,t+1}^j}{NW_{it}}\right] \cdot \Delta_{it}^j \tag{39}$$

Substituting (39) into (36) gives the slope of the demand curve in terms of Δ and Γ, as expressed below, or in equation (28).

$$\frac{dm_t^{j,TOT}}{dp_t^j} = \sum_i \left[-\frac{m_{it}^j}{p_t^j} + \Gamma_{i,t+1}^j \cdot \frac{f''(p_t^j)}{f''(p_{t+1}^j)} + \frac{\Delta_{i,t+1}^j \Delta_{it}^j}{NW_{it}} + \sum_\delta \frac{\partial m_{it}^j}{\partial \pi_{it}^{j\delta}}\frac{\partial \pi_{it}^{j\delta}}{\partial p_t^j}\right] - \frac{\eta_j A}{p_t^{j2}} \tag{40}$$

C Calculations

C.1 LTCM's notional value of the outstanding options

An investor's exposure to changes in volatility is denoted by the Greek letter *vega* (ν). Vega is the derivative of the option price with respect to the implied volatility, or annualized standard deviation of the price of the underlying instrument. LTCM's 5-

year equity option position was about \$100 million per percentage point of volatility (Dunbar, 2000), which implies a vega of \$10 billion. In standard option-pricing notation (see, for example Hull, 2006), the vega of a put or call option is given by:

$$\nu = S_0 \sqrt{T}\, N'(d_1) \tag{41}$$

where

- S_0 is the price (or for a portfolio, total notional value) of the underlying asset
- T is the time to expiry
- $N'(d_1)$ is the probability density function for the standard normal distribution, $N'(d_1) = \frac{1}{2\pi} \exp(-d_1^2/2)$
- $d_1 = \frac{\ln(S_0/K)+(r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$
- K is the strike price of the option
- r is the risk-free rate of interest
- q is the dividend yield on the asset
- σ is the implied volatility, in percentage points per year

LTCM traded at-the-money-forward options, which means that $\ln(S_0/K) = -(r - q)T$, simplifying the expression for d_1. LTCM's vega is thus given by:

$$\nu = S_0 \sqrt{T}\, N'(\sigma\sqrt{T}/2)$$
$$(\$10 \text{ billion}) = S_0 \cdot \sqrt{5} \cdot \frac{1}{2\pi} \exp[-(20\%)^2 \cdot 5/8]$$

Solving for S_0, the notional value of the outstanding options, thus gives $S_0 = \$11.5$ billion.

C.2 LTCM's Γ in the equity markets

Recall that Γ is the derivative of Δ with respect to the price. The Γ of an option is given by Hull (2006) as:

$$\Gamma = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}} \tag{42}$$

where the variables are as defined as in Appendix C.1. We care about the $p^2\Gamma$ of the portfolio. If the portfolio includes options on m shares of the underlying asset, then the total $p^2\Gamma$ in equation (29) is:

$$\begin{aligned}
p^2 m\Gamma &= p^2 m \frac{N'(d_1)}{S_0\sigma\sqrt{T}} \\
&= (p/S_0)\cdot pm \cdot \frac{N'(d_1)}{S_0\sigma\sqrt{T}} \\
&= 1\cdot(\$11.5\text{billion})\cdot \frac{N'(20\%\cdot\sqrt{5}/2)}{20\%\sqrt{5}} \\
&= \$10 \text{ billion}
\end{aligned}$$

C.3 MinMaSS of the market for equity volatility

MinMaSS is calculated using the following parameters:

- $\Delta = \$100$ million per volatility point
- $p = 20$ points
- $NW = \$2.1$ billion
- $\Gamma = 0$

Note that while LTCM's equity in early summer was \$4.5 billion, only \$2.1 billion of this was actually required as margin for trades (MacKenzie, 2003), with the rest as risk capital intended to absorb losses. By early September, LTCM had lost its entire risk capital cushion. The \$2.1 billion is therefore the correct number to consider for the net worth as it was here that liquidations would have been forced. We again assume no changes in LTCM's portfolio balance during a forced liquidation and utilize the left-hand side of equation (29):

$$\text{MinMaSS} = p^2 \cdot \left[NW \cdot \left(\frac{\Delta}{NW}\right)^2 \right] \tag{43}$$

$$= (20)^2 \cdot \left[\$2.1b \cdot \left(\frac{-\$100m}{\$2.1b}\right)^2 \right] \tag{44}$$

$$= \$2 \text{ billion} \tag{45}$$