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by

Alexander Tepper

and

Karol J. Borowiecki

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FURTHER INFORMATION
Department of Business and Economics
Faculty of Business and Social Sciences
University of Southern Denmark
Campusvej 55
DK-5230 Odense M
Denmark

Tel.: +45 6550 3271
Fax: +45 6550 3237
E-mail: lho@sam.sdu.dk
http://www.sdu.dk/ivoe

Accounting for Breakout in Britain: The Industrial Revolution through a Malthusian Lens

Alexander Tepper[1] and Karol Jan Borowiecki*[2]

[1]Federal Reserve Bank of New York and University of Oxford
[2]Department of Business and Economics, University of Southern Denmark

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Abstract: This paper develops a simple dynamic model to examine the breakout from a Malthusian economy to a modern growth regime. It identifies several factors that determine the fastest rate at which the population can grow without engendering declining living standards; this is termed maximum sustainable population growth. We then apply the framework to Britain and find a dramatic increase in sustainable population growth at the time of the Industrial Revolution, well before the beginning of modern levels of income growth. The main contributions to the British breakout were technological improvements and structural change away from agricultural production, while coal, capital, and trade played a minor role.

Keywords: Industrial Revolution, Malthusian Dynamics, Maximum Sustainable Population Growth, Development, Demographics.

JEL Classification Numbers: N13, N33, O1, O41, O52.

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1 Introduction

Since Robert Solow founded modern growth theory 50 years ago, the fashion in economics has been to ascribe growth in living standards to technological progress. While it seems indisputable that technology is an important ingredient in growth, it is not, as most growth models predict, sufficient for growth. Technological progress has been around since the dawn of civilization, but it is only in the last 200 years that we have been fortunate enough to experience a takeoff in living standards. Indeed, the ancient world was full of technological advances that not only improved productivity but, no less than the steam engine or the cotton gin, revolutionized the productive process and opened vast new possibilities to the people of their time. The advent of stone tools, agriculture, the aqueduct, irrigation, the wheel, the bow and arrow, water transport, metal-working, the wheelbarrow, the windmill—none of these resulted in a sustained rise in average living standards. Perhaps a certain level of technology is required for takeoff, but in the modern world, poor countries adopt modern technologies like corrugated steel building materials, cell phones, cars and the use of fossil fuels, yet their economies remain as stagnant as ever. The inspiration for this paper is the natural question these observations invite: why, when both the theory and casual observation suggest that technology is the principal cause of growth, did technology advance for so long without growth, and why do some countries today remain poor despite having access to technology far more advanced than that available during the Industrial Revolution? What caused technology all of a sudden to lead to sustainable per capita income growth?

Economic historians have pointed to a number of factors that potentially explain the first breakout from a Malthusian economy to a modern growth regime that occurred in Britain in the 19th century. Yet, the specific links between these factors and the beginnings of modern growth are often tenuous, and precisely which factors are important is still the subject of considerable debate. This paper develops a simple model that accounts for the existence of both Malthusian and growth economies, and allows us to draw clear links between some of the factors cited by economic historians and the beginning of growth. It further provides a framework that permits a quantitative assessment of the relative contributions of those factors.

In this paper, we will define a modern or growth economy as one where technological progress at society's normal rate implies rising living standards. In a Malthusian economy, technological progress at society's normal rate implies stagnant living standards. We refer to the transition between these two regimes as "takeoff" or equivalently "breakout." This is a more precise statement of Rostow's (1960) notion that "the takeoff is the interval when (...) the forces making for economic progress (...) expand and come to dominate the society. Growth becomes its normal condition" (p. 7).

There exists a modest literature modeling growth in a unified way, to which the three most prominent contributions are Kremer (1993), Galor and Weil (2000), and Hansen and Prescott (2002). Kremer assumes and empirically tests a simplistic relationship between essentially global population size and the global rate of technological progress and shows how this relationship enables emergence from a Malthusian trap. Galor and Weil construct a complex model that includes assumptions similar to Kremer's as well as Beckerian assumptions about child quality-quantity tradeoffs to generate an endogenous demographic transition as societies grow richer. Hansen and Prescott assume exogenous technical progress and fertility decisions but include shifting from a Malthus sector that relies on land for production to a Solow sector that does not. The shift is an equilibrium phenomenon driven by diminishing returns to land as the population grows concurrently with technical progress.

The model developed here shares with those papers an emphasis on a phase change generated by a demographic shift and an increase in sustainable population growth, although this is not their terminology. Sato and Niho (1971) also employ a framework of sustainable population growth in a two-sector model of emergence from Malthusian stagnation in a closed economy, concluding that only technological progress in agriculture leads to breakout. Our model could be viewed as a prequel both to the modern "Unified Growth" literature, which is reviewed by Ashraf and Galor (2011), and to the previous generation of literature of which Sato and Niho is an example.

The model presented here has several advantages over previous attempts to model the transition process. First, it is much simpler and allows easy access to the intuition that can be obscured by a more complex model. While Sato and Niho, and Hansen and Prescott require two sectors, which essentially doubles the number of variables, and Galor and Weil employ four simultaneous difference equations, the present model combines one production function and one differential equation governing population growth in a simple, intuitive way. It thus highlights the most important and fundamental dynamics of the transition from a Malthusian regime to a Solovian one, showing that the many assumptions and complexities of the literature are not necessary to generate both a qualitatively and quantitatively correct story. Second, unlike Galor and Weil, and Kremer, this model's assumptions do not dictate that takeoff can only occur in societies of a certain size or level of technological advancement. Given that the first takeoff actually did occur in Britain, rather than in much larger France or China, this result is to be viewed as an advantage. Third, in this model, most of the underlying causes of growth are exogenous. We view this as an advantage both because it avoids arbitrary and simplistic assumptions about social behavior and because many of the important factors for takeoff are driven by processes that are as much political, historical, or social in nature as they are economic. By remaining agnostic about the relationships between the elements of a breakout, the model can accommodate a rich array of development strategies and can apply to a wider range of societies.

This article makes three main contributions to the growth theory literature. First, it develops the sustainable population growth framework and shows how it acts as a "common currency" in measuring contributions to takeoff. Second, we believe it represents the most elementary possible model in which takeoff can occur. Third, it identifies a heretofore underappreciated channel through which change in the structure of the economy is an important cause of takeoff, in addition to being simply a symptom of it as many authors have assumed.

In the second part of the paper, we apply the model to the breakout that occurred in Britain at the time of the Industrial Revolution, which was characterized by an unprecedented rise in maximum sustainable population growth (MSPG)—the fastest rate at which the population can grow without engendering declining living standards. In the years from the late 17th century to the early 19th century, MSPG increased from firmly Malthusian levels to levels that exceeded not only the peak of British population growth but also any rate of population growth that has ever been recorded. This vast increase in the amount of population growth that the British economy could absorb without engendering declining living standards was possible due to a set of economic changes associated with the Industrial Revolution. We will see that while many of the factors that have been cited by other authors—technology, structural change, coal, and capital—made a contribution to lifting the Malthusian constraint, the process was dominated by technology and structural change.

In addition to solidifying the link between the Industrial Revolution and rising living standards, we make several contributions to the economic history literature. First, this research reconciles the gradualist and limited Crafts-Harley view of the Industrial Revolution with a dramatic and rapid change in Britain's macroeconomic character. Second, it estimates, in an accounting sense, the contributions of various economic factors to the ability of the economy to sustain income growth. Third, it shows that the link between the Industrial Revolution and the economy's ability to sustain a rise in living standards is robust to a wide variety of estimates for the various components of economic growth. Although the pace of transition varies, the qualitative story stays the same. Finally, along the way to compiling MSPG estimates we come up with new estimates for total factor productivity during the Industrial Revolution. These generally follow Crafts and Harley but employ improved estimates of factor shares and natural resource growth. The new estimates point to a somewhat larger role for TFP than the most recent estimates put forth by Crafts and Harley.

The MSPG estimates presented here also carry two further implications that are at variance with the conventional wisdom. First, the effect of trade on the ability of the British economy to transition to modern growth is probably exaggerated. Second, we will see that short- or even medium-term economic growth is a different phenomenon than the ability to sustain a breakout in living standards. If we want to understand the end of the Malthusian era, just asking which factors contributed to growth does not necessarily shed light on this question. Rather, the question we must answer is "What allowed the British economy to transition, possibly fairly abruptly, from a regime where per capita income was trendless to one where it was growing at 1% per year?" The difficult thing to understand is not the "growth" but the "transition."

Finally, it is important to recognize that this analysis does not purport to investigate the ultimate causation of the Industrial Revolution. Rather, it is an accounting exercise in the spirit of Solow, linking the economic changes that were part of the Industrial Revolution to the accompanying rise in living standards. While it would be correct to consider the factors identified in this paper as the direct causes of a breakout, they are likely to be linked to each other as part of the broader underlying process of the Industrial Revolution. Although some cursory thought is given to these links here, a thorough examination of them is beyond the scope of this research.

Section 2 lays out our basic framework for modeling transitions from Malthusian regimes to modern growth. Section 3 explains data sources and estimation methods for the various components of MSPG. Section 4 combines the data to estimate MSPG in Britain. Section 5 discusses some implications of the results and how they relate to the previous literature. Section 6 concludes.

2 The Basic Model

We employ a very simple Malthusian model, with a production function of population and resources.[1] Resources are fixed and population is endogenous. Technology is exogenous, for we wish to examine the role of technology in the transition from a Malthusian economy to a growth economy, and to do this we would like to exogenously vary the rate of technological progress.

We begin with Cobb-Douglas production function (though we shall later present a more general form), and the birth and death rates are taken to be exogenous functions of per capita income:

$$Y = AL^{\alpha}R^{1-\alpha} \tag{1}$$

$$\frac{\dot{L}}{L} = b - d \equiv g(Y/L) \tag{2}$$

The birth rate rises with income until income reaches some critical level and subsequently falls, while the death rate falls with income. Defining per capita income $y = Y/L$, the functions $b(y)$, $d(y)$, and $g(y)$ look like figure 1. The justification for the qualitative functional form of $g(y)$ in Britain is discussed in some detail in appendix A, but is consistent with Lucas' (2004) results on the relationship of

[1] The basic results do not change if endogenous accumulation of capital is included in the model.

population growth to per capita GDP for five regions of the world.

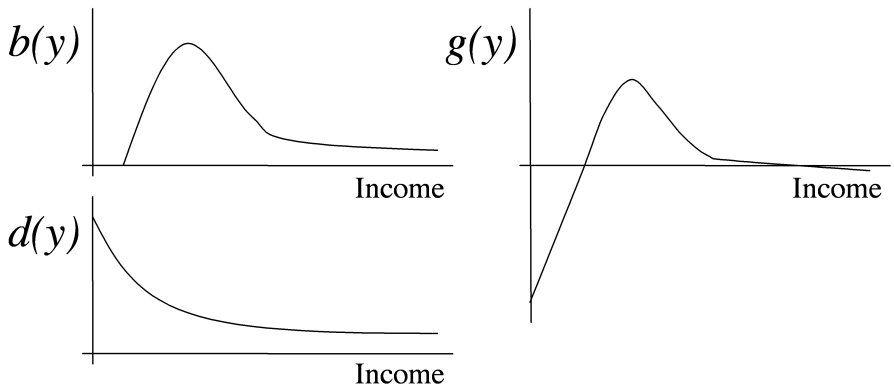

Figure 1: Functional Forms of Birth Rate, Death Rate, and Population Growth

To solve the model, we rewrite it in per capita terms, take logs, and differentiate, which yields:

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + (1-\alpha)\frac{\dot{R}}{R} - (1-\alpha)\frac{\dot{L}}{L} \tag{3}$$

Noting that resources are fixed and substituting for population growth, we have simply that:

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} - (1-\alpha)g(y) \tag{4}$$

This has a simple interpretation: if technological growth is faster than population growth times the resource share, per capita income is rising, and otherwise it is falling.

$\frac{\dot{A}}{A}(1-\alpha)^{-1}$ thus defines the *maximum sustainable population growth*, i.e. the fastest rate at which the population can grow without falling incomes.[2] MSPG can be thought of as the carrying capacity of the economy. If per capita income starts at a low level, then it will eventually rise to a Malthusian equilibrium at point M in figure 2, where the MSPG is precisely equal to $g(y)$: i.e. sustainable population growth equals actual population growth. This equilibrium is stable: if per capita income is hit by a positive shock, the population will grow faster and income will fall again; if hit by a negative shock, population will grow more slowly and income

[2] If the resource base of an economy is expanding, then $MSPG = \frac{\dot{A}}{A}(1-\alpha)^{-1} + \frac{\dot{R}}{R}$.

will rise. The same analysis holds for population shocks.[3]

The mechanism is classically Mathusian: A technology shock leads to higher incomes, causing population growth to rise and encountering diminishing returns to land. Diminishing returns combined with a population that is growing faster than the maximum sustainable population growth cause falling incomes and a return to a Malthusian equilibrium.

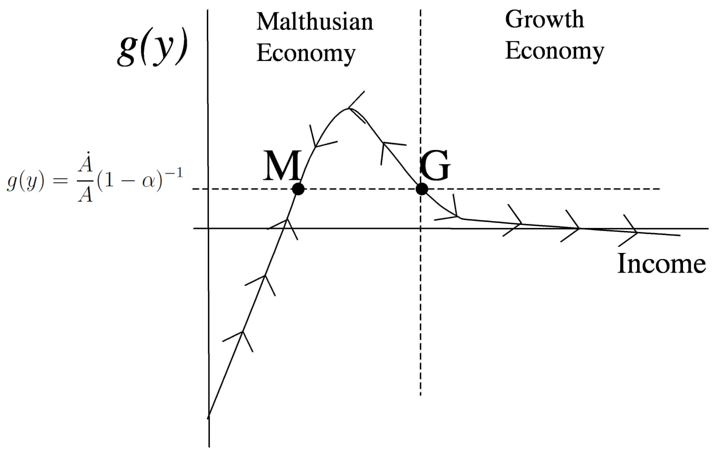

Figure 2: The Dynamics of the Economy

In this economy the *level* of income is determined by the *growth rate* of technology. The situation is analogous to pouring water into a leaky bucket—the faster the water is poured in, the higher the steady-state level of water in the bucket, but if the faucet is turned off, the water leaks out to a lower level. This leads to a prediction that in pre-modern societies, higher income levels should be associated with periods of technological advancement (or high MSPG for other reasons) but should dissipate once the technological advancement ends.

In essence, maximum sustainable population growth is a reservoir that can be used to support a growing population or rising living standards. In a Malthusian soci-

[3]Point G in the figure is also an equilibrium, albeit an unstable one. If the economy finds itself to the right of point G, income will rise, causing fertility to fall resulting in further income rises, fertility falls, and sustained growth. In practice, however, point G tends to occur at a high enough income that it is not attainable by pre-industrial societies. If the economy is to the left of G, it will fall back to the Malthusian equilibrium at point M.

ety, the maximum sustainable population growth rate is below the peak population growth rate. All growth in productivity is used to support a larger population, fully exhausting the reservoir. But once MSPG rises above that peak, people do not want to reproduce fast enough to "use up" all the productivity advances being discovered in the economy. Some of these advances can be used to increase per capita income, effecting the transition to modern growth. Returning to the leaky bucket analogy, modern growth is equivalent to pouring water into the bucket so fast that, despite the leaks, the bucket overflows.

These results do not depend on the Cobb-Douglas functional form or on having just two factors of production. Consider the very general production technology or, even more generally, an income function:

$$Y = F(L, R_i, X_j, s_k) \tag{5}$$

which is assumed to have constant returns to scale in the extensive inputs L, R_i, and X_j. Here, R_i are different types of fixed resources, X_j are variable factors of production such as capital or human capital, and s_k are parameters of the economy, such as the terms of trade and level of technology. Then, if we take logs and differentiate, per capita income growth is given by:

$$\frac{\dot{y}}{y} = \sum_i \eta_i \frac{\dot{R}_i}{R_i} + \sum_j \eta_j \frac{\dot{x}_j}{x_j} + \sum_k \eta_k \frac{\dot{s}_k}{s_k} - \eta_R g(y) \tag{6}$$

where η_i and η_j are the elasticities of income with respect to each of the fixed resources and variable factors of production, and η_k are the elasticities of income with respect to the parameters s_k. Lowercase letters denote per capita amounts or intensive properties of the economy, and $\eta_R = \sum_i \eta_i$ is the total share of output paid to fixed factors. Maximum sustainable population growth is then determined, as before, as the rate of population growth giving rise to constant per capita income:

$$MSPG = \frac{1}{\eta_R} \cdot \left[\sum_i \eta_i \frac{\dot{R}_i}{R_i} + \sum_j \eta_j \frac{\dot{x}_j}{x_j} + \sum_k \eta_k \frac{\dot{s}_k}{s_k} \right] \tag{7}$$

$$= \frac{1}{\eta_R} \frac{\dot{y}}{y} + \frac{\dot{L}}{L} \tag{8}$$

The R_i terms in line (7) are familiar and represent the contribution from an expanding resource base. The terms in x_j are new. They represent the contributions of deepening in other factors of production, particularly capital. Growth in these factors contributes to MSPG proportionately to their shares in output, and inversely with the share of fixed resources in output. Thus, structural change away from land-intensive production and into capital-intensive production raises MSPG by increasing the multiplier on the capital deepening term of equation 8.

The formulation in equation (8) offers a very general reduced form for MSPG that applies to any constant-returns-to-scale economy. Note that this is not a causal relationship but an observational one that allows us to calculate MSPG from economic observables.

Equation (8) highlights the difference between income growth and MSPG. The concepts are related but not identical, and changes that increase one may decrease the other. To see this, let us consider the effect of a small change in the structure of the economy by differentiating equation (8):

$$d(MSPG) = d\left(\frac{\dot{y}}{y}\right) \cdot \frac{1}{\eta_R} + d\left(\frac{1}{\eta_R}\right) \cdot \frac{\dot{y}}{y} + g'(y)dy \tag{9}$$

Equation (9) makes explicit that changes that increase instantaneous income or income growth may or may not increase MSPG depending on what happens to the resource share. If a structural change increases the resource share, it may decrease MSPG even though it increases income or income growth. If it decreases the resource share, it may increase MSPG even in the face of a decline in income or income growth. The intuition is that a lower resource share raises the ultimate steady-state income level even as it decreases the rate of growth toward that equilibrium.

We shall illuminate this effect in more detail as we explore the effect of an opening to trade on the economy. In a world with no trade, a poor country must produce its own food, which requires labor and land. It might have a production function as previously denoted in equation (1). The process of opening to trade *raises the value of* α, reducing the resource share in the economy.[4] This trade effect on the transition to growth can be seen in figure 3.

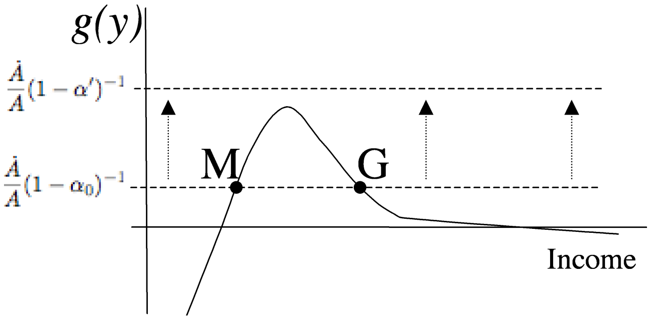

Figure 3: The Effect of Trade on MSPG

The increase in α raises the level of population growth that can be sustained in a Malthusian equilibrium.[5] If the increase is large enough, as shown in the figure, the change can be enough to lift the resource constraint entirely and set off the transition to modern growth.

For large countries, there is an additional complication that a large productivity

[4]It is recognized that as a mathematical matter the production functions for the non-tradeable and tradeable sectors should simply be added. Given that Cobb-Douglas is already an abstraction that does not apply to the real world, it is hoped that the reader will accept the further abstraction that combining two industries with Cobb-Douglas production functions will be taken to yield a Cobb-Douglas with intermediate factor shares. Even an economy that specializes completely into another good requires at least some small share of non-tradeable resources, such as drinking water, living space or land on which to build factories.

[5]This depends on at least some part of technology being non-resource augmenting. Otherwise we have $A^{(1-\alpha)}$ in the production function and moving away from resource-intensive production hurts technological progress as well as lifting the Malthusian constraint, and these effects offset each other.

improvement in the export sector, precisely what occurred in Britain during the Industrial Revolution, will affect the terms of trade. In the appendix B we provide a basic analysis of the case when the terms of trade depend on trade volumes. There we show that if an economy is experiencing income gains, MSPG may increase while income growth decreases. Similarly, in an economy that experiences income declines, MSPG may decrease while income growth increases. Therefore, trade may increase income growth while making it harder to escape from the Malthusian trap, or vice versa. Later, we will see that this odd result is not just theoretical but in fact likely applied to Great Britain during the Industrial Revolution: Britain was experiencing income gains, but was nonetheless suffering immiserating trade that increased the sustainability of its growth.

The effect of trade on the Malthusian economy when terms of trade are not constant is therefore ambiguous. Trade may help or hurt an economy's ability to escape the Malthusian trap, and it provides a channel for breakout to be exported even without the prospect of technology transfer. Even more surprisingly, the analysis provides a graphic demonstration that factors increasing income growth may not necessarily make it easier to escape from Malthusian stagnation.

3 Estimating the Building Blocks of MSPG

We now estimate MSPG in Britain during the period 1300-1850. For this purpose we utilize equation (7), which allows us to include capital in the model as follows:

$$MSPG = \frac{1}{\gamma}\frac{\dot{A}}{A} + \frac{\beta}{\gamma}\frac{\dot{k}}{k} + \frac{\dot{R}}{R} \tag{10}$$

where γ is the resource share in the economy and β is the capital share. We will thus require estimates of total factor producivity A, the factor shares β and γ, effective land area R, and capital intensity k.

3.1 Total Factor Productivity

Estimates of TFP growth prior to the advent of modern statistics are quite unreliable and difficult to come by. The estimates that have been used in this paper are based on those by Allen for the years prior to 1700 and by Crafts for the years after 1700. Allen (2005) estimates total factor productivity in agriculture in 1300, 1500, and 1700, and he also estimates labor productivity for 1400 and 1600 (Allen, 2000). In order to estimate TFP for these latter two years, we assumed that TFP followed a similarly shaped path to labor productivity.[6] Table 1 shows Allen's estimates for TFP and labor productivity, and our interpolated estimates for 1400 and 1600. There is assumed to be no TFP growth outside of agriculture prior to 1700[7], so the agricultural TFP growth rate is multiplied by the share of GDP in agriculture, which is estimated as described in section 3.2. The top three rows of the table present index numbers in the turn of the centuries; the bottom three rows are averages over the century beginning with the date at the column heading.

	1300	1400	1500	1600	1700
Allen TFP	0.83		1.00		1.38
Allen Labor Productivity	0.8	0.92	1.00	0.76	1.15
Interpolated TFP		0.94		0.83	
Estimated Ag TFP CAGR for Century	0.12%	0.06%	-0.18%	0.51%	
Proportion of GDP in Agriculture	46%	43%	41%	35%	
Estimated Economy TFP Growth	0.06%	0.03%	-0.08%	0.18%	

Sources: See text.

Table 1: Calculation of Total Factor Productivity pre-1700

For the years from 1700 to 1860, we use the residual approach of Crafts (1985, p78), making use of Crafts (1985, 1995) and Crafts and Harley's (1992) estimates of

[6]See appendix C for details of this calculation.

[7]This may appear to be an extreme assumption, but in the absence of data we believe it to be the most appropriate. First, it avoids repeating Deane and Cole's (1967) error of assuming that the sector with notable productivity growth (in their case textiles, in this case agriculture) was representative. Second, Crafts' (1985) estimates that industrial and commercial output grew at an annual rate of 0.70% from 1700 to 1760, while the population outside agriculture grew at a rate of 0.64% (Wrigley and Schofield, 1981). This leaves little room for productivity growth even in the 18th century, and there was probably less in earlier times.

real GDP growth, Wrigley and Schofield's (1981) population estimates, Feinstein's (1988) estimates of capital stocks, and our own estimates of natural resource growth and factor shares as described in sections 3.2 and 3.3 below. Combining these figures with the numbers from Allen gives estimates of TFP growth from 1300 to 1860. This time series is shown in table 2, and it will serve as our estimate of TFP growth for the rest of this paper.

	1300-1400	1400-1500	1500-1600	1600-1700	1700-1760	1760-1780	1780-1800	1800-1830	1830-1860
Allen/Tepper and Borowiecki	0.06	0.03	-0.07	0.14					
Crafts/Harley/ Tepper and Borowiecki					0.31	0.04	0.41	0.57	0.88

Sources: see text.

Table 2: Annual Growth Rates of Total Factor Productivity (Percent)

These estimates of TFP growth are close to Crafts' original (1985) estimates and are generally somewhat higher than Crafts and Harley's revised figures. This does not stem from any fundamental disagreement with Crafts and Harley over the progress of the economy but rather reflects the need to treat land and capital as separate factors.

The results indicate that prior to the scientific revolution in the mid-17th century TFP growth was extremely slow and may have been dominated by exogenous events like climate change. The 150 years from 1650, including the agricultural revolution, saw modest but consistently positive rates of TFP growth in England. Then, beginning with the Industrial Revolution in the first half of the 19th century, TFP growth slowly accelerated to modern levels.

3.2 Structural Change

Structural change is linked to breakout because it reduces the dependence of the economy on land and other natural resources that are constrained in the Malthusian sense.[8] While there are many measures of structural change, the one that matters for breakout is the factor shares in the economy. Specifically, the inverse resource share acts as a multiplier on MSPG, so that reducing the importance of fixed factors in the economy can have a large effect on sustainable population growth.

Structural change may encompass not only a shift to less resource-intensive production but also a shift to more capital-intensive production. In equation (10) we observe that as the resource share, γ, falls (if the move is into capital-intensive production), then β increases at the same time, magnifying the effect. Estimating the production elasticities β and γ is fraught with pitfalls. The simplest way to do so, and the approach taken by most authors, is to assume that the pre-Industrial and Industrial Revolution British economy was approximately competitive and to proxy the elasticity of production by the share of GDP paid to each factor. That is the approach taken here, although it is recognized that this is a strong assumption.

We construct our own rent share coefficients prior to 1700. For the rent share after 1700, we obtain two separate series: using Clark et al.'s (2012) and Allen's (2009b) estimates or Broadberry et al.'s (2013) figures.

The estimation methodology is as follows. We assume that all land rents derive from agriculture prior to the 18th century, as rents paid on coal mines even in 1860 were at most 4% of agricultural rents, and were negligible before this time.[9] We then combine data on the share of agricultural income paid to land (Allen, 2005)[10],

[8]Although it is not modeled as part of the main paper, it is easy to create a feedback loop so that structural change is endogenous in the model. Intuitively, if agricultural productivity increases slowly, then the entire productivity growth is absorbed into supporting a larger population. However, if agricultural productivity increases more quickly, then as incomes rises, people consume more non-agricultural goods (the income elasticity of food demand is less than one). This leads to people moving off the land, decreasing the rent share, which increases per capita income growth, leading to a lower share of agriculture in GDP and more people moving off the land, which leads to a lower rent share, in other words a virtuous circle.

[9]See Clark and Jacks (2007) for data on coal rents.

[10]We obtain from Allen (2005) estimates of the agricultural income paid to land for 1300, 1500, and 1700. The share of agricultural income to land in 1400 is assumed to be the same as the estimate for 1500, as all the change in economic structure between 1300 and 1500 is assumed to be due to the Black Death in 1348-51. We further use the share from 1700 for 1650, as it is the closest available estimate. The later assumption is further motivated by the observation that the share

share of labor force in agriculture (Broadberry et al., 2013; Clark, 2013; Clark et al., 2012), and the productivity differential of agriculture and other sectors (Broadberry et al., 2013) to determine the share of output paid as land rents.

Data on the share of agricultural income paid to land is fairly straightforward but there is a range of estimates for the share of English labour force engaged in agriculture prior to 1760. Previous estimates by Wrigley (1985) and Overton and Campbell (1996) have been contested by recent scholarship (Broadberry et al., 2013; Clark, 2013; Clark et al., 2012). However, the estimates provided by Broadberry et al. differ quite significantly from those reported by Clark et al.. Broadberry et al. reconstruct the labor force in the three principal sectors—agriculture, industry and services—for benchmark years using the poll tax returns of 1381, the Muster Rolls of 1522, and re-worked social tables for 1696, 1759, 1801 and 1851. Clark (2013) also uses the poll tax returns of 1381 to reconstruct the share of English labour force engaged in agriculture, but for later dates (1560-79 and 1651-60) uses occupation statements in wills.

The estimated share of the labor force in agriculture for 1381 is essentially the same according to both groups of researchers, which is unsurprising since they rely on the same primary source. Broadberry et al. find that 57% of the workforce was engaged in agriculture, while Clark (2013) provides figures of 56-59%. The next available estimate is provided by Broadberry et al.—58.1% in 1522, which suggests that the farm share in employment remained fairly stable over the course of the 15th century. The agreement ends however here and the estimates provided for the following years diverge. While Clark et al. (2012) report consistently high figures for 1560-79 and 1652-60, which are 61% and 59%, respectively, Broadberry et al. provide significantly lower estimates for 1700, equal to 38.9%, and further decreasing numbers for later periods. If both sets of figures are right, it would imply

of agricultural income to land remains remarkably stable at around 50% throughout the 17th and 18th centuries an implausibly rapid structural change in the English economy from 1660 to 1700. Until new and better evidence becomes available, it is not clear which point of view will stand the test of time (Leunig, 2013), and as such, it is not obvious which estimates are the right ones. Therefore, we construct two separate rent share series for the period of the disagreement.

Based on the earliest estimates, which are provided for 1381 and come closest to 1400, we calculate the average agricultural share in employment for 1400. Since there is no estimate available for the share of workforce in agriculture in 1300, we approximate it with the estimate from 1400, an assumption supported by the near-constant share of agricultural employment from 1381-1650. Broadberry's figure for 1522 is used for 1500 and Clark's estimate for 1652-60 is employed for 1650. From then on their estimates diverge significantly and we construct two separate series for the period post 1700. The first one takes rent share estimates directly from Allen (2009b) for the period after 1760 and calculates the rent share for 1700 as the average between Allen's figure for 1760 and our estimate for 1650. The second series is based on Broadberry's estimates that are closest to 1700, 1760, 1800 and 1841. Table 3 summarizes the calculations.

We turn next to the estimation of productivity differences between the agricultural sector and the rest of the economy. Broadberry et al. (2013) estimate that productivity outside agriculture was higher than productivity in agriculture by a factor 1.6 in 1381, 2.11 in 1522, and 1.74 in 1700. We use these figures to translate our agricultural share in employment into GDP share estimates for 1400 and 1500, and interpolate the productivity difference for 1650. We then multiply the share of GDP in agriculture by the share of agricultural production paid as rent to obtain the share of GDP paid to land. To determine the rent share in 1600, we use Clark's (2002) rent index combined with the assumption that gross land area under cultivation was constant over the half-century from 1600 to 1650. Linked with data on population

	1300	1400	1500	1600	1650	1700	1760	1800	1841
Allen:									
Rent Share of Ag. Inc.	0.39	0.19	0.19		0.51	0.51	0.48	0.51	0.48
Clark/Broadberry/Allen:									
Share of Pop. in Ag.	57%	57%	58%		59%				
Share of GDP in Ag.	46%	46%	40%		46%				
Rent Share in GDP	0.18	0.09	0.08	0.29	0.23	0.23	0.22	0.17	0.11
Broadberry:									
Share of Pop. in Ag.						39%	37%	32%	24%
Share of GDP in Ag.						27%	30%	31%	19%
Rent Share in GDP						0.14	0.14	0.16	0.09

Table 3: Rent Share in National Income Estimates
Sources: Rent share in agricultural income from Allen (2005). Share of population in agriculture from Broadberry et al. (2013); Clark (2013); Clark et al. (2012). Share of GDP in agriculture and rent share in GDP own calculation based on Broadberry's et al. and Clark's et al. data on share of population in agriculture. See text for calculation methods.

growth and the rent share in 1650, this uniquely determines GDP per capita growth, which is estimated to have been 0.02% per annum from 1600 to 1650.[11] Broadberry et al. (2012) report a comparable figure for annual GDP per capita growth rate of -0.04%. Our estimates imply a rent share of 29% in 1600 and, by taking the average of the rent share for 1650 (based on Clark's estimates) and Allen's estimate for 1760, we find a rent share of 23% in 1700. Calculating the rent share using Broadberry's numbers, one would end with a low estimate of 14%.

As a check on these figures, one could conduct a bottom-up approach: multiplying the rent per acre by the number of acres (Clark, 2002) and dividing by nominal GDP (Lindert and Williamson, 1982). This yields a rent share in GDP of 23% in 1700, 24% in 1760 and 13% in 1800. The 1700 estimate is practically the same as the rent share coefficient obtained in our framework when Clark's agricultural share in employment is used; the estimates for the following years come very close to Allen's rent share.

To get averages over a period, we simply average the endpoints of the period. The

[11]See appendix C for the details of this calculation.

final rent share series for use in the MSPG calculation are reported in table 4, along with Allen's figures for capital shares after 1700. Both series show a large reduction of the rent share—by over a factor of two—from the 17th century to the middle of the 19th century. Because many components of MSPG are inversely proportional to the rent share, this structural change in the economy approximately doubled sustainable population growth over the period.

	1300-1400	1400-1500	1500-1600	1600-1700	1700-1760	1760-1780	1780-1800	1800-1830	1830-1860
Clark/Broadberry/Allen: Rent share in GDP	0.13	0.08	0.18	0.25	0.23	0.22	0.19	0.16	0.11
Broadberry: Rent share in GDP				0.22	0.14	0.15	0.15	0.14	0.11
Allen: Capital share in GDP					0.20	0.19	0.20	0.32	0.39

Sources: See text.

Table 4: Factor Shares in National Income

A further check could be conducted by calculating the sum of incomes derived from land, based on the social tables from Lindert and Williamson (1982). We assume that 80% of the incomes of the high titles comes from land, 20% for secular professions, 50% for farmers, cottagers and paupers. We also assume that 80% of the excess of income of freeholders over farmers was derived from landholdings.[12] These estimates translate into a rent share of 28% in 1688. The calculation is summarized in table 13. Once again this figure fits more closely with the series based on Clark's and Allen's figures than that based on Broadberry's.

Both checks support the series based Clark's and Allen's estimates over Broadberry's. For this reason we shall use this series in the baseline specifications when estimating MSPG. We use the rent share based on Broadberry's estimates to demonstrate the robustness of our results.

[12]These shares are based on Holmes (1977, p54-55), Mingay (1963), Mimardiere (1963, p98), Stone (1965, p562), Thompson (1966, p509) and Cooper (1967, p431)

3.3 Coal and Land Improvements

Coal increased MSPG in two ways. Most significantly, it substituted for timber, which required land, i.e. the Malthusian fixed factor, for its cultivation.[13] Second, because rents on coal producing land were proportionately lower than rents on agricultural land, it reduced the factor share of land in the economy.[14] The substitution for timber meant that coal increased the effective amount of land that Britain had to support its population; land that previously was needed to grow timber could be used to grow food, and the demand for energy that would have needed to be met by additional timberland could be met by coal. Determining the effective easing of the land constraint, however, can be slightly more complicated. Coal energy is recognized to have been of a lower quality than charcoal (coal sold at a discount to charcoal), and coal mines are not as versatile as land in production of final goods. Additionally, because coal mining is a different production process than agriculture, rent typically made up a much lower share of gross produce. Exactly what adjustments should be made for these differences is open to debate.

The baseline approach taken here aims at determining the acreage necessary to produce timber of the same value as British coal output. This methodology has the advantages that it implicitly takes into account a measure of land quality, and it reflects Britain's ability to trade coal for timber on the world markets. Appendix D discusses three alternative estimation strategies:

1. a simple calculation of the amount of woodland that would be required to

[13]It may be argued that once nearly all energy came from coal, coal mines could no longer substitute for agricultural land because people cannot eat coal. However, coal could be and was exported and thus held down prices of land-intensive goods on the world market, goods which Britain imported.

[14]Adam Smith comments on this in *The Wealth of Nations* (p272): "Rent, even where coals afford one, has a generally smaller share in their price than in that of most other parts of the rude produce of land. The rent of an estate above ground commonly amounts to what is supposed to be a third of the gross produce... In coal mines a fifth of the gross produce is a very great rent; a tenth the common rent... Thirty years' purchase is considered as a moderate price for the property of a landed estate, [but] ten years' purchase is regarded as a good price for that of a coal mine." Clark and Jacks (2007) also figure that coal rents were typically just 7-10% of the pithead production.

produce the same quantity of energy as British coal;

2. a comparison of the total output of the coal industry to the agricultural output; and

3. a comparison of land rents in agriculture to site rents for coal mines.

We will later use the results from these methods to estimate the range of possible estimates of MSPG, but now we return to our baseline approach.

According to Clark's price indexes, coal sold for about 4.1 shillings/ton in 1800, which agrees with Crafts' (1985) estimate that British coal production was worth £2.7 million in 1800. A cord of firewood sold for 31 s. At 1 cord/acre (Allen, 2009a), an acre's worth of firewood production cost as much as 7.6 tons of coal.[15] Combined with improvements in land quality as discussed by

1300-1500	1500-1700	1700-1750	1750-1775	1775-1800	1800-1830	1830-1860
0.04%	0.16%	0.26%	0.37%	0.39%	0.18%	0.53%

Sources: See text for sources and calculation method.

Table 5: Effective p.a. Land Area Growth

Effective land area growth contributes to sustainable population growth on a one-for-one basis, so while coal and land area growth contributed moderately to MSPG, they were not sufficient to cause a dramatic breakout. Appendix D conducts a number of estimates based on different methodological approaches. We present the low and high estimates from those alternative calculations in table 10. It is only at the very highest end of the estimates, and even then only beginning in the mid-19th century, that coal could itself have been sufficient to raise MSPG above the threshold for breakout. Even then, other contributions to MSPG were of equal or greater magnitudes.

[15] We assume 82 cu. ft. of solid wood per cord after accounting for air space, which is what is implied by figures in Allen (2005).

3.4 Capital Deepening

The technological change of the Industrial Revolution brought with it an increase in investment and therefore significant capital deepening. We use estimates of capital stocks from

	1700-60	1760-80	1780-1800	1800-30	1830-60
$\Delta K/K$	0.7%	0.63%	1.30%	1.73%	2.48%
$\Delta k/k$	0.37%	-0.05%	0.26%	0.29 %	1.30%

Sources: See text.

Table 6: Growth in Capital Stock and Capital Per Worker

Prior to 1700, capital per worker is assumed to be constant outside of agriculture. In agriculture, any GDP changes due to capital per worker show up as increases in TFP.

4 Calculating MSPG in Britain

An important question about the Industrial Revolution is how British industrialization relates to the beginning of the sustained rise in living standards that occurred at approximately the same time. To the extent that the Industrial Revolution refers to a shift in production away from agriculture into industry, and into more technologically advanced industrial production processes requiring higher capital intensity and relying on coal for energy, we are now able to draw a clear link between these economic changes and the beginning of a sustained rise in living standards.

Using the estimates made in the previous section we now plot British MSPG (figure 4), calculated according to equation (10), and population growth. In the appendix E we report alternative estimates based on the rent share derived from Broadberry's data.[16] In post-medieval British history, we can see that it was not until the last

[16]As we will see, the qualitative story is not sensitive to which estimates one chooses. We also show this in this section by estimating low, best and high estimates of MSPG.

two decades of the 18th century that sustainable population growth significantly exceeded the peak in the population growth function for any sustained period, but by the middle of the 19th century, the economy was able to sustain a population growing at around 13% per year—nine times the maximum observed rate of population growth and more than six times the rate that could have been sustained at any time prior to the Industrial Revolution.

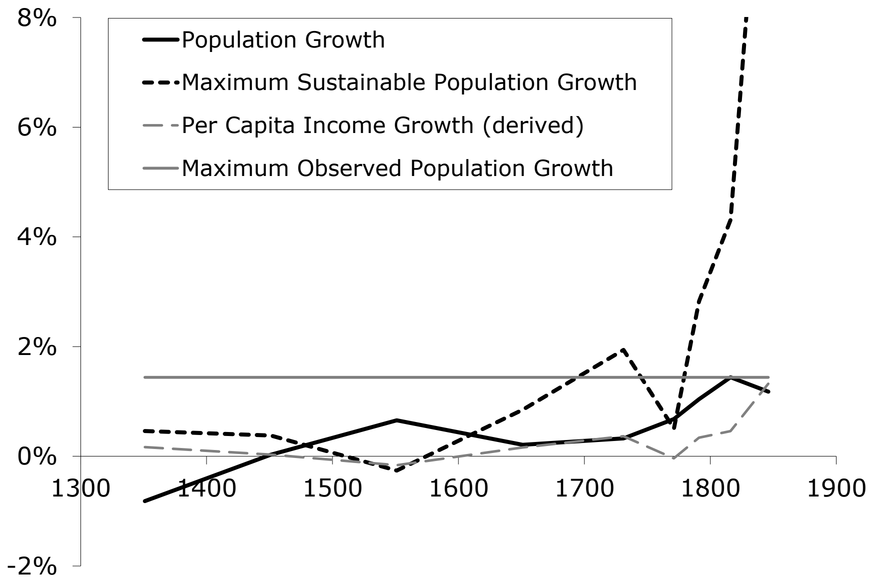

Figure 4: Maximum Sustainable Population Growth from 1300 to 1860

These results naturally explain Crafts and Mills' (2009) empirical finding that the British economy exhibited strong homeostasis prior to 1645 and extremely weak homeostasis thereafter. Crafts and Mills' result is likely a consequence of their having tested an equilibrium version of a Malthusian model where wages and population are always in their Malthusian equilibrium. By contrast, the present model is a dynamic one that explains the behavior of the economy even in out-of-equilibrium states. Prior to 1645, MSPG was close to zero, so a traditional Malthusian model linking wages to population should perform relatively well. After 1645, MSPG was near the threshold for breakout, meaning the traditional Malthusian mechanism operated only very weakly.

The chart also suggests that from the 17th century onwards the British economy was slowly and smoothly building toward the ability to achieve breakout, with the "knee" of the curve reached at the Industrial Revolution. While at first glance the chart shows a dramatic change at the end of the 18th century, it could just as easily be interpreted as a smooth curve from 1600 onwards with technological progress in agriculture flowing into technological progress in industry as part of a single phenomenon, save only for a depression from 1760 to 1780. That depression was characterized by a slowdown in TFP growth that may have been related both to the Seven Years War from 1756 to 1763 and the American Revolution from 1775 to 1781.

In any event, by the beginning of the 19th century, the Malthusian constraint had been completely and—depending on which view of the 1760s and 1770s one takes—suddenly eliminated. This finding is consistent with Lee and Anderson (2002, p217), who find a "very sharp and discontinuous rise in the rate of increase [of labor demand] starting around 1810." Their definition of the rate of increase of labor demand is the rate at which new labor can be absorbed by the economy; this is MSPG viewed through the econometrician's lens.

Maximum sustainable population growth is the only measure of capacity we are aware of that shows such a dramatic change over precisely the time period of the Industrial Revolution and the beginning of the increase in living standards. TFP growth, for example, did not reach truly unprecedented levels until the middle of the 19th century. The MSPG framework thus reconciles the Crafts-Harley limited view of the Industrial Revolution with a large and discontinuous change in the carrying capacity of the economy, which was linked to the beginning of a sustained increase in living standards.

Viewing the takeoff through the lens of MSPG also reconciles the timing of the economic change and expansion in output—the last two decades of the 18th century—with the timing of real wage growth, which did not begin in earnest until a few decades later. The model naturally produces an Engels Pause as the economy moves over the hump in the population growth function, which causes rapid growth in output but much lower growth in output per worker.

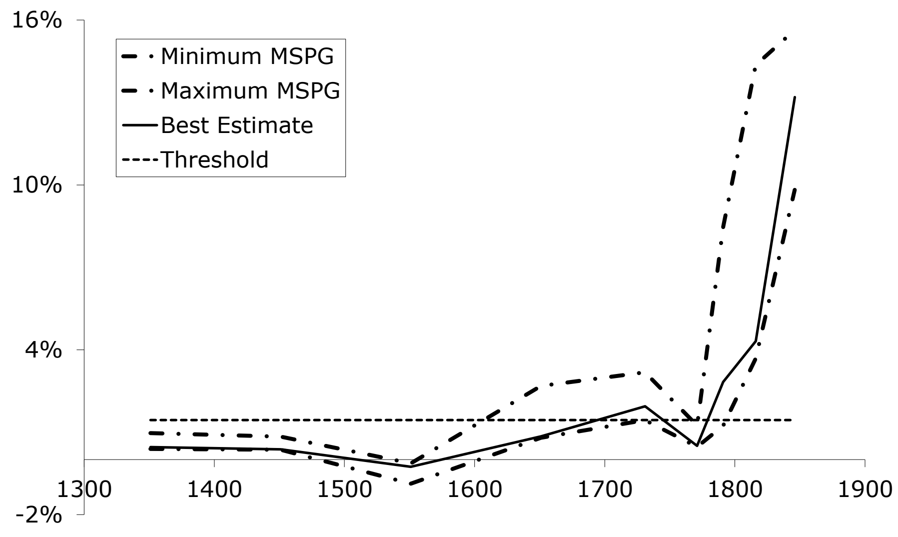

Figure 5: Range of MSPG Estimates, 1300-1860

The estimates upon which MSPG is based are open to debate and measurement error. Fortunately, the broad, qualitative story of breakout and how and when it occurred does not change if different estimates are used. Figure 5 plots a low, high, and best estimate of MSPG from 1300 to 1860. The high estimate uses the highest estimates of GDP growth (Deane and Cole's), assumes that productivity growth in the non-agricultural part of the economy mirrored productivity growth in agriculture prior to 1700, employs the energy-equivalent method for coal mining (see appendix D) with Warde's figure of 0.38 tons of coal per acre, and uses the lowest rent share estimates (based on Broadberry et al.'s share of agriculture in employment). The low estimate uses the lowest estimates of TFP growth we found (from Antras and Voth, 2003), the rent-equivalent method for coal mining (implying an equivalence of 50 tons per acre), and the highest rent share estimates available (based on Clark et al.'s share of agriculture in employment and Allen's rent share for post-1760). It should be recognized that the range presented does not represent a confidence interval in any sense. Rather, it is intended to show that the overall story is not sensitive to which "view" of the Industrial Revolution one takes. The narrow interval in the early years of the chart is indicative not of strong confidence in the figures but of a paucity of varying estimates in the literature.

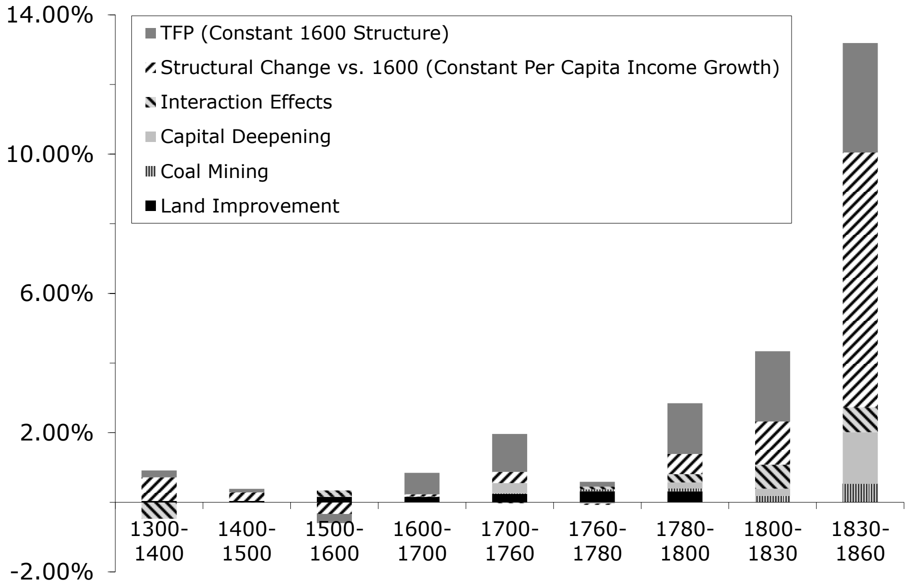

Figure 6: Contributions of Various Factors to MSPG

Figure 6 plots the contributions of each of the factors we consider to MSPG. These

	1300-1400	1400-1500	1500-1600	1600-1700	1700-1760	1760-1780	1780-1800	1800-1830	1830-1860
TFP (Constant 1600 structure)	0.20	0.10	-0.27	0.62	1.09	0.14	1.46	2.02	3.15
Structural Change vs. 1600 (Constant Per Capita Income Growth)	0.68	0.25	-0.33	0.07	0.32	-0.04	0.58	1.24	7.32
Interaction Effects	-0.47	-0.01	0.18	0.01	-0.03	0.08	0.24	0.69	0.72
Capital Deepening					0.30	-0.04	0.18	0.21	1.48
Coal Mining	0.00	0.00	0.01	0.01	0.02	0.06	0.08	0.17	0.52
Land Improvement	0.04	0.04	0.15	0.15	0.24	0.32	0.31	0.01	0.01
MSPG	0.46	0.38	-0.25	0.83	1.94	0.51	2.83	4.31	13.19

Sources: The utilized rent share estimates are based on Clark et al.'s share of workforce in agriculture. See text for remaining sources and calculation method.

Table 7: Contributions to MSPG (Percentage Points Per Year)

are also shown in table 7. The figure shows that during the Industrial Revolution, MSPG was largely attributable to technological advances and structural change in the economy and, by the middle of the 19th century, to a shift to more capital-intensive production. The shift to more capital-intensive forms of production played a rather minor role prior to 1830. At least in an accounting sense, coal played only a minor part in the takeoff.

5 Discussion

The key to breakout is the ability to sustain non-immiserating population growth; MSPG is the measure of this ability. Figure 7 illustrates that the Industrial Revolution, with its technological improvements and growth in less resource-dependent sectors, caused a clear and unprecedented increase in maximum sustainable population growth as early as 1780, well before a large effect was seen in wages and GDP per capita. The proposed framework explains how such a phase change can take place without unprecedented levels of per capita income growth. Indeed, even though the rates of income growth seen from 1780 to 1830 were at most a tenth of

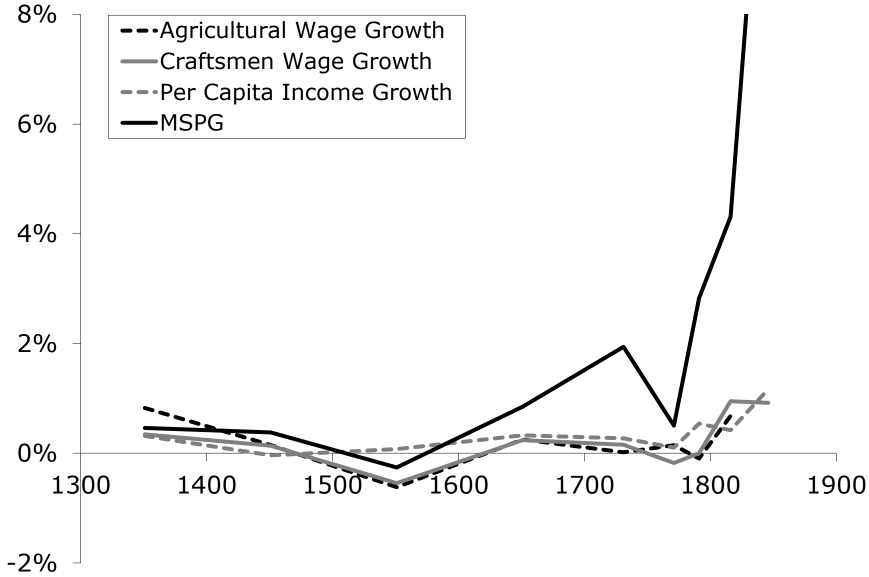

Figure 7: MSPG, Real Wage and Real GDP, 1300-1860
Sources: The utilized rent share estimates are based on Clark et al.'s share of workforce in agriculture. Real wage series for craftsmen from Clark (2005) and real GDP per capita series from Broadberry et al. (2012).

a percentage point greater than those seen a century earlier, the massive change in MSPG shows that the growth in the Industrial Revolution period was part of a new and unprecedented process of breakout.

Structural change, i.e. the movement of the labor force from agriculture into industry and from the country to the city, is often remarked upon as a feature of industrialization and the beginning of modern growth. Most of the literature, however, focuses on structural change as a *result* of the growth in living standards or of factors driving that growth.

The most obvious link between structural change and growth is that as agricultural productivity improves, fewer people are needed to produce the food supply for a society. The surplus labor that emerges tends to migrate to cities and towns and find employment in less resource-intensive sectors. While there is debate about what exactly drives people off the land (see, for example,Weisdorf, 2006, and Crafts and Harley, 2002, for different views on the topic), structural change is viewed as a consequence of or accompaniment to growth rather than an important ingredient in itself.

Some authors have discussed structural change as a driver of growth, but usually through more complex channels than those modeled here. While Mendels (1972) argued that "proto-industrialization", which is the move into handicrafts rather than (or in addition to) food production, set the stage for the growth of modern industry, his arguments are based on cost pressures spurring technological improvement. Mathias (1989) similarly argues that proto-industrialization allowed the development of industrial and entrepreneurial skills, a commercial infrastructure, and capital. Harley (1994) discusses how Britain's focus on rapidly-advancing industry became an engine of growth. These arguments, however, tend to be made in broad brush strokes and offer a tentative rather than firm link between structural change and growth. The nature of the hypotheses are difficult to support with hard evidence. More importantly, they do not make it clear whether these channels were an idiosyncrasy of the British experience or whether they somehow represent an integral part of the process. Here, we show how structural change is an important cause of breakout because it reduces the economy's dependence on resource-intensive production, raising MSPG.

Coal's importance in the Industrial Revolution remains a subject of debate. While the New Economic History tends to view coal as playing a smaller role (e.g. Clark and Jacks, 2007, who also provide a review of the current state of the debate), coal remains to be seen by many scholars as a major driver of industrialization. In fact the industrial revolution is sometimes defined through the lens of coal as "the escape from the constraints of an organic economy" (Wrigley, 2010, p239).

According to Griffin coal was *the* factor that has mitigated the "age-old constraints which had placed a ceiling to the growth of industry and population" (Griffin, 2010, p107).

This analysis concludes that while coal could theoretically have played a large role in easing the Malthusian constraint, a quantitative investigation indicates that it probably did not do so, at least directly. However, coal may have contributed to the Industrial Revolution in ways beyond the scope of this model. In addition to easing the land constraint, the discovery of coal provided the areas of Britain in the vicinity of the coalfields with extraordinarily cheap energy because of the lower transport costs (Allen, 2009a). It is thus no surprise that energy-intensive industry did in fact spring up around the coalfields. Independent of its contribution to increasing effective land area, economic growth, or import revenues, coal contributed to structural change in the British economy. This structural change was another significant factor in raising MSPG.

So far, little attention in the applied analysis has been devoted to the role of trade in easing the Malthusian constraint. Economic historians tend to agree that trade played an important role in the British Industrial Revolution, but there the agreement ends. Acemoglu et al. (2005) argue that trade helped create and sustain the institutions necessary for economic growth. Harley and Crafts (2000) and Crafts and Harley (2002) claim that trade was a critical ingredient in the exceptional structural change that took place in Great Britain away from agriculture. Pomeranz (2000) disputes the claim that trade with (or exploitation of) the New World provided the raw materials to ease the Malthusian constraint. Davis (1979) suggests that trade was not a primary factor in triggering the industrial revolution but was critical in sustaining demand that allowed it to continue. While the present theory examines the direct role of trade, any indirect effect may coexist with those described here.

Our framework allows for two competing effects of trade on MSPG. First, trade is associated with specialization, and in the case of Britain, this meant specialization in industry over agriculture. This kind of specialization reduces $(1-\alpha)$, the resource share, thereby raising MSPG. Second, if a country's terms of trade worsen as trade increases, then trade may decrease income growth and MSPG, *ceteris paribus*.

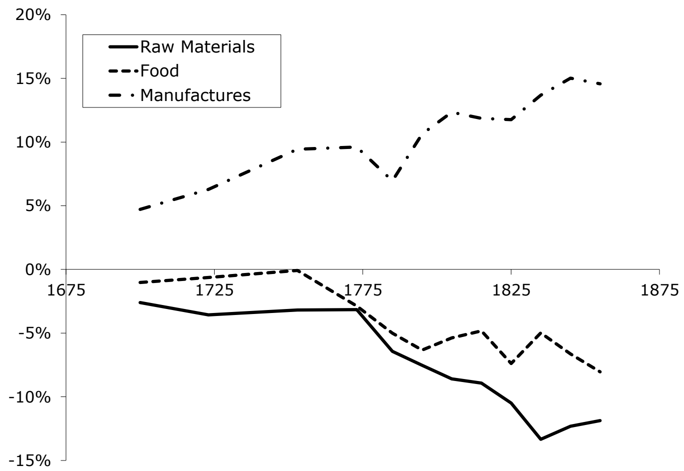

Figure 8: Net Exports/GDP by Sector

The contribution of trade to structural change in Britain was likely significant, a point that has been made many times in the literature. Figure 8 shows net exports as a share of GDP by sector from 1700 to 1855.[17] By the middle of the 19th century, Britain was importing more than 15% of its GDP in resource-intensive food and raw materials and using non-resource-intensive manufactures to pay for them. This likely resulted in a significant reduction in the all-important rent share in the British economy, although estimates of this are quite sensitive to assumptions about the counterfactual situation. In the late 18th century, this effect was smaller but likely still significant.

[17]Trade data are from Davis (1962, 1979). Export-to-GDP ratio from Crafts (1985, p131), with interpolation.

The simplest way to analyze the effect of trade on the rent share is to simply compare the British economy under autarky to the economy with trade, holding the consumption basket constant. While this makes a useful benchmark, it is not correct for two reasons. First, under autarky the price of agricultural goods would rise relative to that of manufactured goods. In response, consumers would substitute away from agricultural goods so that the consumption basket would consist of a higher proportion of manufactured goods. Second, a shift to more agricultural production at home would raise the marginal product of land and hence land rents. These two effects work in opposite directions, with the first tending to cause the simple comparison to overstate the effect of trade on the rent share and the second tending to cause the simple comparison to understate the effect of trade on the rent share. Table 8 shows the effect of trade on the rent share from 1700 to 1850 under this methodology.

Simply comparing trade with autarky holding the consumption basket constant suggests that trade did not have much of an effect on enabling takeoff in Britain. At the time when the breakout occurred at the turn of the 19th century, trade reduced the rent share in the British economy by an estimated 3-4 percentage points, enough to increase MSPG from 3.8% to 4.3% in the first third of the 19th century.[18] However, this was still a small proportion of MSPG at this time.

1700	1723	1753	1773	1785	1795	1815	1845
-0.8%	-0.7%	-0.4%	-1.8%	-3.1%	-3.9%	-3.3%	-4.5%

Sources: Trade figures from Davis (1962, 1979). See text for calculation method.

Table 8: Effect of Trade on the Rent Share (Percentage Points)

For another approach examining the effect of trade on MSPG, we can begin with Crafts:2002) computable general equilibrium model, which simulates the effect of trade on the British economy from 1770 to 1841. This model takes into account

[18]Rents in agriculture are assumed to be 50% of trade values, rents in raw materials are assumed to be 10% of trade values, and rents in industry are assumed to be zero. This is in line with Allen (2005) and Clark and Jacks (2007).

both relative price effects and substitution effects described above, and when its results are put into the MSPG framework, they are even more interesting.

The Crafts-Harley simulation considers the effect in 1841 of freezing agricultural imports at the 1770 level. Surprisingly, they find that when all effects are considered, the restriction of trade would have actually *increased* the 1841 national income per capita by 8% and, hence, its 1770-1841 growth rate by 0.1 percentage points per year. This is because the deterioration in British terms of trade over the period more than offset the production gains from specialization.[19] However, Crafts and Harley also find a large effect on factor prices, enough to increase the rent share in national income by a multiple of 1.55, from 11% to 17%. The net result of restricting trade would have been a decrease of 0.2 percentage points in MSPG to 3.8% for the first third of the 19th century and a decrease of 3.5 percentage points to 9.5% for the second third of the century.[20] Nonetheless, in both periods MSPG with restricted trade would still have been well in excess of the peak in population growth.

We are thus left with the extremely odd result that freer trade from 1770 to 1841 decreased per capita income but nonetheless facilitated the transition to a modern growth regime. The intuition for this is that the key to escaping the Malthusian trap is reducing the economy's dependence on natural resources, combined with technological growth. Trade can ease the dependence on natural resources, but a large economy must pay a price for this in terms of worsening terms of trade. The ability to escape the Malthusian trap is inversely proportional to the rent share in production in the economy, while income growth is not so simply related to the rent share.

In contrast to the conventional wisdom, then, the direct effects of trade were likely not a major factor in effecting the transition to a modern growth economy until the middle of the 19th century, by which time modern growth was well underway.

[19] See Harley (1994, p305) for British terms of trade during the 19th century.

[20] These calculations are carried out using equation (8).

Equally important, we have in trade an example of why it is essential to ask precise questions about the industrial revolution: Freer trade decreased per capita income in England but still contributed somewhat to its ability to escape the Malthusian trap. This is a graphic demonstration of the fact that "What factors increased economic growth?" and "What cause d the economy to transition from a Malthusian regime to a growth regime?" are not really the same question. A tie between an economic phenomenon and general growth is not sufficient to establish that that phenomenon aids in shaking off Malthusian constraints.

6 Conclusions

It seems obvious that there was in some sense a fundamental change in the British economy at the turn of the 19th century, a change that, at least in the popular imagination, has long been characterized by a small number of trends and inventions: the steam engine, cotton gin, coal, the reorganization of production into factories, and migration off the land and into cities. Yet one of the great mysteries of the New Economic History is that as prime causes of wealth, these icons of the Industrial Revolution have proved ethereal when investigated quantitatively (e.g. Harley, 2012).

The framework of sustainable population growth reconciles the older and popular notion of a cataclysmic, revolutionary change in the economy with the relatively limited changes in most economic variables. During the Industrial Revolution, the rate of population growth that the British economy could sustain without declining living standards increased from less than 2% to almost 13%. This change meant the virtually complete elimination of Malthusian constraints, so that technological advances and capital investments could be used to increase incomes rather than population. Interestingly, however, contributions to income growth and contributions to takeoff are not necessarily equivalent, which points to the importance of asking the right questions when we seek to understand what changed in the British economy.

In an accounting sense, the proximate causes of the increase in the carrying capacity of the British economy were the twofold increase of total factor productivity growth combined with an even greater decline in the economy's dependence on land from 1780 to 1860. The commonly cited factors of coal and trade, despite the drastic increases in volumes, do not appear to have nearly the same immediate impact on MSPG. Trade does not seem to have had a large effect until the middle of the 19th century, by which time the process of modern growth was well underway, while the direct contribution of coal to the economy never appreciably increased sustainable population growth. Those wishing to argue in favor of a strong role for coal must therefore argue that coal had an impact far in excess of what coal producers were compensated for.

An important corollary of this study is that there is no one prescription for development; an economy does not necessarily need to look a certain way, or have a certain quantity of capital, level of education, set of industries, nor arrangement of institutions to achieve takeoff. Rather, wherever the economy starts, modern growth is touched off by sustained but potentially temporary rapid productivity gains or other increases in maximum sustainable population growth. These gains can be achieved by capital accumulation, technological transfer, education, exploitation of natural resources, or structural change in any combination sufficient to raise maximum sustainable population growth above the peak in the Malthusian population-income function. They can be achieved in combination with factors like population control that act to lower the threshold.

While this paper demonstrates which proximate causes of the British breakout were most important and how those causes contributed to the ability of the economy to sustain population growth, it tells us little about the underlying causes of the broad change. The Industrial Revolution was a set of interrelated changes—the takeoff of coal mining, increases in capital per worker, rise in international trade, changing structure of the economy, and increases in TFP—that all occurred roughly contemporaneously; it is hard to think of this as a coincidence. This observation highlights some important open questions. What is the connection between the changes in the economy that occurred at the time of the Industrial Revolution? As the agricultural revolution enabled more efficient food production, what caused the new commodities not also to be land-intensive? And finally, what caused the acceleration of TFP growth around 1800, or around 1650 if the process is viewed as having been continuous with the agricultural revolution?

7 Appendix

A The Fertility Function

The theoretical section of this paper has proposed a definite functional relationship $g(y)$ between per capita income and population growth, presented as a manifestation of the Malthusian mechanism by which higher incomes enable higher fertility and reduce mortality. The strength of the Malthusian relationship in Britain varies significantly depending on the timescale examined. Over the short term in Pre-Industrial Britain, all authors we are aware of who attempt to test a Malthusian framework find a positive relationship between wages and population growth.[21] In the long term, however, population growth is dominated by factors other than wages and incomes. In other words, while $g(y)$ appears to be well defined over short timescales, the functional relationship itself changes over longer ones. However, despite the finding that population growth may vary exogenously to wages, the literature concludes that the "homeostatic" Malthusian feedback mechanism placed a constraint on the Pre-Industrial British economy that prevented a sustained simultaneous increase in both population and living standards. While we rely on previous findings as to the existence of the Malthusian causal relationship between wages and population growth, we provide a "sanity check" by plotting a raw estimate of the $g(y)$ function based upon data from 1541 to the present. The left panel in figure 9 plots population growth from Wrigley and Schofield (1981) against GDP per capita from Broadberry et al. (2012) for 1541 to 1861 and from Maddison (2001) for 1871 to 1981. The right panel shows the relationship between population growth and the real wage from Clark (2005) from 1541 to 2000. Both plots show a relationship of the form proposed in section 2, though the period from 1550 to 1630 appears to depart from the functional relationship in the GDP per capita panel and the period from 1580 to 1630 in the wage panel.

The long tail indicates that if a country became stuck in the Malthusian era (at point M in figure 2), a simple shock to income would have to be very large in order to push the economy over the hump and past point G into the modern growth regime. Indeed, figure 9 suggests that even with a relatively high Malthusian MSPG of around 1% (which was not seen until the beginning of the 18th century and certainly did not exist in the aftermath of the Black Death), per capita income would have to double in order to effect the transition to growth.

[21] All these investigations use wages rather than GDP per capita as a measure of living standards (even recent ones, e.g., Møller and Sharp, 2012). Here we can use Broadberry et al.'s (2012) new GDP estimates, but for comprehensiveness we additionally report the functional relationship between wages and population growth.

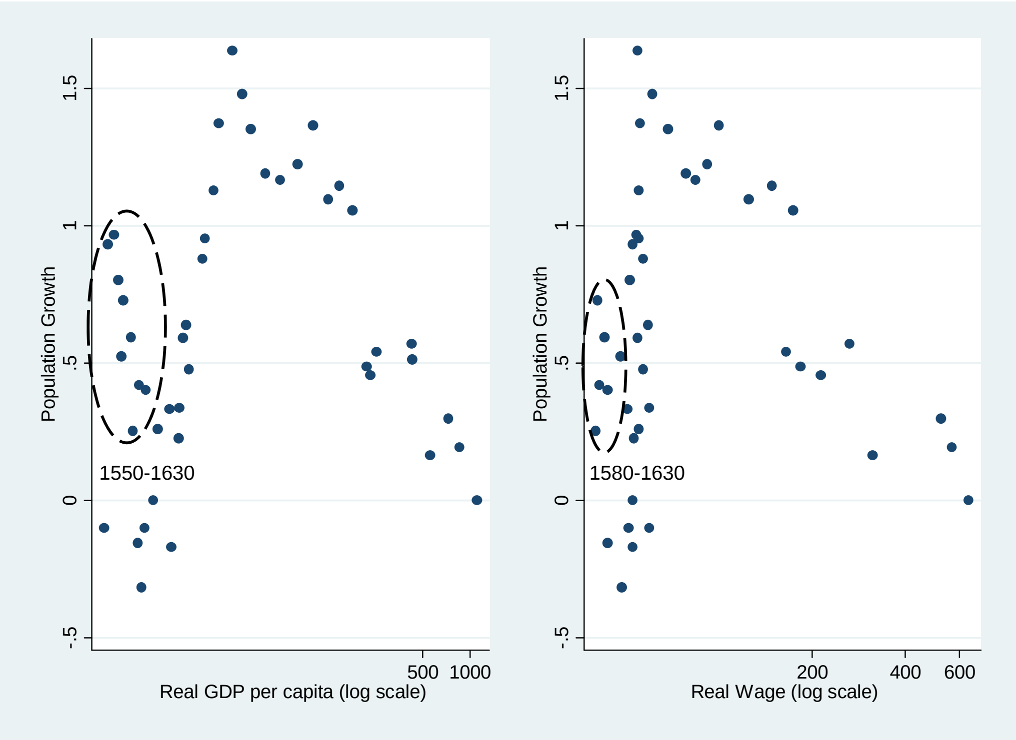

Figure 9: The Relationship Between Income and Population Growth, 1541-1981
Sources: See text.

B The Role of Trade and Structural Change

Here we present a more detailed analysis of the effect on the economy when it opens to trade. When the terms of trade depend on trade volumes, it is no longer the case that specialization away from (or into) resource-intensive production necessarily eases (or inhibits) the transition to growth. Despite this complication, the situation can be analyzed in the same basic manner. Trade is still to be thought of as an alternative production function:

$$Y = \frac{p(\text{Home Production})}{p(\text{Consumption Basket})} \cdot AL^{\alpha'} R^{1-\alpha'} \tag{11}$$

The consumption basket will consist partially of home production and partially of imports. In a country like Britain that specialized in industry, imports will be resource-intensive, meaning $\alpha' > \alpha$. For Britain's trading partners, exports will be resource-intensive, meaning $\alpha' < \alpha$. We also expect that the terms of trade will be worsening for the economy whose output is expanding faster.

Normalize the price of home produced goods to 1, and replace the price of the consumption basket with a weighted average of the prices of home production and imports:

$$p_C = \beta + (1-\beta)p$$

β is the share of consumption produced at home, so lower β indicates a more open economy. Price of imports in terms of home production, p, implies that its increase leads to worsening terms of trade. Taking logs and writing the production function in per capita terms, we have:

$$\ln y = -\ln[\beta + (1-\beta)p] + \ln A - (1-\alpha')\ln L + (1-\alpha')\ln R \tag{12}$$

Differentiating and simplifying:

$$\frac{\dot{y}}{y} = -\frac{\beta(1-p)}{\beta + p(1-\beta)} \cdot \frac{\dot{\beta}}{\beta} - \frac{p(1-\beta)}{\beta + p(1-\beta)} \cdot \frac{\dot{p}}{p} + \frac{\dot{A}}{A} - (1-\alpha')g(y) \tag{13}$$

implying that

$$MSPG = \frac{1}{1-\alpha'}\left[-\frac{\beta(1-p)}{\beta + p(1-\beta)} \cdot \frac{\dot{\beta}}{\beta} - \frac{p(1-\beta)}{\beta + p(1-\beta)} \cdot \frac{\dot{p}}{p} + \frac{\dot{A}}{A}\right] \tag{14}$$

There are thus three effects. The $\frac{\dot{p}}{p}$ term shows that importing products with declining relative prices increases per capita income growth and maximum sustainable population growth. Importing products with rising relative prices decreases per capita income growth and MSPG. The $\frac{\dot{\beta}}{\beta}$ term shows that if imports are cheap compared to home goods, more trade increases income and MSPG, while if imports are expensive, the opposite is true. The $(1-\alpha')$ term shows that in the absence of a downward sloping demand curve for an economy's products, specializing in a resource-intensive industry exacerbates the resource constraint, decreasing per capita income growth and MSPG, while specializing in a resource-light industry increases them. If all the effects are present, the price effect and the increasing trade effect will tend to work in the same direction, and the resource share effect will tend to work in the opposite direction. This is because we typically think of new industries as being the growth ones.

In order to simplify notation, we let

$$X = -\frac{\beta(1-p)}{\beta + p(1-\beta)} \cdot \frac{\dot{\beta}}{\beta} - \frac{p(1-\beta)}{\beta + p(1-\beta)} \cdot \frac{\dot{p}}{p} + \frac{\dot{A}}{A}$$

Roughly speaking, X represents the drivers of income growth: changes in trade volumes, terms of trade, and productivity. Equations (13) and (14) then become

$$\frac{\dot{y}}{y} = X - (1-\alpha')g(y) \tag{15}$$

$$MSPG = \frac{X}{1-\alpha'} \tag{16}$$

Note that these equations are dynamic and are true both in and out of steady state.

We can see that reduction of trade barriers will increase per capita income growth if

$$\Delta X > (1-\alpha')\Delta g(y) - (\alpha' - \alpha)g(y) \tag{17}$$

while the condition for it to increase MSPG is different:

$$\Delta X > -X \cdot \left[1 - \frac{1-\alpha'}{1-\alpha}\right] \tag{18}$$

Let us consider the simplified case where trade has no immediate effect on current income y. Conditions (17) and (18) then simplify to:

$$\frac{\Delta X}{\alpha' - \alpha} > -g(y) \qquad \text{(income growth) (19)}$$

$$\frac{\Delta X}{\alpha' - \alpha} > -\frac{X}{1-\alpha} \qquad \text{(MSPG) (20)}$$

and The right hand sides of conditions (19) and (20) are only equal in steady state. (This can be seen by reference to equation 15.) If an economy is experiencing income gains, MSPG may increase while income growth decreases. If an economy experiences income declines, MSPG may decrease while income growth increases. Thus, trade may increase income growth while making it harder to escape from the Malthusian trap, or vice versa. Section 5 provides a discussion that this seemingly odd result is not just theoretical but is also supported by the case of Great Britain during the Industrial Revolution: Britain saw income gains, but was nonetheless suffering immiserating trade that increased the sustainability of its growth.

C Notes on Data Sources

Where available, we have used data sources for England and Wales rather than only England or Great Britain as a whole. However, because we are primarily interested in growth rates rather than absolute levels, we have endeavored to find the longest time series possible and have tried hard to avoid switching geographical coverage in the middle of a series. We have therefore sacrificed some geographical exactness in the name of achieving estimates that are comparable across time, on the belief that growth rates for Great Britain depend mainly on England, and the effects of geographical mismatch are not likely to be large compared to the already tentative nature of the data.

Post-1541 population data are from Wrigley and Schofield (1981, p208). Pre-1541 data are from Broadberry et al. (2012). Both are for England only.

Except where otherwise noted, real wage data are from Clark (2005) and are the average of the two wage series he computes for England.

Total factor productivity in agriculture for 1300, 1500, and 1700 comes from Allen

(2005) and is for England and Wales. We used TFP that was adjusted for labor inputs and land quality but not for capital. Thus prior to 1700 any productivity increases due to capital are included in TFP. We assumed that the course of TFP followed labor productivity (from Allen, 2000) in order to get estimates for 1400 and 1600. The calculation for 1400 is as follows:

$$\frac{TFP_{1400}}{LP_{1400}} = \sqrt{\frac{TFP_{1300}}{LP_{1300}} \cdot \frac{TFP_{1500}}{LP_{1500}}} \tag{21}$$

where LP_i is labor productivity in year i. The calculation for 1600 is analogous.

Estimates of British GDP per capita growth pre-1861 are from Broadberry et al. (2012) and for post-1871 from Maddison (2001). 1861 GDP, which has been used to determine the share of coal in GDP for this date, is from Deane and Cole (1967).

Proportion of the English workforce in agriculture comes from Broadberry et al. (2013) or Clark et al. (2012) and Clark (2013).

We use Broadberry et al.'s (2013) estimates of how much more productive non-agricultural production was than agricultural production in Britain for 1381, 1522, and 1700, and we interpolate for years in between.

The method to calculate the rent share for 1600 was rather involved. Consider first the equation for income growth:

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} - \eta_R \frac{\dot{L}}{L} \tag{22}$$

which, consistent with our earlier assumptions, absorbs any capital effects into TFP over the period from 1600 to 1650. η_R is the rent share and is given by $(\rho R/Ly)$ where ρ is rent per acre, which we take from Clark (2002). Thus, we have data on all variables in this differential equation at the beginning and the middle of the 17th century except GDP per capita y in 1600. There is thus a unique level of y in 1600 that is consistent with this differential equation, which in turn determines the rent share in this year.

To get averages over a period for the share of output in agriculture, we simply average the endpoints of the period.

Estimates of factor shares post-1760 come from Allen (2009b) and are for Great Britain.

The capital share for 1700 is calculated by assuming that the labor share was constant from 1700 to 1760 so that any decreases in factor payments to land were absorbed by capital.

We use net capital stocks for Great Britain from Feinstein (1988, table XIII).[22]

[22] Crafts and Harley (1992) write that they take their figures from Feinstein's table XVI. However, the growth rates they use come closest to matching the rates from table XIII.

Effective land area for English and Welsh agriculture is from Allen (2005, table 1), which gives estimates for 1300, 1500, 1700, 1750, 1800, and 1850. The estimate for 1850 is used for 1860; estimates for other dates are interpolated.

Coal production for England and Wales comes from Hatcher (1993) for 1560 to 1700, Flinn (1984) for 1700 to 1830, and Church (1986) from 1830 onwards. Pithead prices are from Clark and Jacks (2007), and industry revenue is given as pithead price times production.

Data on trade volumes post-1780 are for Great Britain and come from Davis (1979). Pre-1780 data are for England only and are sourced from Davis (1962). Because we only consider trade on a proportionate-to-GDP basis, this does not lead to severe data inconsistencies. The export-to-GDP ratio for Great Britain is from Crafts (1985, p131) and is interpolated and rounded to the nearest half a point for intervening years. This ratio is then used with Davis' import data to calculate the trade balance in each sector.

D Coal and Timber Data and Alternative Coal-Land Equivalance Estimates

In addition to the approach presented in this paper, where we estimate the acreage required to produce timber of the same value as British coal output, we use three additional strategies to estimate the equivalence between coal and land.

The simplest method is to calculate the amount of woodland that would be required to produce the same quantity of energy as British coal. To do so, we obtain estimates of the energy content of wood by weight, the density of wood from British forests, and the timber yield by volume of woodland. We can multiply these figures to obtain the energy content per acre. We then compare this to the energy content of a ton of coal.

For our estimate of energy content by weight, we rely on estimates from a variety of sources[23] that one pound of air-dried wood (20% moisture content) contains about 6,400 BTU of energy, a figure which varies little by species of wood.

For estimate of density, we note that British coppice was largely populated with oak, ash, beech, hornbeam, and hazel, which are relatively dense woods ranging from 40-50 lbs./cu. ft. of solid wood when air dried.[24] We take the midpoint of this range.

For our estimate of timber yield per acre, we assume that an acre of woodland yielded 82 cubic feet of dry wood per year, which we obtain by using Allen's (2009a) estimate that an acre of wood yielded one cord (a stack 4 feet by 4 feet by 8 feet, not necessarily tightly packed) of green wood per year. Allen's energy content figures imply that the wood was stacked with a density of 40 lbs./cu. ft., which when dried

[23]See, for example, the U.S. Department of Energy (2009).

[24]See, for example, Glover (2003), or Chimney Sweep (2009).

becomes 28.8 lbs./cu. ft. We combine this with our estimate above that solid, dry English coppice weighed 45 lbs./cu. ft. to obtain a figure of 82 cubic feet per acre for the yield of woodland. This is in line with the high end of modern coppice estimates (Crockford and Savill, 1991). By contrast, Warde (2007) notes that the Forest of Dean in the 1690s yielded 4 cubic meters per hectare, which corresponds to 57.2 cubic feet of green wood per acre, while himself using a lower figure of 3.3 cubic meters per hectare. Hammersley (1973) claims that an acre of woodland could produce up to 100 cubic feet per year. Using Warde's yield figure would imply a produce of woodland of only about 15 shillings/year, which is much less than Allen's (2005) figure of £2 per year for agricultural produce and even less than his figure of £1 per year for agricultural rents. Our figure implies that the gross produce of woodland was 31 shillings/year, which makes it less productive than intensively farmed agricultural land, but not too much so.

Our figures imply that woodland produced about 23.6 MBTU of energy per year. We follow Warde in assuming that British coal, which was largely bituminous, yielded 27.7 MBTU per ton, in line with modern sources. An acre of woodland thus yielded as much energy as 0.85 tons of coal.

Other estimates of this figure vary widely. Hatcher (1993) estimates 0.5-1 ton per acre, Clark and Jacks (2007) estimate 0.9 tons per acre, and Allen (2009a) estimates 1.3 tons per acre, though Allen's figure appears to be due to a typographical error (his figures appear to imply 1.3 acres per ton rather than tons per acre), which, when corrected for, becomes 0.76 tons per acre. Warde's (2007) figures imply an equivalence of 0.38 tons per acre. All of these figures appear to have some problems in their approach, though Clark and Jacks' figures appear to be the least problematic. Their combination of energy content and density figures suggests they are using bone dry or oven dry wood, which is lighter and contains more energy on a per pound basis. However, their estimate of yields per acre is therefore at the extreme upper end of the range, especially as wood tends to shrink somewhat as it dries. The assumptions of the various researchers are presented in table 9 below.

	Allen (Green Wood)	Allen (Dry Wood)	Clark and Jacks (Dry Wood)	Warde (Green Wood)	Tepper and Borowiecki (Dry Wood)
Wood Yield (cu. ft./acre)	128.0	107.6	97.5	47.2	81.8
Energy Content (BTU/cu. ft.)	140,000	166,500	251,000	185,000	288,000
Energy Content (BTU/lb.)	3,493	4,859	8,600		6,400
Density (lbs./cu. ft.)	40.1	34.3	29.2		45
Coal Energy Content (MBTU/cu. ft.)	23.6	23.6	26.9	27.7	27.7

Sources: Allen (2003), Allen (2009a), Clark and Jacks (2007), Warde (2007).

Table 9: Assumptions Underlying Various Coal/Timber Equivalence Estimates.

In any event, our figure of 0.85 tons of coal per year as equivalent to one acre of land implies that English and Welsh coal production by 1860 was equivalent to 88 million acres, assuming an annual output of the coal mining industry of 75 million tons (Church, 1986). Combined with advances in agriculture, coal could thus be argued to have increased the effective land area of England and Wales from 17.5 million acres in 1300 to 232 million acres by 1860.[25] By this method, British effective land area increased faster than population over this period. At least at first glance, there is a case to be made that coal and land improvement helped to lift the Malthusian constraint facing Britain and allowed it to emerge as the first growth economy, even without technological progress.

This approach does not take into account that coal was a less desirable fuel than charcoal and therefore sold at a discount, nor does it permit adustments to account for land quality, nor does it reflect Britain's ability to trade coal for timber on the world markets; the acreage equivalence was not purely a theoretical matter. Therefore, we view the above approach as inferior to the one chosen in the main body of the paper.

A third method is to compare the total output of the coal industry to agricultural output. In 1860, coal output for Great Britain (including Scotland) was approximately £24 million[26], or 3.6% of GDP[27], compared to 18% of GDP in agriculture (Deane and Cole). Thus, English and Welsh coal mines produced as much additional GDP as did 7 million acres of agricultural land, implying a coal-land equivalence of 11 tons per acre. This method yields estimates of 8-10 tons per acre for earlier periods.

Finally, we can directly compare land rents in agriculture to site rents for coal mines. Agricultural rents per acre ranged from 35 times coal site rents per ton in the 1710s to 70 times coal site rents in the 1860s (Clark and Jacks, 2007, for coal rents; Clark, 2002, for agricultural rents). This method would therefore imply an equivalence of 35-70 tons of coal per year per acre. This method could be justified by the principle that land value should be viewed as the best measure of land productivity in an efficient market. In our preferred method, however, we use the sale price of coal for comparison, as it covers the entire production process that is a substitute for the agricultural production process.

	Effective Land Area (Exc. Coal)	Coal Production (Tons MM)	Coal Equivalent Acreage			Total Effective Land Area			Effective p.a. Land Area Growth (following period)		
			(Low Est.)	(Best Est.)	(High Est.)	(Low Est.)	(Best Est.)	(High Est.)	(Low Est.)	(Best Est.)	(High Est.)
1300	17.5					17.5	17.5	17.5	0.04%	0.04%	0.04%
1500	18.9	0.1	0.0	0.0	0.2	18.9	18.9	19.1	0.15	0.16	0.26
1700	25.6	2.5	0.1	0.3	6.7	25.7	25.9	32.3	0.25	0.26	0.47
1750	28.9	4.5	0.1	0.6	11.9	29.0	29.5	40.8	0.33	0.37	0.98
1775		7.9	0.2	1.0	20.7	31.5	32.4	52.0	0.34	0.39	1.10
1800	34.0	13.0	0.3	1.7	34.3	34.3	35.7	68.3	0.04	0.18	1.48
1830		27.4	0.5	3.6	72.0	34.7	37.7	106	0.10	0.53	2.64
1860	34.2	75.1	1.5	9.9	198	35.7	44.2	232			

Sources: Land area from Allen (2005). Coal production from Hatcher (1993), Flinn (1984) and Church (1986). See text for conversion factors.

Table 10: Coal Production and Land Area in England and Wales

[25] This calculation uses Warde's 0.38 tons per acre equivalence and constitutes our upper-bound. See Allen (2005, table 1) for effective land area calculations.

[26] This figure combines the pithead price of coal from Clark and Jacks (2007) with output figures from Church (1986).

[27] GDP is sourced from Deane and Cole (1967).

Table 10 shows the contributions of coal and land improvement to effective land area for three different estimates, along with intermediate calculations. The best estimate figure considers 7.6 tons of coal as equivalent to an acre of woodland, as described. The high estimate uses Warde's figure of 0.38 tons of coal as producing energy equivalent to an acre of woodland, without any adjustment for prices. The low estimate compares the land with coal mines on the basis of 1800 rents, with an equivalence of 50 tons per acre.

E Alternative MSPG Estimations

For robustness, we present alternative MSPG estimations based on Broadberry et al.'s (2013) estimates for the agricultural labor force. Table 11 shows the contributions of each of the factors we consider to MSPG derived from this alternative data source.

	1300-1400	1400-1500	1500-1600	1600-1700	1700-1760	1760-1780	*1780-1800	1800-1830	1830-1860
TFP (Constant structure)	0.20	0.10	-0.27	0.62	1.09	0.14	1.46	2.02	3.15
Structural Change (Constant Per Capita Income Growth)	0.68	0.25	-0.33	0.20	1.30	-0.05	1.05	1.70	7.64
Interaction Effects	-0.46	-0.01	0.18	0.01	-0.06	0.14	0.30	0.75	0.73
Capital Deepening					0.33	-0.03	0.18	0.21	1.48
Coal Mining	0.00	0.00	0.01	0.01	0.02	0.06	0.08	0.17	0.52
Land Improvement	0.00	0.04	0.15	0.15	0.24	0.32	0.31	0.01	0.01
MSPG	0.46	0.38	-0.26	0.99	2.93	0.57	3.38	4.86	13.52

Sources: The utilized rent share estimates are based on Broadberry et al.'s (2013) share of workforce in agriculture. See text for remaining sources and calculation method.

Table 11: The Components of MSPG (Percentage Points Per Year)

Before we turn over to the graphical visualisation of these results, we estimate an additional robustness test using estimates on sectoral shares in GDP from Broadberry et al. (2012). The obtained GDP data is then combined with the share of agricultural income that accrued to land from Allen (2005) to obtain headline estimates of MSPG based on the new figures in the framework of equation (8). This is shown in Table 12.

Figure 10 presents all results for the calculated MSPG series. The solid black line shows again our baseline MSPG specification, based on Clark et al.'s agriculture share in employment and Allen's rent share, and estimated as an aggregate of all individual factors. The dashed black line presents the MSPG series based on Broadberry et al.'s (2013) numbers for the agriculture share in employment using the baseline methodology. The two series diverge during the late 17th century with MSPG based on Broadberry et al.'s estimates being approximately greater by half

	1400-1450	1450-1480	1480-1553	1553-1600	1600-1650	1650-1700	1700-1760	1760-1780	1780-1801	1801-1830	1830-1861
Per Capita Income Growth	-0.07	-0.07	0.03	0.12	-0.04	0.69	0.27	0.10	0.54	0.42	1.17
Population Growth	-0.14	0.29	0.54	0.67	0.45	-0.08	0.34	0.74	1.09	1.44	1.17
Rent Share	8	8	8	7	17	15	13	14	13	12	7
MSPG	-1.04	-0.59	0.93	2.33	0.22	4.52	2.36	1.46	5.11	5.07	17.13
Addenda:											
Sectoral Share in Agriculture	40	41	40	38	34	30	27	28	27	23	15
Share of Agricultural Income to Land	19	19	19	19	51	51	51	48	48	51	48

Sources: Broadberry et al. (2012), Allen (2005), authors' calculations. All figures in percentage points or percentage points per year, as appropriate.

Table 12: MSPG Using Broadberry et al. (2012) Data, 1400-1860.

than the series based on Clark et al.'s data. This is due to Broadberry's lower agricultural share in employment.

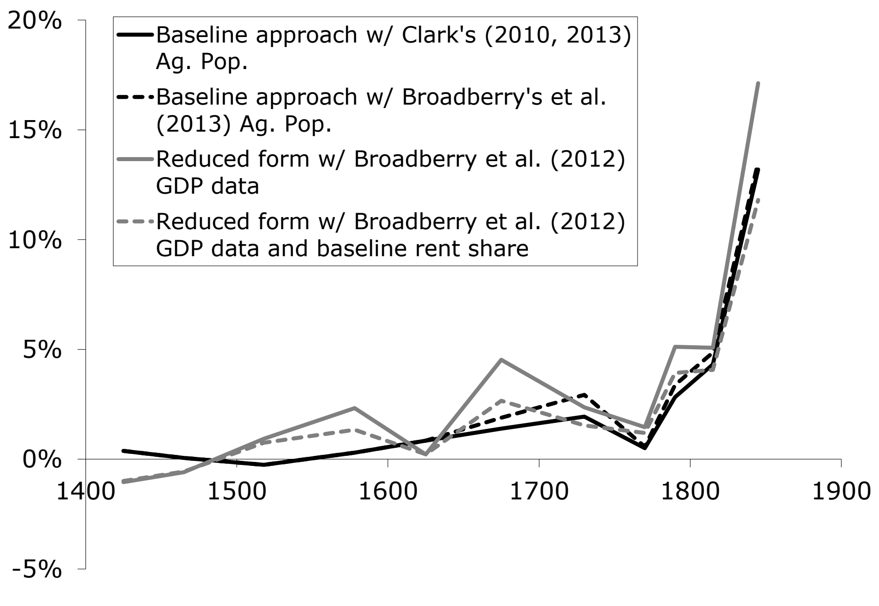

Figure 10: Robustness of MSPG

The solid grey line in figure 10 reports MSPG obtained from Broadberry et al.'s (2012) share of GDP in agriculture estimates and employing the reduced form for MSPG. Broadberry et al. (2012) find considerably less change in the sectoral breakdown of GDP over the period from 1380-1840, with the share of GDP in agriculture estimated to be as low as 45% as early as 1380 and falling only somewhat to 31% of GDP by 1800. This is due both to a finding of a smaller share of the workforce in agriculture and a finding that labor productivity in agriculture was much lower than in services and industry. This leads to lower estimates of the rent share in national income, and as a result, implies MSPG that is both higher and more volatile than our baseline estimates. The dashed grey line shows an estimate that uses Broadberry et al.'s GDP figures with the rent share from out baseline specification: this shows that the rent share accounts for most of the difference between the two series. The other important source of variance between Broadberry et al. (2012) estimates and ours is that Broadberry et al. take a more optimistic view of the 16th and 17th centuries than we do. We find that living standards fell during the 16th century and then retraced these losses during the 17th century, while Broadberry et al. find flat living standards during the 16th century and a rise of nearly 40% during the 17th century. While an old puzzle, this is somewhat difficult to reconcile with evidence that real wages fell 25% or more from 1500 to 1700.

The central finding of this paper, i.e. that the decades around the turn of the 19th century saw a dramatic lifting of the Malthusian constraint, reflected as an explosion of MSPG, is unaltered. Our results hold for each of the used data sources and are also robust to the employment of different estimation methodologies.

F Additional Tables and Figures

	Families	Average Income	Share of income from land	Total income from land
High titles:				
Temporal Lords	200	6060	80%	969,600
Temporal Lords	200	6060	80%	969,600
Spiritual Lords	26	1300	80%	27,040
Baronets	800	1500	80%	960,000
Knights	600	800	80%	384,000
Esquires	3000	562.5	80%	1,350,000
Gentlemen	15000	280	80%	3,360,000
Secular professions:				
Persons in offices (greater)	5000	240	20%	240,000
Persons in offices (lesser)	5000	120	20%	120,000
Persons in the Law	8062	154	20%	248,310
Agriculture:				
Freeholders, greater	27568	91	66%	1,655,458
Freeholders, lesser	96490	55	57%	3,015,313
Farmers	103382	42.5	50%	2,196,868
Cottagers and paupers	313183	6.5	50%	1,017,845
Total		54,440,248		15,544,433

Sources: Lindert and Williamson (1982).

Table 13: Income derived from land

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