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A bioeconomic MPA study based on cellular automata population growth and distribution

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ABSTRACT

This paper investigates possible biological and economic effects of using marine sanctuaries as a management tool, employing cellular automata techniques to model biological growth and area distribution, assuming open access to the fish stock resources outside the protected area. The cellular automata model incorporates a fish harvest model based on standard assumptions. In agreement with previous studies this study confirms that large protected areas are necessary for significant impact on stock conservation, given standard assumptions. The conclusion may however not be equally unambiguous when employing more realistic scenarios, assuming non-uniform distribution of biomass and fishing effort. This study shows that significant stock conserving effects could be obtained even when less that 10% of the total distribution area of the stock is protected from fishing activities.

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1. Introduction

As the word suggests, the idea behind Marine Protected Area (MPA) is to protect marine ecosystems from human activities which may cause destruction or other undesired impacts on environment or species located in the area. Such destructions may have negative economic consequences not only for the environment and ecosystem, but also for fisheries, tourism or other industries depending on the natural resources. How these industries are affected depend on the biological properties of the ecosystem, how the MPA is implemented, control and surveillance, as well as the economic dynamics of industries exploiting the natural system. Controversial issues may be placement and size of the protected area, type of protection, and restrictions on economic activities within the protected area. Given the complex and dynamic interrelation between natural systems and economic activities and the uniqueness of each such relation, it is challenging to establish general solutions fitting all cases. This paper presents a theoretical study of an idealised system including fish migration along a coastline where a part of the coast line may be closed for fishing while there are open access fisheries elsewhere.

Marine protected areas involves different issues within different disciplines, consequently a number of different modelling approaches are found. A vast number of publications on MPA related issues are found in resource economics and conservation biology, but there are few influential contributions aiming

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to merge the different disciplinary traditions. Several papers refer however to these differences in modelling approaches. Fisheries biologists have been criticised for including harvest in a simplified manner, excluding economic dynamics (Smith and Wilen, 2003), while economists may express biological growth and spatial dynamics too simplified. Grafton et al. (2005) points at some of these problems. Interesting attempts of cross-over models exists, as by Hilborn et al. (2006), including fleet dynamics though excluding economic behaviour, or Smith and Wilen (2003) who include spatial distribution of biomasses, but not the important interaction between spatially distributed fishing activities and the spatial distribution of fish biomass. While biologists of obvious reasons emphasise the biological dynamics, the interests of economists of equally obvious reasons are on the economic dynamics (or rather equilibriums). The first (biological dynamics) may be studied in greater details when simplifying the fishing activity to a single fishing mortality rate, while the latter (economic dynamics) may lead to conventional biological modelling within the framework of standard bioeconomics.

Most bioeconomic publications on MPA issues assume MPA to be no-take zones. Several studies based on deterministic models, indicate that MPA in fact has limited value as a management tool (Hannesson, 1998; Conrad, 1999). The use of MPA reduces the net revenues, and the conservation effect seems to be weak unless very large areas are included in the marine sanctuary. Conrad (1999) argues however that deterministic models are less useful to reveal the true value of MPA regulation. He therefore introduced a model with stochastic growth and found the variance of fish stock biomasses to be reduced after introducing marine sanctuaries.

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Hannesson (2002) arrived at the same conclusion, using a stochastic spatial distribution of stock components inside and outside the MPA.

From a methodological point of view the spatial component appears to be most challenging when studying the economic effects of MPA regulation. Nevertheless the spatial dimensions in most bioeconomic studies has been largely ignored or highly simplified. The most common modelling approach has been to define two homogeneous stock components, one within the MPA and the other within the open area; assuming an interaction between the two areas through density driven migration (metapopulation modelling). Different solutions exist on how to handle the environmental carrying capacity of the two stock components and differences in the final conclusions seem in some extent to reflect differences in modelling approach and basic assumptions. Previous bioeconomic studies include continuous and discrete time models and investigate the effects of open access fisheries as well as regulated fisheries maximising present value of net revenues over time.

A even wider range of modelling approaches are found in biological studies on stock-harvest interaction in cases of protected areas, also including the use of cellular automata modelling techniques. Moustakas et al. (2006) utilise cellular automata methodology to model spatial distribution while including stochastic elements, to evaluate the effects of closed areas. Fishing is included as a learning process by vessel movements between neighbouring cells based of fish stock densities. Apart from harvest production, economic modelling is not included and the fleet dynamics is controlled by presence of fish, willingness to move and fleet density constraints, not by economic performance. A recent study by Silvert and Moustakas (2011) discusses the performance of using one large versus several small MPAs while employing a 1D cellular automata model with similarities to the one presented in this study. Moustakas and Silvert (2011) presents a corresponding 2D model assuming, as in Moustakas et al. (2006), the objective of the fishing fleet to be to maximise catch. Based on bioeconomic theory and assuming an open access fishery (Gordon, 1954), the objective of each fishing unit is in this study assumed to be to maximise net revenue.

Different modelling traditions have developed alongside each other in fisheries biology and economics, both dealing with the same fisheries management issues, including the use of MPA, from the angle of different disciplines. The relationship between the two regarding the impact of MPA regulations has been the focus of several papers. Grafton et al. (2005) aims to bridge the divide between the two traditions, but do not address the modelling challenge of spatial distribution in particular, even though this part appears to be one of the core problems of bridging the gap between the traditions. The spatial behaviour is however the main focus of Smith and Wilen (2003) in their case study of a sea urchin diving fishery. They criticise biological models for not including economic behaviour and illustrate in their case study how the effect of economically rational behaviour differs from the results while ignoring these dynamics. In an empirical study of a snow crab fishery Swain and Wade (2003) describe how fishing effort are more intense in areas closer to home port, than in other areas with similar or higher densities of crab. Obviously they are observing an effect of economically rational behaviour. Salthaug and Aanes (2003) also discuss and model the spatial distribution of a fleet on the basis of the relation between catchability, fish density and fleet concentration, rather than including economic measures directly. There is an assumption of rational behaviour, but these models have not been capable of expressing this in economic terms. Unfortunately they have not been offered much help from the bioeconomic literature, since the modelling tradition there more is into dynamic optimisation and metapopulation modelling than integrating more complex biological dynamics and sophisticated spatial distribution models.

Cellular automata methodology is a relatively new approach in the modelling of complex systems, utilising certain arbitrary rules specifying how the automaton develops (Wolfram, 2002). The cellular automaton consists of cells and an initial state of each cell. The state variables of a cell evolve over time due to predefined rules and the initial state of the cell and its neighbouring cells. Simple rules may create complex patterns as the automaton defined by a fixed number of cells evolves by each computational step. The theoretical idea of cellular automata (CA) dates back to von Neumann, 1966 and was developed further by Stephen Wolfram and others in the early 1980s as a discrete time model with stages of evolvement in space and state (Wolfram, 2002). The basic idea of stepwise development through computational calculations also opens for the inclusion of continuous state variables within each cell, referred to as continuous cellular automata (CCA; see Wolfram, 2002).

Darwen and Green (1996) claims that cellular automata methodology is a better approach to model a population in a landscape than models based on partial differential equations are. CA and CCA models are consistent with empirical experiences of spatially heterogeneous population densities with local extinctions and local booms (Darwen and Green, 1996). The simplicity of cellular automata methodology also reduces the computing time compared with corresponding models based on numerically solving differential equations. Balzer et al. (1998) provides an overview over the first decades of development of cellular automata ecological models. During recent years there has been a vast number of publications within this field and cellular automata models now are virtually covering all areas of biological modelling as well as, in fact, most other modelling areas. This development also offers new possibilities of including economic rational behaviour of harvesting units, utilising economic theory and merging the two modelling traditions.

This study is an early attempt on moving in this direction, presenting deterministic models where the fish stock biomass essentially develops through simple CCA rules, also influenced by fishing activities. Fleet dynamics is related to economic performance, following standard assumptions of bioeconomic dynamics where the marginal changes of fishing effort are proportional to net revenue of previous time period.

The two alternative biological growth models are presented (1) a model proposed by Wolfram (2002; page 157, here referred to as CCA) and (2) a straight forward cellular automata representation of a discrete time logistic growth equation (here referred to as LCA). A slightly different representation of logistic growth was also studied by a cellular automata rule in Darwen and Green (1996). Their model was however a 2D lattice while the models presented here is a 1D row of cells evolving over discrete computational steps.

In this study the expression MPA is used on an area where fishing activities are prohibited, while open access to the fish resources is assumed outside the protected area. This simplification means that MPA here is employed in its most extreme version, together with the other extreme, the open access fishery. The term MPA is chosen because it is commonly used in the literature, but the only type of MPA considered in this study is marine sanctuaries.

2. Biological model

Assume a finite number of cells in a row and connect the first and last cell, resulting in a circular representation of cells as illustrated in Fig. 1. The cells may represent the coast line of an island. Each cell holds a specific fish biomass, being the state variable of the cell. The total stock biomass then is the sum of biomasses in the finite



Fig. 1. A finite number of cells (here 25 cells) in a circular connection (above) illustrates the spatial distribution and relationship in the model. In the following graphics the circle is represented by a row of cells (below), assuming the first and last cell to be connected.

number of cells. The initial biomass vector with *n* elements (cells) is

$$\mathbb{b} = (b_1, b_2, b_3, \dots, b_n) \tag{1}$$

b evolves over time as a function of a simple CA rule involving a growth rate (g) and assumed diffusion pattern. The diffusion pattern is controlled by the range parameter r which determines the number of neighbouring cells involved when calculating the new state variable of each cell (Wolfram, 1984). If r for example equals 1, the biomasses of 1 neighbouring cell on each side of the current cell, together with the current cell itself, determine the new biomasses of the cells. More specifically implies the CCA rule employed here that biomass b_2 (state variable of cell 2) next step with range r = 1, is one third of $b_1 + b_2 + b_3$ plus the growth given by the growth rate g. In the CCA model only the fractional part of the new biomass will remain in the cell, therefore

$$0 \le b_i \le 1 \tag{2}$$

for $1 \le i \le n$. The growth rate (g) gives the percentage growth per unit of time. The biomass growth then is expressed by

$$b_{i,t+1} = \left\{ \frac{g+1}{2r+1} \sum_{j=i-r}^{i+r} b_{j,t} \right\},$$
(3)

{_} representing the fractional part of the expression within the brackets $(0 \le \{.\} < 1;$ example: $\{1.2\} = 0.2$; while $b_{n+1,t} = b_{1,t}$ (connecting the circle, see Fig. 1), $g \ge 0$ (a non-negative growth rate) and $r \ge 0$. As the growth rate (g) is constant, biomass growth is linear until the collapse level of b = 1 is reached, where only the fractional biomass value remains within the cell. Hence the long term average biomass (in absence of fisheries mortality) is 1/2.

Natural mortality is expressed indirectly by the remaining fractional part, reflecting a density dependent mortality determined on cell level (local collapses). The biomass vector is a discrete function of time at given initial biomass value (b_0), here on the basis of (3) expressed by the continuous cellular automata rule

$$\mathbf{b}_t = CCA(\mathbf{b}_{t-1}) \tag{4}$$

The corresponding discrete logistic growth equation is

$$b_{i,t+1} = \frac{g+1}{2r+1} \left(1 - 2\sum_{j=i-r}^{i+r} b_{j,t} \right) \sum_{j=i-r}^{i+r} b_{j,t}$$
(5)

represented by the cellular automata rule

/

$$\mathbf{b}_t = LCA(\mathbf{b}_{t-1}) \tag{6}$$

Total biomass at time *t* is

$$B_t = \sum_{i=1}^n b_{i,t} \tag{7}$$

According to Wolfram (2002) CCA has its equilibrium biomass $B_{\infty} = n/2$ when r > 0. Biomasses calculated by Model (4) are presented in Table 1 for the case of 13 cells, g = 1/2 and an initial biomass of 1 is placed in the mid cell. Table 2 displays the corresponding biomasses of Model (6) distributed on 7 cells and with g = 3/5.

Graphical examples of Model (4) when varying growth rate (g) and diffusion pattern (r), are shown in Fig. 2 when a initial biomass in a single cell of a total of 99 cells develops over 100 time steps for different growth and range parameters. One of the cases displayed in Fig. 2 (r=1 and g=1/2) is shown numerically in Table 1.

Growth Model (4) is displayed in Fig. 2 both as biomass patterns (left hand panel) and as total biomass development over time (B_t) in a more traditional way (right hand panel). Corresponding biomass developments are shown in Figs. 3 and 4 for Models (4) and (6), with a randomly distributed initial low biomass (in Fig. 2 the initial biomass is placed in a single cell). The figures show how diffusion and growth properties affect biomass variations over time for 100 (Fig. 3) and one million (Fig. 4) cells.

If the number of cells (n) is increased and time span (t) prolonged, the graphical presentation displayed in the left hand panel of Fig. 2 soon proves to be less useful than more traditional graphical presentations of the development of the total stock biomass over time, as shown in the right hand panel of Fig. 2. The solid curves in Figs. 3 and 4 represent the CCA model, while the dashed curves represent the logistic growth model LCA. The figures reveal lower stock biomasses in Model (4) than in Model (6) for r = 0, while increased biomass fluctuations seem to be associated with increasing r values. These fluctuations may further to be damped by increasing growth rates (g). For r=0 the stock biomasses of Model (4) do not reach the levels of Model (6) unless g is having very high values (beyond 0.75 in the given parameter setting). The simple explanation is that without diffusion (r=0) the effect of additional growth (based on a fixed percentage) in cells with low biomasses in biomass terms are less than the corresponding growth of cells with large biomasses. While diffusion efficiently levels out this asymmetry, it becomes apparent when there is no diffusion. It may be regarded more as a theoretical issue than a real problem since there will always be some diffusion, not only due to the biological properties of the species, but there will also be physical reasons for diffusion to certain extent be present. While Fig. 3 displays total biomass development over time with a spatial distribution over 100 cells, Fig. 4

Table 1

Cellular biomasses calculated by Model (4) for g = 1/2, r = 1 and b_0 given by the first row (t = 0). The table corresponds to the first period of the pattern shown in the mid column of the first row of Fig. 2.

t	b_1	b_2	<i>b</i> ₃	b_4	b_5	b_6	<i>b</i> ₇	b ₈	b_9	<i>b</i> ₁₀	<i>b</i> ₁₁	<i>b</i> ₁₂	b ₁₃	В
0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
1	0	0	0	0	0	1/2	1/2	1/2	0	0	0	0	0	3/2
2	0	0	0	0	1/4	1/2	3/4	1/2	1/4	0	0	0	0	9/4
3	0	0	0	1/8	3/8	3/4	7/8	3/4	3/8	1/8	0	0	0	27/8
4	0	0	1/16	1/4	5/8	0	3/16	0	5/8	1/4	1/16	0	0	33/16
5	0	1/32	5/32	15/32	7/16	13/32	3/32	13/32	7/16	15/32	5/32	1/32	0	99/32
6	1/64	3/32	21/64	17/32	21/32	15/32	29/64	15/32	21/32	17/32	21/64	3/32	1/64	297/64

Table 2

Parameter values used in the simulations displayed in Figs. 6 and 7.

Parameter	CCA and LCA	Description
r	2	Range, number of affected neighbouring cells on each side
g	0.5	Biological growth rate
п	100	Number of cells
q	1	Catchability coefficient
d	0, 1, 8	Parameter controlling fishing effort distribution
р	10	Unit price of harvest
С	5	Unit cost of effort
a	0.002	Fishing effort stiffness parameter

displays the corresponding picture in the case of one million cells. The impacts of increasing growth rates and diffusion ranges are striking. Increasing the number of cells makes the curves smoother, comparing Fig. 3 with Fig. 4. Increased diffusion increases biomass fluctuation, while mean biomass equilibrium is not affected by the diffusion properties, except in the case of r = 0 (no diffusion).

From a biological point of view diffusion of fish stocks may be explained as behavioural adaptation to varying environmental conditions, as the experienced prey densities may increase and (or) the corresponding predator densities decline by proper migratory behaviour. Both causes (targeting food and avoiding predators) involves relations to fish of the same species as well as well as to other species. As previously indicated the only significant difference in overall performance of the two models is found in the case of no diffusion (r = 0). Increasing positive r values increased the rate of adaptation and adapting capacity, but in order to link this to the dispersal abilities of different species, it has to be related to the physical basic cell size, since the range number refers to number cells on each side of the cell which initially holds the migrating biomass.

Models (4) and (6) express biomass growth as discrete time processes on micro level, within and between cells. The total biomass is indirectly determined by the growth functions at micro level, but in a non-transparent and pseudo-random manner, though the average biomass of neighbouring cells in the long run may be a good proxy of the overall total biomass of the stock (at least when harvest is not included). Diffusion (determined by the range parameter r) contributes in the long run to level out biomass differences between cell clusters placed in different areas, reaching a relatively stable total biomass level. In the present study all cells are assumed to share the same local carrying capacity of 0.5, with a collapse limit of 1.



Fig. 2. CCA Model (4) of varying diffusion properties at constant growth with an initial condition of one single biomass ($b_{49} = 1$) in the centre cell of 99 cells (n = 99). The growth rate (g) is 0.5 and the diffusion property given by the range parameter r, indicating number of influenced neighbouring cells. The figure includes 100 computational steps (t = 100) displayed in the horizontal axes. The left panel shows the biomass of each cells distributed vertically, while the right hand panel shows the total biomass of each computational step in the nine cases.



Fig. 3. Biomasses over time in a CCA Model (4) (solid curves) and LCA Model (7) (dashed curves) of varying diffusion properties (*r*) at growth rates (*g*) with the same random initial cell biomasses and 100 cells (*n* = 100). The vertical axes of each case measures biomass and the horizontal axis time. The figure includes 100 time steps (*t* = 100).

3. Fishing regulated by closed area

The CCA and LCA models presented above represent biological growth and spatial distribution. Now fishing is introduced to the model, restricted by the marine protected area (MPA) and market economic constraints. The control of compliance with the regulations is assumed to be perfect and no cost of control and surveillance are considered. The stock biomass within a MPA is given as a subset of the biomass vector $\ensuremath{\mathbb{b}}$,

$$\mathbb{b}_{\mathsf{MPA}} = (b_s, \ldots, b_{s+m-1})$$

where *s* is the first cell and *m* is the number of cells included in the MPA. Absence of protected area is regarded being a special case of MPA regulation (no closed area; m = 0). The model circularity



Fig. 4. Biomasses over time in a CCA Model (4) (solid curves) and LCA Model (7) (dashed curves) of varying diffusion properties (*r*) at growth rates (*g*) with the same random initial cell biomasses and 1,000,000 cells (*n* = 1,000,000). The vertical axes of each case measures biomass and the horizontal axis time. The figure includes 100 time steps (*t* = 100).

makes the choice of *s*-value insignificant, hence s = 1 is assumed in the following. The MPA biomass vector then simplifies to

$$\mathbb{b}_{MPA} = (b_1, \dots, b_m), \\ 0 \le m \le n \text{ and } s = 0 \text{ when } m = 0$$

$$(8)$$

Fishing activities target biomasses in the non-protected area (NPA), represented by the complementary subset of \mathbb{b}_{MPA} in the biomass vector \mathbb{b}

$$\mathbb{b}_{\text{NPA}} = (b_{m+1}, \dots, b_n). \tag{9}$$

Total targeted biomass at time t then is

$$B_{\text{NPA},t} = \sum_{i=m+1}^{n} b_{i,t} \tag{10}$$

A common assumption in harvest production models is that the stock-output elasticity is equal one (Eide et al., 2003). A useful property which follows as a consequence of this assumption is a linear relationship between the fishing mortality rate and fishing effort per unit of time. Most studies on stock–effort–harvest relationships indicate however that this relationship seldom is linear, and empirical studies suggest that the stock-output elasticities are more likely to be around 1/2 than close to 1 (Hannesson, 1983; Eide et al., 2003). Fish harvest production is therefore in this study assumed to have a stock output elasticity of 1/2, while harvest (*h*) is assumed to be linear in fishing effort

$$h_{i,t} = q e_{i,t} \sqrt{b_{i,t}},\tag{11}$$

when $b_i \in \mathbb{B}_{NPA}$ and e_i is the fishing effort of cell *i*. Total fishing effort is the sum of the fishing effort of all cells

$$E_t = \sum_{i=m+1}^{n} e_{i,t}$$
(12)

and total harvest is given by

$$H_t = \sum_{i=1+m}^n h_{i,t} \tag{13}$$

The spatial dimension in the cellular automaton opens for different ways of spatial distribution of fishing effort. A distribution rule based on stock biomass distribution is given by the expression

$$e_{i,t} = \frac{b_{i,t}d}{\sum_{i=1+m}^{n} b_{i,t}d} E_t,$$
(14)

where distribution parameter d (where $d \ge 0$ is expected) controls to what extent biomass distribution affects the distribution of fishing effort. The special case d=0 gives a uniform distribution of fishing effort independent of biomass distribution. The case of d=0is in line with most of the published bioeconomic MPA studies published, usually separating the stock into two components (open and closed to fishing). Uniform distribution of fishing effort (d=0) simplifies the expression of the fishing effort with each cell to the constant ratio

$$e_{i,t} = \frac{b_{i,t}0}{\sum_{i=1+m}^{n} b_{i,t}0}, \ E_t = \frac{E_t}{n-m}$$
(15)

while d = 1 gives an effort distribution perfectly reflecting the stock biomass distribution. The cases of d > 1 which could be called smart fishing, where the distribution of fishing effort takes advantage of the biomass clustering, successfully targeting the most biomass dense cells, In the extreme situation of $d = +\infty$ all fishing effort is placed in the single cell holding the highest biomass.

More sophisticated distribution rules (or simple cellular automatons, for example as proposed by Moustakas et al., 2006) may take into consideration distribution histories (of biomass and/or effort) and include constraints on local densities and growth of effort. It could be argued that the distribution model proposed above (Eq. (14)) connects to history through the time path of total fishing effort, as will be explained in the following.

Including harvest Model (11) in the biological growth Model (3) yields

$$b_{i,t+1} = \left\{ \frac{g+1}{2r+1} \sum_{j=i-r}^{i+r} b_{j,t} \right\} - h_{i,t}$$
(16)

Eq. (4) is adjusted accordingly and the complete CCA model including harvest (by the fishing effort E) and MPA regulation (by the MPA size variable m), is expressed by

$$\mathbb{b}_t = \mathsf{CCA}(\mathbb{b}_{t-1}, E_{t-1}, m) \tag{17}$$

m being the number of MPA cells and E the total fishing effort. Corresponding expression in the logistic case is from Eq. (5) found to be

$$b_{i,t+1} = \left(\frac{g+1}{2r+1}\left(1 - 2\sum_{j=i-r}^{i+r} b_{i,t}\right)\sum_{j=i-r}^{i+r} b_{j,t}\right) - h_{i,t}$$
(18)

and rule (6) is modified accordingly,

$$b_t = \text{LCA}(b_{t-1}, E_{t-1}, m).$$
(19)

4. Economic model

The harvest Eq. (11) involves fishing effort (*E*) which is assumed to have a fixed unit cost *c*. The unit cost *c* also is assumed to include the sum of opportunity costs of all input factors in the production of fishing effort. Further a constant unit price of harvest (*p*) is assumed. The net revenue of harvest (*NR*) then is

$$NR = pH - cE. \tag{20}$$

Since normal profit is included in the unit cost of effort, *NR* more precisely is the total economic rent (abnormal profit) obtained in the fishery. If an equilibrium solution exists, *NR* represents the resource rent of that equilibrium; outside equilibrium a large part of the rent obtained may be labelled *quasi rent* following the definition by Marshall (1893).

The dynamics of an open access fishery is determined by the economic performance of the fleet and how fast it adjusts its fishing effort and how fast the stock biomass adjusts to the changes in fishing effort. The marginal changes in fishing effort may be positive or negative depending on positive or negative fleet net revenue. Since normal profit already is included in the total cost of effort, normal profit is obtained from the harvest production process when NR = 0 while economic rent is earned when NR > 0. In case of the latter the fishing effort should increase, as input factors earns higher pay-offs in the fishery than outside. In this study marginal changes in fishing effort is assumed to be proportional to NR, expressed as a discrete time process by

$$E_{t+1} = E_t(1 + aNR_t) \tag{21}$$

a is the adjustment (stiffness) parameter and represents an intrinsic rate of change in effort. The value of *a* is determined by a number of factors, first of all at which rate capital can be replaced to adjust for market perturbations. A further discussion regarding the parameterisation of *a* is however outside the scope of this study.

Cost of fishing effort (c) is decomposed on cells by fishing effort (Eq. (14)) and net revenue of the cell. Hence net revenue (economic rent) of cell i is

$$nr_i = ph_i - \frac{b_i^d}{\sum_{i=1+m}^n b_i^d} E$$
⁽²²⁾

or simply

$$nr_i = ph_i - \frac{E}{n-m}$$

in case of d=0 (also seen from Eq. (15)). Global net revenue is expressed by

$$NR = \sum_{i=1}^{n} nr_i \tag{23}$$

The parameter values used in the simulations are shown in Table 2. Initial biomass vector in all the performed simulations includes 100 cells evolving over 2000 time steps. Time unit and cell size is not defined, hence the simulations may cover a wide range of different annual growth rates and actual migratory performance (Fig. 5).

5. Results

The biological models presented above (CCA and LCA, Eqs. (17) and (19)) include the biological and spatial parameters g, r, d and n. The economic part of the bioeconomic model includes the parameters q, p, c and a, while the MPA-size parameter m represents the management tool of this study.

Let the two state variables (biomass vector \mathbb{b} and total fishing effort *E*) have initial the values \mathbb{b}_0 and E_0 . Fig. 6 (CCA) and Fig. 7 (LCA) display how fishing effort (horizontal axes) develop over time due to changing economic rent (vertical axes), for different MPA sizes and effort distributions by two different values of *d*; 0, representing a uniform distribution of fishing effort and 8, representing smart fishing or effort clustering.

Fig. 8 presents some statistical properties of the time series of effort, harvest, biomass and net revenue, from Figs. 6 and 7 (for d=0 and d=8 while r=2), as Box–Whisker plots with connected median values. The figure indicates a trend of increasing biomass by increased percentage closure (increasing MPA size) in an open access fisheries and a corresponding decline in fishing effort and harvest in both the CCA and the LCA model. The changes seem to be more pronounced in the case of LCA and more vague in the CCA case. In addition to the two r/d combinations (2/0 and 2/8) displayed in Fig. 8, Fig. 9 also includes combinations (0/0) and (2/1). From the figure it is easy to see that in the LCA case of r=0 and d=0the open access fishing effort becomes zero at a MPA sizes beyond 70%, and a corresponding negative biomass impact on MPA sizes just below this level. In all other cases the open access solutions implies positive fishing efforts and a almost a linear increase in stock biomass by increasing MPA size in an open access fishery. While the values on r and d seem to have a significant impact on the slope of this linear relationship in the CCA cases, this is not so in the LCA cases (except for the special case of r = 0 and d = 0). Interestingly, though not unexpected since uniform effort distribution of both biomass and effort is assumed in non-spatial models, the biomass development of the LCA and the CCA cases of d=1 (where effort distribution exactly is reflecting the distribution of biomass) show the same pattern.

Figs. 10 and 11 display how respectively biomass and resource rent cluster according to MPA size. The chosen values of r and d also connects to the simulation results shown in Fig. 9. The clusters are presented as dendrogram plots involving different *r-d-*combinations of the CCA model (the upper panels) and the LCA model (the lower panels).

Further analysis of the model, also in other areas of the parameter space, could be done by utilising the online version of the model, published at Wolfram Demonstration Project (http://demonstrations.wolfram.com/CellularAutomataModel-OfAnMPAFishery/).

6. Discussion

This paper presents a deterministic spatially heterogeneous model (according to the categorisation proposed by Sumaila and Charles, 2002) based on simple principles of cellular automata modelling combined with a more traditional harvest production model under the assumptions of open access to the fish stock resources and MPA management. Fishing effort is distributed on the basis of biomass distribution, controlled by a knowledge or smartness parameter, *d*.

Standard deviation of the average biomass (shown for the CCA model in Fig. 9, but visible for both the CCA and LCA model in Figs. 7 and 8) is decreasing by increasing MPA size, suggesting larger stock biomass fluctuations by smaller MPA size. This is consistent with the findings by Conrad (1999) for the stochastic model he applied, where also variance differences between open and closed areas are discussed. It is not obvious how the effect of increased smart fishing (higher d values) is in this respect. Probably there are two counteracting effects, since increased *d*-value reduces the fishing area by concentrating the fishing activity, but over time the fleet may cover a large area since the biomass density varies, also as a function of previous fishing. The open access fishery also creates effort fluctuations since no equilibrium solution is established, also influencing the causes of this. The fleet increases and decreases as a function of profitability in the fishery, which generates biological consequences in different areas. As the area open to fishery diminishes the fluctuations also are reduced, as a function of the reduced probability space of fluctuating effort and biomass. Decreasing biomass variance related to increasing MPA size in particular causes corresponding changes in fluctuations of harvests and net revenues (economic rent) in the CCA case. The average values remain however quite stable over large ranges of MPA sizes, different from in the LCA case (see Figs. 7–9).

Fleet dynamics related to net revenue fluctuations causes limitcycle patterns in the LCA Model (19) (Fig. 7), while the CCA Model (17) displays pseudo-random patterns (Fig. 6), with decreasing fluctuations by increasing MPA sizes. Similarly the limit-cycles displayed in Fig. 7 increase in ranges by increasing MPA size up to a closed area of about 25%, from which the limit-cycles contract by increases closed area. Fig. 8 suggests that a similar pattern may be found in the CCA Model (17), though less visible due to the pseudo-random feature of CCA.

Figs. 6 and 8 also display interesting internal differences in the open access dynamics between the two distributions of effort (d = 0 and 8). While d = 8 fishing activities effectively are targeting areas with high biomass densities aiming to almost maintain the level of fishing effort by increasing MPA size. In the case of a uniform distribution of fishing effort (d = 0) the open access effort exhibits larger fluctuations and lower average values. Obviously the assumption regarding effort distribution is critical for the model results. It should be noted, as previously mentioned, that results obtained by models which are not including any spatial dimension in principle corresponds to the current results of d = 1, rather than d = 0 (if r > 0), since the lacking spatial dimension also implies a uniform distribution of biomass.

Optimal size of the protected area has been the focus of many authors, both from a biological (conservation) perspective and from



Fig. 5. Basic flow of Models (17) and (19), based on parameter setting and the initial value of the state variables.

an economic point of view. As biologists tend to emphasise the biological system in empirical studies and since economists put more weight on human behaviour and theoretical models, it seems to be more common in biological MPA studies to discuss the actual physical size of the protected area (typically in terms of km², see for example Claudet et al., 2008 and Halpern, 2003), while economists more or less in unison measure MPA size in percent of total area. A biodiversity study by Rodrigues and Gaston (2001) links the two concepts of physical and relative size to number of species and number of sites. Their study is based on bird distribution, but the complex relationships between absolute size, relative size, number of species and local communities they describe, are probably equally relevant for aquatic ecosystem. Economic utilisation of such ecosystem further complicates the picture.

It appears to be a considerable gap between the empirical biological MPA studies and bioeconomic studies based on metapopulation dynamics modelled by systems of differential equations. To include all this aspects in a modelling exercise is however neither easy nor particularly useful. A range of modelling approaches, with varying focuses and simplifications, may all represent valid and useful ways forward, depending of the aim of the exercise. There is no need for including all aspects, in fact it would not lead to a useful model. But when spatial issues are the focus of a study, it appears to be a doubtful approach to simplify the spatial aspect to only be represented by two biomass points (units) interacting with each other through density driven migration. Nevertheless this appears to be the most common approach in bioeconomic MPA studies (see Conrad, 1999; Armstrong and Reithe, 2001; Hannesson, 1998, 2002; Lauck et al., 1998). Migration between the two areas is not a border phenomena propagating into the two areas causing gradients, biomass clusters and empty locations, but is modelled as if it was liquid placed into two connected chambers. When removing some liquid from one chamber, its filled in from the other by gravitation rather than by individual decision behaviour by the elements (in the liquid molecules which in an aquatic system could be individual fishes, constituting the total stock). Vessel distribution becomes no issue, as the open area in the model has no distribution.

It seems to be a rather robust conclusion of a number of bioeconomic MPA studies utilising the modelling approach described above, that the MPA area needs to be rather large in order to benefit the soundness of the stock and the economy of the fishery. Armstrong and Reithe (2001) and Boncoeur et al. (2002) are examples of this, Beattie et al. (2002) present a review over several other studies arriving at the same conclusion, suggesting large MPAs. The hypothesis of the modelling exercise presented in this paper, is that the finding that MPAs need to be large to do the work, to a large degree could be influenced by the lack of spatial resolution in the applied models. This hypothesis could not be rejected by this study. On the contrary, this study shows that in cases where biomass and fishing effort is equally distributed, both the CCA and the LCA models arrive at the same conclusion. But in other cases this conclusion is altered.

The spatial distribution of biomass in the cellular automata models are controlled by the range parameter r. In the case of no biomass diffusion between cells (r=0) it is clearly seen from Figs. 2–4 that the stable biomass level of the CCA model never reaches the theoretical equilibrium level of number of cells divided by two (0.5 in each cell) as in the LCA model.

Uniform effort distribution combined with a heterogeneous distribution of biomass may have devastating negative stock effects, causing even small MPA sizes to have significant conservation effects, as seen in the CCA model of this study and reflected both in Figs. 8 and 9. The negative stock effect of covering the complete open area by fishing activities is shown to be dramatic, but equally unrealistic, first of all by economic reasons. Technological development makes the fleet increasingly efficient in catching efficiency and fish finding capacity, the latter is probably increasing the value of *d* far beyond 1, as indicated by other studies (Swain and Wade, 2003; Ellis and Wang, 2007).

Increasing growth rate (g) improves the fit of the CCA model, moving the stable biomass closer towards the equilibrium level and reducing the time of recovery. The LCA model seems to be more robust towards changes in fishing patterns controlled by the parameter d (Figs. 6 and 7), while the CCA model displays significant changes by varying values of d. It is simply easier to take advantage of increased knowledge about biomass distribution when this distribution is non-uniform. The biomass distribution of the LCA model is a priori expected to be less heterogeneous than what is found in the CCA model since it follows a continuous growth pattern within each cell, while the CCA model embeds local collapses and recoveries (booms and blasts). These expectations are also confirmed in the performed simulations. This difference between the two models vanishes in the case of d = 1 where the distribution of



Fig. 6. CCA Model (17) on varying percentage of MPA cells and two fishing effort distributions, d = 0 (upper panel) and d = 8 (lower panel). The range parameter r is 2 in all cases. Parameter values used are displayed in Table 2. In all graphs the horizontal axis measure total fishing effort (*E*, covering the range of 0–60), while net revenue (NR, the range of -250-150) is measured by the vertical axis. The period displayed in each graph counts 2000 time steps.

fishing effort perfectly reflects biomass distribution and the average biomasses in the CCA model get very close to the biomasses obtained by the LCA model. For other values of *d* the CCA model seems however always to come up with larger biomass values than those found by the LCA model. This result may question the effect of partial area closure as an effective mean of stock conservation, often put forward as one of the main concerns when implementing MPA regulation. The stock conservation effect of MPA regulation comes out to be less, and in some cases almost negligible in the CCA model compared with the LCA model. Smart fishing and high expertise in targeting areas of high fish density (d > 0) contributes in stabilising the stock and reducing the risk of stock depletion, increasingly so with increasing values of *d*. In addition to the positive stock effects of effort clustering, there are also economic benefits up to a certain point. As the value of *d* is approaching infinity, the whole fishing fleet will be placed into the single cell currently holding the largest biomass. Hence the total catch could not exceed the stock biomass of this cell, illustrating that increased fish finding capacity not necessarily leads to increased catches, even in the short run. The immediate (short



Fig. 7. LCA Model (19) on varying percentage of MPA cells and two fishing effort distributions, d = 0 (upper panel) and d = 8 (lower panel). The range parameter r is 2 in all cases. Parameter values used are displayed in Table 2. In all graphs the horizontal axis measure total fishing effort (*E*, covering the range of 0–15), while net revenue (NR, covering the range of –20–20) is measured by the vertical axis. The period displayed in each graph counts 2000 time steps.

run) catch therefore reaches a maximum value at some value of d within the range $0 < d < \infty$. It should be noted that there also is a positive stock effect in the long run even though the economic benefits contribute in increasing the level of open access effort and hence increasing the stock pressure.

The seemingly stock conserving effect of increased ability to target and approach the fish dense area (cells) compared with uniform distribution of fishing effort origins from the fact that the latter represents a situation where all cells are targeted, even though the fishing effort distributed on each cell is low. When the fishing effort



MPA size in percent of total area

Fig. 8. Box–Whisker plots of the time series presented in Figs. 6 and 7, showing trends and variances in fishing effort, harvest, biomass and net revenue for different MPA size in Models (17) and (19), respectively the CCA and the LCA model. The range parameter r = 2 in all the simulations. The curves connecting different MPA sizes in each plot shows median values, while the mean values are indicated by horizontal black lines. Each box includes 50% of the sample and the range covered by box and lines includes 95% of the data sample.

is concentrated in some cells, biomass growth in the other cells is not negatively affected. The distributing of increased biomass from these cells contributes to recovery in the cells whit depleted biomass by previously high fishing pressure. The increased fishing pressure in some cells caused by smart fishing also leads to reduced fishing pressure in less attractive areas. The total effect of this needs to be studied in greater details and it seems to be crucial to include fishing effort distribution in fisheries models not only because of the biological effects, but also for the impact it may have on the economic performance of the fleet. In a new not yet published study by this author, the distribution of fishing effort is linked to the economic performance in a 2D model, including biomass distribution (relevant for income) as well as distance from port and other costs of fishing. The same pattern is obtained while increasing the value of d, as described above. This present study indicates that the performance of MPA regulation in both aspects (stock conservation and economic benefits) has to be understood as functions of fish and fishers' behaviour, the latter including fish finding activities and effort clustering in areas with high fish densities.

As further seen from Figs. 6 and 7, as well as in Fig. 8, net revenues fluctuate between positive and negative values (quasi rent due to the definition by Marshall, 1893). In the long run periods of positive rent may more than compensate for the losses in the periods of negative rent. The stiffness parameter *a* determines the rate at which the fishing effort adjusts according to the economic performance of last period. The adjustment rate depends on how easily input factor in production may be moved from one place to another. As labour increasingly is substituted by capital in effort production, the stiffness parameter is expected to decline, since labour more easily is moved than capital items (vessel, gears, etc.) Changing the stiffness parameter *a* within a realistic range of course changes the absolute values, but the main pattern caused by the permanent adjustment to changing economic performance remains as long as a > 0.

The fluctuations in net revenues are reduced by increased MPA size. Since an open access equilibrium is characterised by no rent (as only normal profit is obtained), the rent obtained in the open access dynamics is accidental rent when the level of fishing effort is adjusting to constantly changing stock biomass. This quasi rent causes the effort to increase or decrease with a rate determined by how fast capital could be moved into or out off the fishery. In the models presented here this rate is given by the stiffness parameter a. assuming equal entry and exit rates. In the real world it is probably easier to enter than to leave a fishery, as the entrance may come from all capital sources, while the capital already bound into a specific fishery has limited possibilities of other placements in the short run. Based on the findings in Eide (2007) a higher entry rate than exit rate seems however not to alter the finding of this study. Fluctuating biomasses give reason for corresponding changes in fishing effort, depending on these entry and exit rates. Even slow growing species outstrip the speed of growth in effort both when increasing and decreasing biomass. Since the fishing effort in some sense is running after the stock biomass, the effort is less than corresponding open access effort when biomass level is peaking (causing



Fig. 9. Average values of biomass, harvest, effort and net revenue of each of the four cases (columns) obtained by CCA Model (17) (solid curves) and LCA Model (19) (dashed curves) from simulations running over 500 time steps. The shaded areas indicate the standard deviations of simulations by the CCA Model (19). Parameter values are found in Table 2, except for *r* and *d* which values are indicated in the figure.

positive rent) and higher when the biomass is reaching low level (causing negative rent). The negative rent in the real world of economic rational behaviour however has a minimum level where the contribution margin becomes zero and the fleet is better off staying in harbour. While this possibility is considered in Eide (2007), it is not included in this study. By correcting for this error the quasi rent obtained in the open access fisheries of this study could be even higher.

The cluster analyses displayed in Figs. 10 and 11 represent inputs in the discussion on determining the optimal MPA size. As previously mentioned most bioeconomic studies on this issue conclude that the MPA size needs to be considerable to give stock conservation effects corresponding to rent maximisation (see for example Lauck et al., 1998; Armstrong and Reithe, 2001; Boncoeur et al., 2002; Beattie et al., 2002; Helvey, 2004). These results are confirmed in Figs. 10 and 11 for the LCA model in cases fitting the common assumptions of uniform effort distribution (r=0) as well as in the case of r=2 and d=1, which actually also represents the case of uniform distribution of effort (since the biomass is actually uniformly distributed even at range 2 in the LCA model). In both these cases there are two large clusters divided at MPA sizes below and above 70-80% of the total area, which confirms the findings of the previous bioeconomic studies mentioned above. Also the CCA model displays a similar pattern for r=0 and d=0, under which parameter setting the CCA model has an almost uniform

distribution of biomass and effort. In this case the stock biomasses and net revenues in the CCA model cluster at the extreme MPA sizes of below and above 80–90% of the total area.

More interesting though is that except for the rather unrealistic cases referred to above, none of the other findings from the CCA model support the hypothesis that a large fraction of the total area need to be protected in order to obtain an optimal economic performance while utilising MPA as a way of regulating the fishery. On the contrary, the effect of closing relatively small areas (3–15% of the total area), is significant on both stock biomass and economic performance. This indicates that the common assumption of uniform distribution of stock biomass and effort outside the protected area dramatically affect the evaluation MPA sizes.

The concept of cellular automata represents a simple way of modelling spatial distributions. A range of other ways of modelling spatial distribution exists, and a number of previous MPA studies have utilised some of these methods. There is however another special feature of cellular automata modelling which is not equally easy achieved by other modelling techniques. Cellular automata implements micro dynamics at cell levels and is a bottom-up modelling approach. The biological models proposed in this study could also include cell specific parameter values in addition to the state variables. Varying environmental capacity of different cells could be implemented by varying cell specific growth rates and/or saturation levels (biomass maximum). A further study of the robustness



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Fig. 10. Dendrogram plots showing clustering on MPA size (horizontal axis) of the biomass data from the simulations presented in Fig. 9. The numbers at the horizontal axes represent the percentage area closed by MPA regulation. The upper panel shows clustering on MPA size in the CCA Model (17), while the lower panel shows the corresponding clustering pattern in the LCA Model (19). The simulation includes 500 time steps (data set length) and 100 independent runs of each of the 8 cases shown in the graph. The vertical axes indicate the squared Euclidian distance between the measured data sets.



Fig. 11. Dendrogram plots showing clustering on MPA size (horizontal axis) of the resource rent data (including catch and effort data) from the simulations presented in Fig. 9. The numbers at the horizontal axes represent the percentage area closed by MPA regulation. The upper panel shows clustering on MPA size in the CCA Model (17), while the lower panel shows the corresponding clustering pattern in the LCA Model (19). The simulation includes 500 time steps (data set length) and 100 independent runs of each of the 8 cases shown in the graph. The vertical axes indicate the squared Euclidian distance between the measured data sets.

of MPA regulation under different environmental conditions and distribution rules should also include spatial environmental variation. The models presented in this study represent a possible way forward to perform such investigations. The theoretical study presented in this paper is not linked to any specific real fishery. The aim has been to investigate some theoretical concepts regarding model design and common issues regarding implementation of MPA regulation in relation to fisheries, the issue of MPA size being the most important. In principle any real fishery could be represented by the model, taking in consideration that the parameter space is larger than the few single points investigated in the simulations presented here. A large range of different fisheries could however be covered by the fraction of the parameter space utilised in this study, first of all since cell size and time unit is not specified here. A time unit of several years could in principle cover slow growing species. Cell size combined with the range parameter *r* could be specified to fit the theoretical physiological and physical diffusion range a species may have within the chosen time frame, or a more accurate distribution area based on empirical studies may be used to specify cell size. The total distribution area is covered by increasing the number of cells sufficiently.

A further development of the model is now in progress. An extension from a 1D row to a 2D lattice also involves a shift in the effort distribution proxy from being biomass distribution to be theoretical (possible) net revenue distribution. This follows from the fact that there will be differences in cost by approaching different areas in the 2D lattice, by varying distance from home port. By such minor changes a very flexible and general model may be available, which because of its simplicity makes it possible to investigate issues which normally represent major modelling challenges, like seasonal growth, seasonal fisheries and seasonal variations in markets.

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