

**Intertemporal Choice of
Marine Ecosystem Exploitation**

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Abstract

The term “Fishing Down Marine Food Webs” describes the gradual transition in landing from marine ecosystems towards organisms lower in the food web. To address this issue and the need to manage the marine ecosystem in a broader perspective, Ecosystem Management is recommended. Ecosystem Management, however, requires models that can link the ecosystem level to the operation level, so this paper examines an ecosystem production model and shows that it is suitable for applying ground rent theory. This model is the simplest possible that incorporates the principles of size as the main determinant of the predator–prey interaction, the inclusion of mass balance in the predator–prey allocation, and mortality and somatic growth as consequences of the predator–prey allocation.

The model needs to be parameterized for the specific ecosystem and the price and cost functions must be established empirically before drawing the conclusion that Fishing Down Marine Food Webs is economically detrimental can be established directly. Nevertheless, the model does reveal a need for intertemporal balance with respect to both fish size and harvest volume. These aspects are not addressed in any systematic way at the ecosystem level in the present management. Therefore, economic predictions for an ecosystem managed as a common pool resource must be that the exploitation probably are conducted at lower sized than optimum. In addition, given its population stock approach, the present management probably overlooks the ability of an ecosystem to sustain total volume of harvest.

Given the two aspects of intertemporal choice revealed by the model, the conclusion must be that the Fishing Down Marine Food Webs is probably driven by the current management’s inability to conduct adequate intertemporal balancing; therefore, it is probably detrimental from an economic point of view. The marine ecosystem therefore requires an ecosystem management for economic reasons; in this context, models like the one presented here can serve as useful planning tools.

Contents

1	Introduction	7
2	Methods and Results	11
	2.1 The size-based ecosystem model	11
	2.2 Ground Rent Theory	16
	2.3 Methods	21
	2.4 Experiments	22
3	Discussion	28
	References	33
A	Technical details	35
	A.1 Details of the Control space	37
	A.2 Details related to output space	38
	A.3 Partial derivatives	40
	A.4 Capital value calculations	40

1 Introduction

The marine ecosystem seems to be degenerating; Pauly et al. (1998) found a decline in the mean trophic level of global landings reported to FAO in the period 1950–1994. The term they used for the gradual transition in the composition of these landings from long-lived, high trophic¹ piscivorous fish to short-lived, low trophic level invertebrate and planktivorous fish was “Fishing Down Marine Food Webs.” Based on their models, Christensen et al. (2003) established that catches of predator fish in the North Atlantic increased in the late 1960s from 2.4 to 4.7 million tonnes annually but then declined to below 2 million tonnes annually in the late 1990s. The biomass of high trophic fish in the North Atlantic declined by two-thirds during the last 50 years and is now a ninth the size it was a century ago. In addition to this decline in the biomass of high trophic fish, other unintended consequences of fishing, such as habitat destruction, incidental mortality of non-target species, evolutionary shifts in population demographics, and changes in the function and structure of ecosystems, are becoming increasingly recognized (Pikitch et al., 2004). To address the degrading of the marine ecosystem, a management of the marine ecosystem in a broader perspective, Ecosystem-Based Fishery Management, is recommended by Pikitch et al. (2004) and seven quoted references. Many other terms refer to the same idea, and Garcia et al. (2003) discusses some of these terms, including Fishery Management, Ecosystem Management, Ecosystem Approach, Ecosystem-Based Fisheries Management, Ecosystem Approach to Fisheries, and Integrated Management.

The ecosystem concept is inseparable from the idea of an organizational hierarchy in nature. Along with management comes the idea of strategic planning, where long-term, large scale plans dominate short-term, more detailed plans. The present paper uses the term “Ecosystem Management” to refer to the management of the marine ecosystem in a broader context, and interprets Ecosystem Management as a management theory that merges the idea of an organization hierarchy in nature and the idea of strategic planning based on a planning hierarchy using the ecosystem as the strategic planning level.²

In the marine ecosystem, operations are executed by fishers catching fish at an aggregate level not bigger than a shoal. A prerequisite for successful Ecosystem Management is the ability to create a quantifiable link from the strategic level, the ecosystem, to the operation level, fish at an aggregated level not bigger than a shoal.² In this

¹Trophic means “relating or pertaining to nutrition and feeding”. Trophic level refers to the position in the food chain.

²The arguments in this paragraph, as well as other arguments referring to this footnote, build on a more comprehensive discussion in Ravn-Jensen (2009a) that will be supplied upon request to the author.

paper, the model of Benoît and Rochet (2004) is extended with economics and numerics to create this link.³ A renewable resource, such as a forest or a marine ecosystem, gives its management a choice between exploiting the resource by, for example, cutting a tree or catching a fish, or leaving that tree or fish to grow (or become food for a larger fish). Exploiting the resource generates an immediate income, whereas refraining from exploiting the resource allows it to be exploited in the future. If refraining from exploiting the resource improves future income, the act can be seen as an investment, and there is an intertemporal choice between exploiting or investing. Consequently, the forest or the marine ecosystem must be treated as capital (in the sense of production machinery) in that it may improve overall productivity if it is not used immediately. This view, where the regenerating resource is treated as capital, is known within forestry as the “Bodenreinertragslehre” (ground rent theory) and goes back to Faustmann (1849) and Pressler (1860); Clark and Munro (1975) applied a similar capital view in the analysis of fish stock. Because the hierarchies of plans under strategic management must link both space and time, a model for strategic ecosystem planning has to provide this hierarchical link and handle intertemporal balancing. The model can therefore only be appropriated if its structure allows the ground rent theory to be applied.

In response to the concern regarding Fishing Down Marine Food Webs expressed by Pauly et al. (1998), Hannesson (2002) simplifies the ecosystem to a “one predator, one prey” system and analyzes a model of the Lotka–Volterra type. Whether the fishing down of the marine food chain is detrimental from an economic point of view is a question of the relative prices of prey and predator species. The global optimum, however, will only be reached through the cooperative management of stocks. Hannesson (2002) intended the model only for a discussion of principles, and his point is crucial: If the ecosystem can be exploited at different levels of the trophic system, there must be some exploitation patterns that are better than others; there will be an economic optimum and related problems when managing the fishery as a common pool resource. To be able to determine this optimum and thus to determine if Fishing Down Marine Food Webs is detrimental from an economic point of view, models of the ecosystem need to be suitable for economic analysis.

For practical applications, the Lotka–Volterra model used by Hannesson (2002) is “woefully inadequate” (Hannesson, 2002, p. 755). The point of Hannesson, that the optimum is a question of relative prices between prey and predator species, stem in

³Only the principles of the model will be described in present article; the details of the model as well, as the numerical implementation and parameterization are documented in Ravn-Jensen (2009b) that will be supplied by the author upon request. An experiment with the ecosystem exploited by a population of adaptive fishers is also described in that paper.

the model from prey abundance has a positive influence on predator growth, whereas predator abundance has a negative influence on prey growth. This seems superficial to be an adequate approach, but in a true economic analysis the model has to reflect the opportunity cost of catching the prey. The alternative to catching the prey is letting it remain in the sea as prey for the predator. In other words, the model has to reflect the production in the ecosystem when predators consuming prey, and predators grow as a result of the consumption.

One other problem with the use of population models as production models is the life story and predator–prey relationship in the marine ecosystem. In the marine ecosystem, most species begin with a small egg that turn into a larva with a mass typically well below 1 g, succeeded by somatic growth⁴ and reaches, in some cases, a size larger than 100 kg. Two fish of the same size but of different species are much more alike with respect to food preference and predator risk than, for example, two fish of the same species but of different sizes (Jennings et al., 2001; Scharf et al., 2000). Because all species come in a continuum of size and because the predator–prey relationship in the marine ecosystem is to a large extent determined by size, it seems futile to construct a production model based on species. Furthermore, population models traditionally operate with population growth, which is a combination of an increase in the number of individuals and the somatic growth of the individual fish. In a production model, predation results in the mortality of the prey and the somatic growth of the predator. The somatic growth leads to individuals growing in size which in turn leads to a dietary shift towards larger prey. In a simple two compartment predator–prey model, the prey will, following somatic growth, advance and become the predator. This flow caused by somatic growth is another opportunity for catching the prey, and a production model must incorporate it.²

Few other attempts to build ecosystem models suitable for economic analysis exist. Ecopath, the model behind both the Pauly et al. (1998) and Christensen et al. (2003) articles, does use the predator–prey interaction to drive the dynamics of the model. Ecopath, however, focuses on modelling the population-community and therefore attempts to build a population–process model. As discussed above, using the species as the division criteria for production functionality seems to be problematic, and a model of this type probably cannot give long term predictions with a reasonably small attractor.² The intention of the Ecopath model is indeed to estimate the energy flow in the ecosystem (Christensen, 1995) rather than to perform long-term economic analysis.

⁴Somatic growth means bodily growth and is different from population growth, which is used in population and community models. Population growth is a combination of an increase in numbers and the somatic growth of the individuals.

Two other examples are Finnoff and Tschirhart (2003), where general equilibrium theory is applied to the predator–prey interaction in a species community model, and Sanchirico et al. (2005), where portfolio investment theory is applied to the stocks of populations. Both of these are attempts to apply economic models to ecosystem components. The Finnoff and Tschirhart general equilibrium model is built on the concept of an input–output matrix in the predator–prey interaction and in this way takes a production view of the ecosystem. There do not, however, seem to have been any attempts to apply capital theory to the model. The model of Sanchirico et al. treats the different populations as investment portfolio objects. The model does not, however, build on a theory of production in the ecosystem. Like the Ecopath model, both models are tied to the population–community view of ecosystem.

The approach of this paper is to perform economic analysis of a model that stems from a theory of production in the ecosystem. As such, this is, to my knowledge, the first attempt to apply ground rent theory to a model of a marine ecosystem.

The model is a size-based production model developed to examine the consequences of fishing in a marine ecosystem as a trophic system. This model’s structure is quite simple, and the only attributes of the fish taken into consideration are size and abundance (density). The model cannot, however, be analyzed analytically by rigorous mathematics but must rely on experiments made using a numerical implementation of the model. The model is, however, so simple in structure that the causal chains are transparent.

At the heart of economic analyse is the intertemporal balancing of exploitation and investment. This aspect can, for a solo owner, be managed by applying ground rent theory. The intertemporal balancing issue is, however, interesting not only for the purpose of finding the optimal point of exploitation given a solo owner with a specific discount rate, but also because it gives insight into the economics of the dynamic side of ecosystems and thereby the economic forces that drive the degrading of an ecosystem under common pool management.

The purpose of this article, then, is twofold. First, the intertemporal balancing of the exploitation of the marine ecosystem is analyzed in the context of the trophic level of exploitation. Second, the details behind the economics are analyzed with the purpose of obtaining a better understanding of the driving forces behind the degrading of marine ecosystem. The structure of the paper is as follows. The size-based ecosystem model, the ground rent theory, the applied method and the results are presented in section 2. Only the main feature of the model is described in the next section. The more technical details of the model are given in a separate paper that will be supplied by the author upon request, but some technical details related to the analytical methods in this paper are given in the appendix. In section 3, the results and their consequences

are discussed along with an attempt to deduce some consequences for management based on the findings of this paper.

An economic analysis can in principle show two extremes. Either it is economically reasonable to fish high in the trophic system, or it is reasonable to fish low in the trophic system. The management of the ecosystem is, however, facing other problems besides the decline in high trophic level fish, including habitat destruction, shifts in population demographics, loss of biodiversity and changes in the function and structure of ecosystems. Many of these problems are related to high fishing pressure. If it is economically sensible to fish high in the trophic system, an economically sensible management will reduce fishing pressure, and many of the other problems facing management and related to high fishing pressure will to some extent be remedied by economically sensible management. On the other hand, the situation is more complicated if an economic analysis shows that the optimum is low in the trophic system because then the collateral effects of fisheries are costs that have to be evaluated or those fisheries may have to be restricted for the protection of the ecosystem. An economic analysis is important in order to know if the current management is economically unwise or if it is economically sensible but fails to value or consider the collateral effects. Accordingly, this paper focuses on the economics related to the exploiting of a marine ecosystem by a fishery.

There are many other anthropic impacts on the marine ecosystem that an ecosystem manager has to consider, such as pollution and nutrition load. This paper, however, is limited to a discussion of the fishery.

2 Methods and Results

2.1 The size-based ecosystem model

The marine ecosystem has as its atomic production unit the individual fish,⁵ and the production itself is the somatic growth of the fish. In order to produce, the fish has to consume other organisms like, for example, other fish. The fish is then a product as well; it can be caught by humans or be internally distributed between other production units. Thus the atomic product of marine ecosystem is the individual fish, and this product may be internally allocated by a predator–prey interaction, or it may be caught by humans as an outlet from the ecosystem. This duality, the fish as both the product and production unit, is a common feature of renewable resources.

⁵The model does not distinguish according to phylogeny, so that a fish can be any organism.

It is impossible to model every single organism in an ecosystem, and the fish has to be stratified appropriately. When the fish is stratified, the internal allocation between production units can be described as an input–output matrix with the strata as both rows and columns. Stratified *appropriately* then means stratified in a manner such the predator–prey interaction matrix is predictable. This predictability can be expected if the fish is stratified according to its function in the trophic system.² In the marine ecosystem, the function of the individual fish, as seen in a trophic context, is closely related to the size of the fish. Two fish of the same size but of different species are more alike with respect to food preferences and predator risk than, for example, two fish of the same species but of different sizes (Jennings et al., 2001; Scharf et al., 2000). Furthermore, the predators in the marine ecosystem are generally considerably larger than their prey, and body size is therefore a rough indicator of trophic level (Borgmann, 1987). In other words, the distribution of individuals with respect to size can be seen as a mapping of the trophic system.

Consequently, the organisms in the sea are stratified in the model according to their body mass m , referred to as their size. The strata, or bins, are made infinitesimally small, transmuting the strata into a continuum of m . The model’s state variable is concerned with the number of fish in the sea of a given size. The state variable $N(t, m)$ gives the density of fish of size m at time t and is referred to as the spectrum. Strictly speaking, density refers to both volume and mass. The density with respect to volume just signifies that the model reflects one representative cubic meter of water, and its unit therefore is per cubic meter of sea. The density with respect to mass signifies that in order to know the number of fish in a size interval between, for example, m_1 and m_2 , the density has to be integrated: $\int_{m_1}^{m_2} N dm$.

In figure 1 the processes of the model are illustrated. The diagram illustrates the population spectrum with size as the abscissa and the density as ordinate; the black line then illustrates $N(m)$, and the pink illustrates that the N is a density and that to know the distribution of the fish, N has to be integrated, hence the area under the curve. The N is drawn as a line; if the two axis are both logarithmic, a pristine ecosystem without a fishery is expected to form a straight line with a slope of approximately -2 (Andersen and Beyer, 2006).

Two black curved arrows go from top to bottom, marked μN at the top and EN at the bottom. These arrows represent the driver of the dynamics in the ecosystem: the predator–prey interaction. This interaction creates a mortality of μN for the prey and leads to the consumption of EN by the predator. To be a production model, the predator–prey interaction must adhere to the principle of conservation of mass. This is in a discrete model secured by an input–output matrix; as the strata turn into a continuum of m , the predator–prey interaction matrix consequently turns into a two-

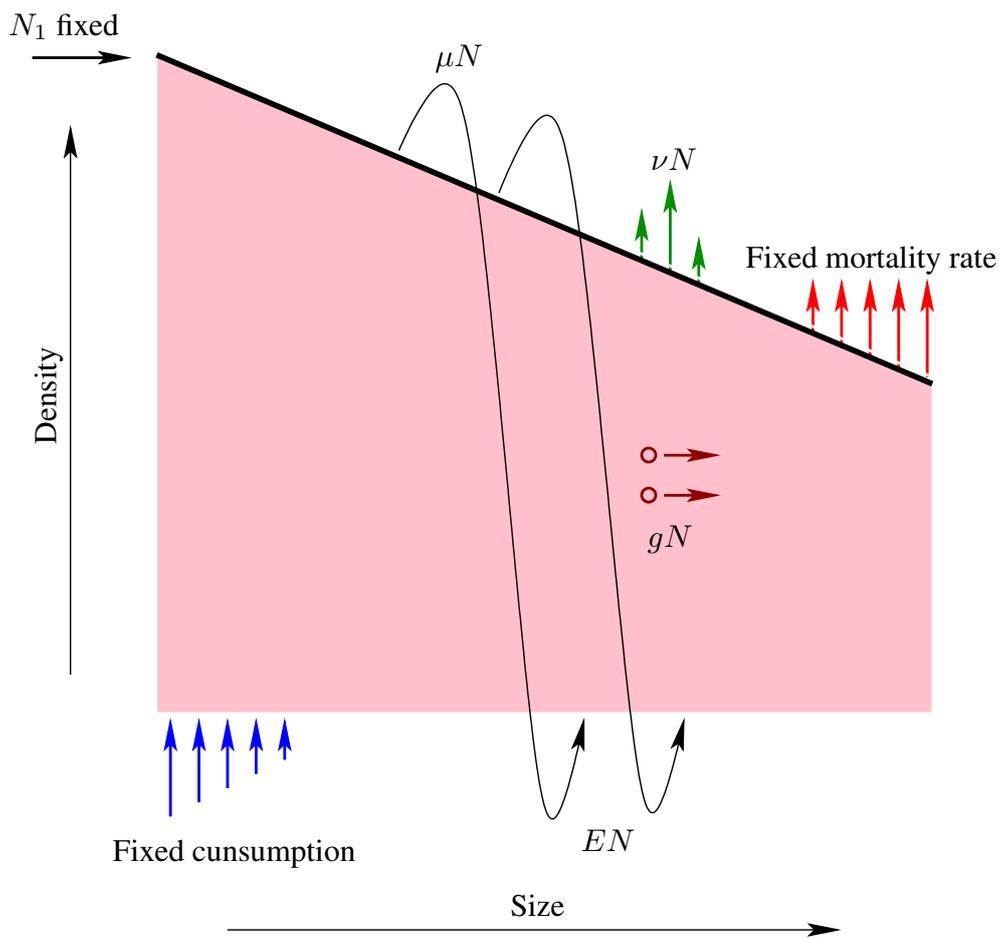


Figure 1: The elements of the model. See the text for explanation.

dimensional interaction density. This approach ensures the conservation of mass in the system so that for every consumed fish there is a corresponding demise.

Consumption leads to the somatic growth g of the predator. As a fish's only attribute in this model is its size, a fish is a point in the m -dimension. When a fish grows and therefore increases in size, this point moves up the m -dimension with a speed of g . The total effect of all the fish with somatic growth is the flow of particles with a flux, the number of particles passing a given point, of gN . This is illustrated in the figure 1 by the dark red points and arrows marked gN . In the model, growth is a consequence of consuming prey $g = g(E)$. The somatic growth is always smaller than the consumed food, and the difference represents fecal waste, respiration and reproduction.

At the top of the figure, green arrows marked νN represent the mortality due to the fishery, and the same mass balance principle applies to this interaction. What is caught has to equal what leaves the ecosystem.

From the predator–prey interaction and fishery interaction, the growth and mortality are derived, leading to the dynamics of the spectrum. As growth leads to a flux gN of particles in the spectrum, the dynamics in the spectrum can be described with a flow equation controlling the state variable:

$$\frac{\partial N}{\partial t} = -\frac{\partial gN}{\partial m} - \mu N - \nu N \quad (1)$$

The partial differential equation (1), known as the Kendrick–von Foerster equation, is the main engine in this model. Turning the equation into a model requires assumptions regarding the predator–prey interaction, the growth function and how the fishery extracts fish from the system—these assumptions follow Benoît and Rochet (2004) and Andersen and Beyer (2006).³ The equation system describes a spectrum with no ends; however, to turn it into a model, the spectrum has to be limited, and the process related to the parts of the spectrum below and above the defined bounds need to be accounted as external factors. In the model, there are three external impacts:

1. Organisms at the lower end of the spectrum will have their main prey outside the spectrum. This is addressed by applying a fixed extra consumption to the lower end of the spectrum, represented in figure 1 by blue arrows.
2. The first point N_1 in the spectrum has to be supplied at every iteration cycle as a boundary condition. In the numeric implementation, it is the equilibrium level without a fishery.
3. Organisms at the upper end of the spectrum will have their main predators outside the spectrum. This is addressed by applying a fixed extra mortality rate to

the upper end of the spectrum, represented in figure 1 by red arrows.

The purpose of this model is to weigh the trade-offs related to fisheries targeting different sizes in the trophic system. The economic inferences made therefore have to relate solely to the dynamics caused by the predator–prey interaction and not to changes in external inputs. Therefore, the inputs to the model are fixed by items 1 and 2 in the list above, and different fisheries cannot affect these inputs. The restrictions on the global inputs to a fixed consumption input at the lower end of the spectrum is equivalent to an assumption that the primary production be fixed and independent of the abundance at higher trophic levels.

The model represents a trophic system and, as such, there is a flow through the system. Inputs enter in the lower end and, in an ecosystem without a fishery, individuals exit through an idle outlet in the upper end in the form of externally-caused mortality. The fishery changes the outlet towards human needs and brings a smaller external mortality. Applying a fixed rate of mortality in item 3, rather just a fixed mortality, implies that when the population density drops as a consequence of a fishery, so does the external mortality.

The fishing mortality in the model is controlled by an effort V and a target size m_f . The effort V is proportional to fishing vessels, and they are fishing *as if* they are targeting fish of a size near m_f . They cannot, however, target the size exactly. The fish are caught around the m_f with a fishing mortality rate proportional to a Gaussian function with respect to the logarithm of m . The breadth parameter of the Gaussian density function is kept fixed at 1, and in the present analysis only one target is allowed in each experiment.

The numerical implementation³ follows Benoît and Rochet (2004) and transforms the size variable logarithmically so that the size dimension in the model is transformed into x , where $\exp(x) = m$. At the same time, the target for the fishery m_f is transformed into ξ , where $\exp(\xi) = m_f$. This transformation is necessary not only for a proper numerical implementation, but also because it mirrors a view with a long tradition in size spectrum analysis that relative size differences are more important than absolute size differences. Henceforth, the size of fish will be referred to as x and the target of fishing as ξ . Furthermore, in the numerical analysis where the analysis relies on small changes in the target, the changes will be small with respect to the logarithmic scale.

Figure 2 is a sketch of how the model is used for experiments. The model has two controls: the target size ξ and the effort V of the fishery. In the present analysis, they are only allowed to be single-valued. When the model is run with a set of controls, the output is a new population spectrum and the harvest summarized by the volume H and

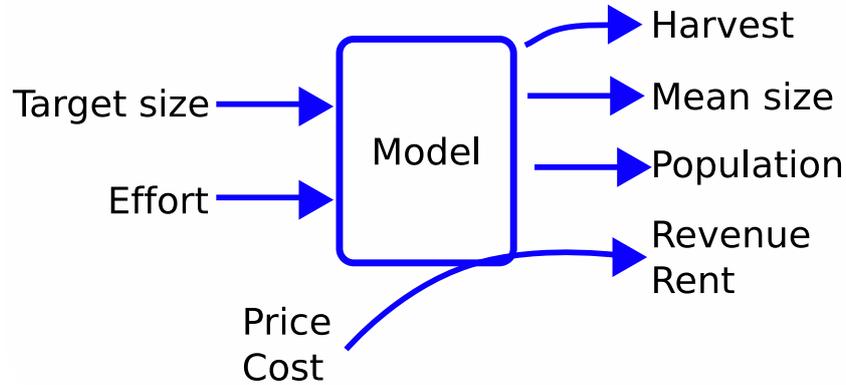


Figure 2: Principle of experiment design

the mean size \bar{x} . When a price and cost function are applied, the model also outputs the revenue and the rent π . The employed price function $P = P(m)$ is a sigmoid function as illustrated in figure 3. The cost is a fixed cost per unit effort, meaning that the total cost is proportional to effort and the harvest is proportional to effort and the density of fish.

An equally spaced grid is defined in the $\xi \times V$ space. For each of the gridpoints, the model is run until it converges to a steady state, and the population structure, the volume of harvest H and the mean target \bar{x} of the catch are recorded together with the rent denoted by $\tilde{\pi}$ for the sustainable yield rent. This steady-state situation is the point of departure for all further analysis. In figure 4, the sustainable yield rent $\tilde{\pi}$ as a function of the controls is shown, whereas in figure 5, the rent as a function of sustainable harvest volume and mean size is shown.

2.2 Ground Rent Theory

In classic economics, the production function has two factors of production: capital K and variable factors L . An increase in K is called an investment, and what is left of the revenues from production after the costs of variable factors and the maintenance of capital are defrayed, is called rent. If this view is applied to renewable resources, the two factors of production are the renewable resource as a capital and other factors as variable input. In the case of the marine ecosystem, the ecosystem is the capital, and the fishing vessels with crews and supplies are the input factors. When the ex-

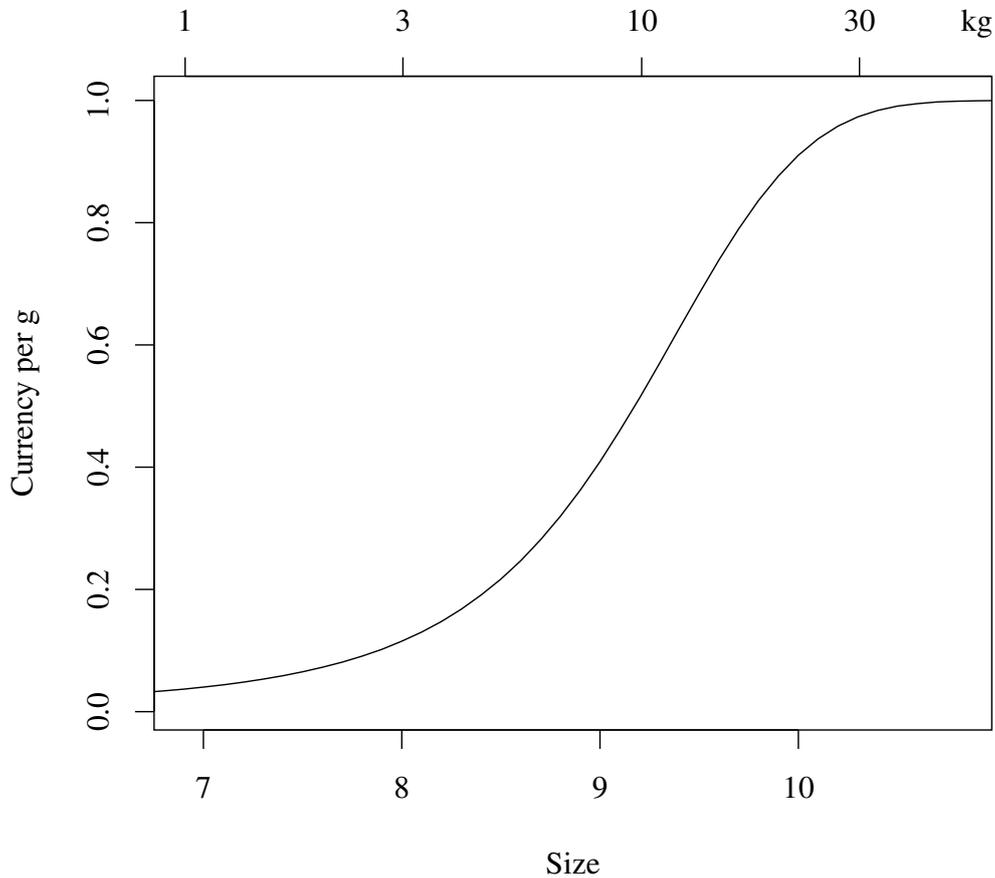


Figure 3: The employed price function.

tracted renewable resources are sold and the cost of the variable factors are defrayed, what is left is a rent from the capital; the resource rent. As renewable resources have the product–production unit duality, investment in renewable resources can, in many cases, be the same as refraining from exploiting the resource. If exploiting the resource is postponed, the future rent may be larger. If the increase in rent in the future is larger than the forgone rent now, the decision whether to postpone extraction is an intertemporal choice.

Faustmann (1849) introduces present value calculation as a tool for the economic valuation of capital,⁶ in Faustmann’s case, the value of an immature forest stand.

⁶The present value calculation was known before Faustmann, see Viitala (2006) for a review of earlier

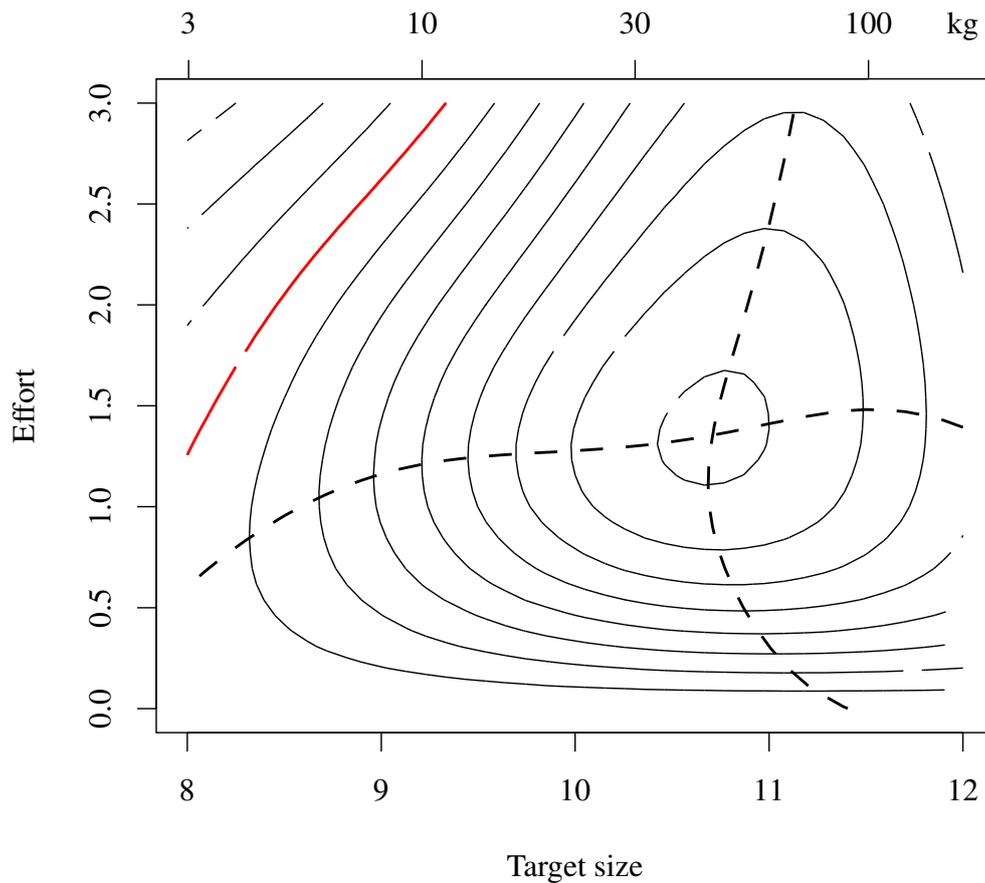


Figure 4: Sustainable rent surface. This diagram shows rent contours in control space. The red line is the zero rent contour, and the stationary point is the maximum rent point. The vertical dashed line indicates the points of maximum rent given a fixed effort, and the horizontal dashed line indicates the points of maximum rent given a fixed target size.

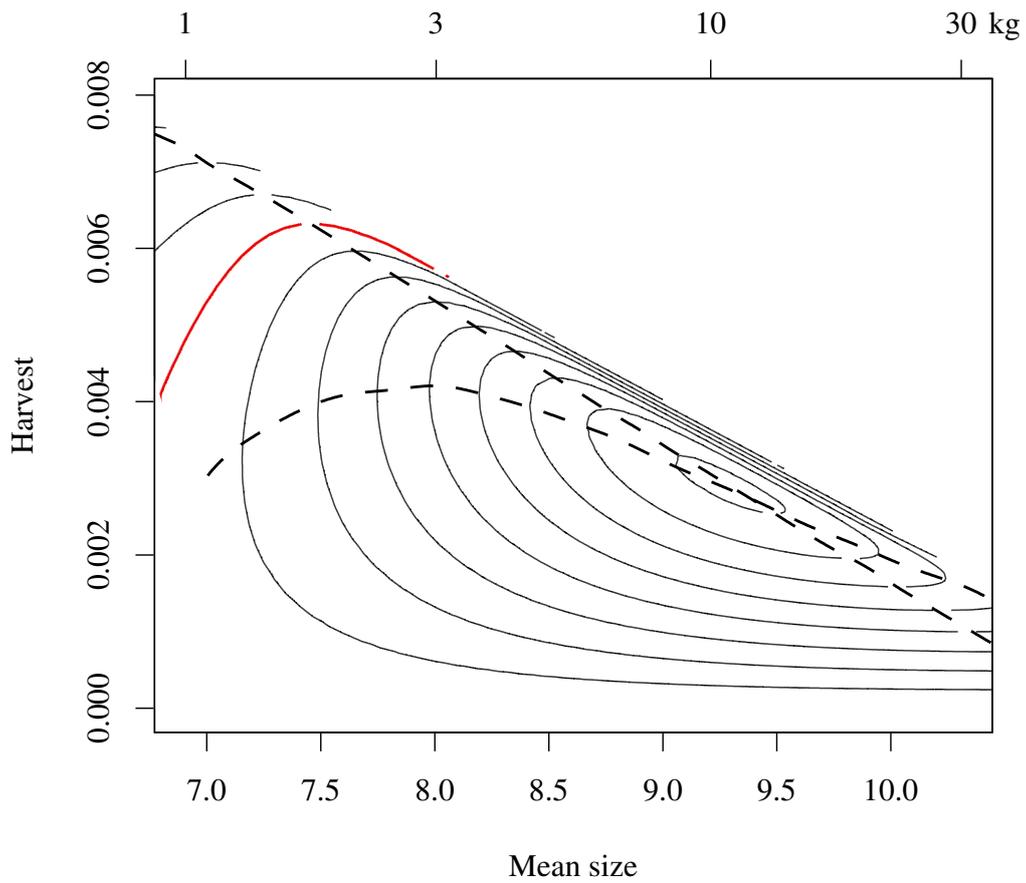


Figure 5: Rent contours in sustainable yield space. The red line is the zero rent contour, and the stationary point is maximum rent. The dashed line from the upper-left corner to the lower-right corner indicates the points of maximum rent given a fixed harvest, whereas the other dashed line indicates points of maximum rent given a fixed mean size.

Faustmann defines the capital value of the resource \mathbb{C} as the present value of all future rent flows:

$$\mathbb{C} = \int_0^{\infty} e^{-\rho t} \pi(t) dt \quad (2)$$

where ρ is the discount rate and π is the rent. The insight Faustmann conveyed by applying (2) was that the value of the forest consists of the expected rent in the future rather than just the net value of the trees. A consequence of this view is that the value of bare land is the present value of the rent that could be earned from a forest planted on the land (or an alternative use of the land). The concept of present value for capital valuation entered general economic theory by Fisher (1906) and represents today a paradigm within economic theory.

After the concept of capital value, the concept of maximizing the capital value as a management objective follows naturally:

$$\text{Objective: } \max_{\mathbf{y}} \mathbb{C} \quad (3)$$

Here \mathbf{y} represents a vector of all possible management controls. This idea was first introduced and solved by Pressler (1860).

The consequences of the Faustmann's view is that the scarce resource is the land where the forest is situated, or more precisely, the time that the forest occupies the land. Therefore the management objective (3) will in forestry lead to an optimal rotation age for a forest stand. Pressler did not find the optimal rotation age by solving the maximization problem directly, instead he argued for the calculation of the Weiser rate⁷ w (indicator rate) as the relative rate of increase in the capital value of a specific stand next year. This indicator rate w is then compared with the discount rate ρ ; if $w > \rho$, the stand is accumulating value faster than the discount rate and is better spared. When $w = \rho$, it will be time for felling. A stand with a $w < \rho$ indicates a overmature stand.

Before Pressler (1860), the ideal of a forest management was the normal forest defined as a forest composed of as many stands as the rotation age used in the forest. One stand is then cleared each year, and in this way there will be one stand of each different age from newly cultivated to mature stands. As one stand of the same size and age is harvested each year, the outlet from the normal forest is the same each year. The Pressler (1860) management objective freed the forest management from

contributions.

⁷Pressler (1860) use the term Weiser-Prozent (indicator percent) .

the normal forest, but the concept of the normal forest is still often used as a model forest. Together with the management objective (3), the normal forest represents the perception of sustainable forestry: A relatively stable production system, a continuous flow of rent and the maximization of the capital value of the production system.

It is the “Bodenreinertragslehre”, the land rent theory described above, that I intend to apply to the marine ecosystem. Instead of the term land rent theory, I shall use the term ground rent theory. Here, “ground” means “the basis,” specifically the basis for the production of the ecosystem; precipitation, sun light, wind, soil, nutrition, water, and so on, together with the pool of genetic information. It is on this ground that the rent originates. As some of the ground is tied closely to a specific site, ground can also mean both the site and the fishing ground; thereby indicating the need to focus on a well-defined physical area.

The paper therefore adopts the management objective as formulated in (3). To be specific, the objective is (3), where the rent π is subjected to the size-based ecosystem model and \mathbf{y} represents the control parameters of the model. It is assumed that the optimum exists with a fixed set of controls \mathbf{y} as opposed to a set of controls that can change over time $\mathbf{y}(t)$. The size-based ecosystem model will, when a fixed set of controls is applied, approach a steady state with a constant outlet. The desired situation is therefore a normal ecosystem, a stable system with respect to production and structure, and a continuous outlet and rent.

2.3 Methods

The first order conditions for the optimization problem (3) are

$$\frac{\partial \mathbb{C}}{\partial y_i} = 0 \quad (4)$$

for all y_i in the control vector. In other word, at the optimum, the change in capital value will be zero for all (infinitesimal) changes in controls. The present model has two controls: ξ and V . According to (4), the set of controls that gives the maximum capital value can be found as:

$$\operatorname{argmax}_{(\xi, V)} \mathbb{C} \in \left\{ (\xi, V) \left| \begin{array}{c} \frac{\partial}{\partial \xi} \int_0^\infty e^{-\rho t} \pi(t) dt = 0 \\ \wedge \\ \frac{\partial}{\partial V} \int_0^\infty e^{-\rho t} \pi(t) dt = 0 \end{array} \right. \right\} \quad (5)$$

I intend perform a comprehensive analysis of the model system and find the optimum points for all discount rates $\rho > 0$. Instead of finding the optimum point for a

given discount rate, an indicator w similar to Pressler's Weiser rate is calculated. As an indicator rate w , the internal rates of return for small changes in ξ and V are used. For all interesting points in the $\xi \times V$ space, the indicator rates are found as:

$$w_\xi = \rho \left| \frac{\partial}{\partial \xi} \int_0^\infty e^{-\rho t} \pi(t) dt = 0 \right. \quad (6)$$

$$w_V = \rho \left| \frac{\partial}{\partial V} \int_0^\infty e^{-\rho t} \pi(t) dt = 0 \right. \quad (7)$$

The controls of the optimum point given the discount rate ρ can then be found as the point where the indicator rate with respect to both controls equals the discount rate:

$$\operatorname{argmax}_{(\xi, V)} \mathbb{C}_\rho \in \left\{ (\xi, V) \left| \begin{array}{l} w_\xi = \rho \\ \wedge \\ w_V = \rho \end{array} \right. \right\} \quad (8)$$

If equation (6) and (7) are substituted into (8), the result is (5) and thus points that satisfies the first order condition (4). The equation (8) will therefore find all relevant candidates for the management object (3).

The use of an internal rent of return has its limitations. If the rent flow is not normal,⁸ there may be more than one root for (6) or (7); a discussion of where normal rent flows can be expected is given in appendix.

2.4 Experiments

As the point of departure for all experiments, a convergence to a steady state is found for every point in a grid covering an appropriate part of the control space $\xi \times V$. From this steady state, an experiment A is run with small but fixed changes in controls for 100 years. The rent $\pi_t(A)$ is recorded for each time step. Another experiment B is performed in the same manner but with the opposite sign on the change in controls. The indicator rate w is calculated as

$$w = \rho \left| \mathbb{C}_\rho(A) = \mathbb{C}_\rho(B) \right. \quad (9)$$

where $\mathbb{C}_\rho(A)$ is the capital value of the rent flow $\pi_t(A)$ calculated with the discount rate ρ .

⁸A normal rent flow only crosses the zero rent line once; running either from negative to positive, or the reverse for disinvestment.

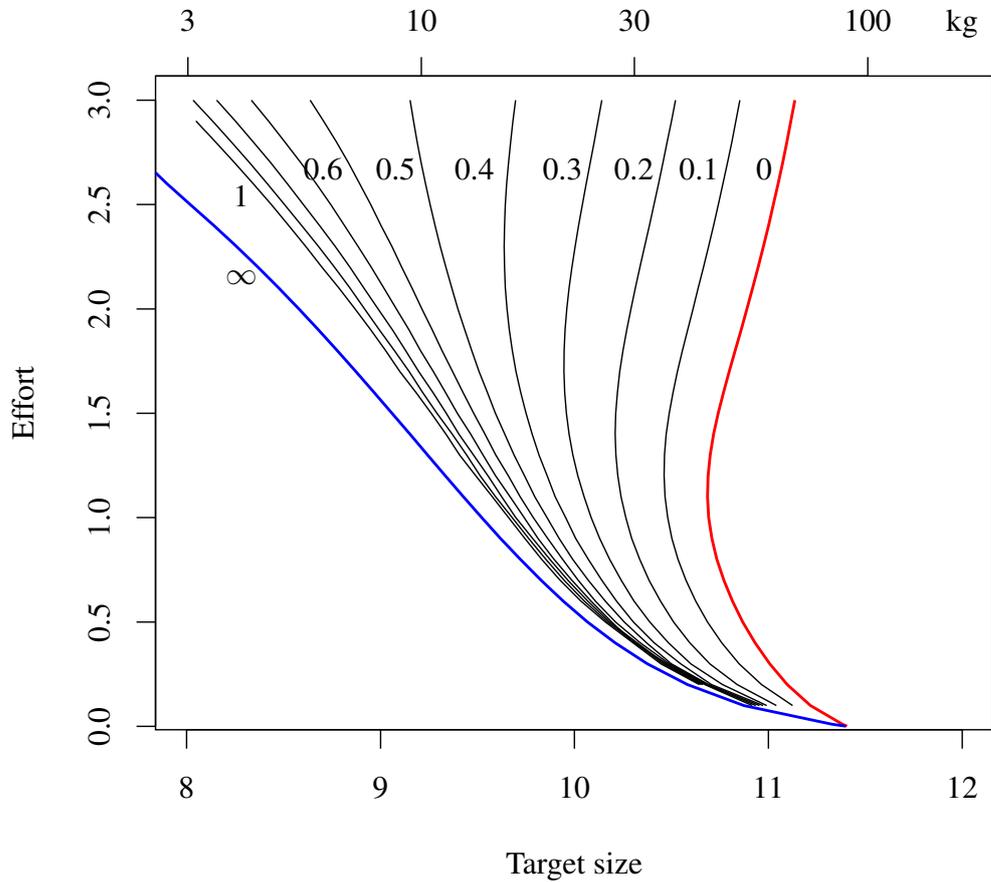


Figure 6: Internal rate of return contours for an increase in target size. The lines indicate points where the internal rate of return w_ξ for an increase in target size given a fixed effort are the same. The numbers indicates w_ξ and the contours lines are in the text labelled l_w^ξ

Control space

First, V is fixed and ξ variable, and the internal rate of return w_ξ is found for all relevant grid points. By interpolation, the lines l_w^ξ through points with same w_ξ are generated. The lines l_w^ξ indicate the points in the control space where an increase in target size is an investment where an internal rent of return of w_ξ is generated by the ecosystem—in figure 6 these lines are shown. To the right of the red line, l_0^ξ an increase in target size will lead to both a short-term decrease of rent and a decrease in long-term

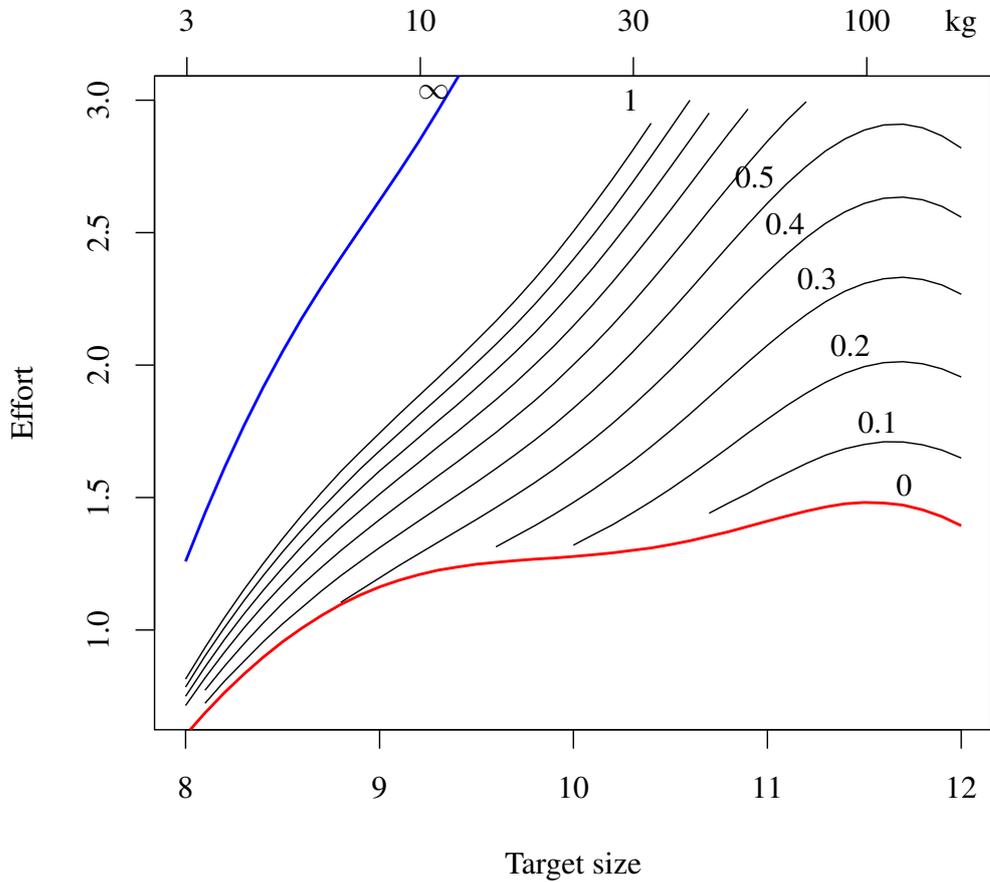


Figure 7: Internal rate of return contours for an increase in effort. The lines indicate points where the internal rates of return w_V to an increase in effort given a fixed target size are the same. The numbers indicate w_V , and the contour lines are labelled l_w^V in the text.

sustainable yield rent as well. The increase in size is then not an investment and there is no intertemporal choice. To the left of the blue line l_∞^ξ , an increase in target will lead to both short and long-term gains. Increase in size is then not an investment, and there is no intertemporal choice.

Next, ξ is fixed and V variable, and internal rate of return is found for all relevant grid points. By interpolation, the lines l_w^V through points with same w_V are generated. The lines l_w^V indicate the points in the control space where a decrease in effort is an investment where an internal rent of return of w_V is generated by the ecosystem—in

figure 7 the lines are shown. Below the red line l_0^V , a decrease in effort will lead to both a short-term decrease of rent and a long-term decrease in sustainable yield rent. The decrease in effort is then not an investment, and there is no intertemporal choice. To the left of the blue line l_∞^V , a decrease in effort will lead to both short and long-term gains. Decrease in effort is then not an investment, and there is no intertemporal choice.

The point where the lines l_w^V and l_w^ξ crosses is, according to (8), a candidate for the optimum point given the discount rate $\rho = w$. There is only one such point for each discount rate, and the monotonicity of w_ξ and w_V reflect the concavity of the capital values. The point where the lines l_w^V and l_w^ξ cross is therefore an optimum point. In figure 8, the blue line indicates the optimum points given discount rates from zero to infinity, with each point marking some specific ρ .

Sustainable yield space

The control space may be a rather academic concept. It is, after all, only variables in a model that are the results of decisions made by fishers and are in practice not measurable. What can be measured is the output; the harvest volume and the size. This is summarized in the model by the output parameters H , the total volume of the harvest, and \bar{x} , the mean size of harvest fish. It may be of more use to know the economics of changes in the sustainable yield harvest volume and mean size. For every gridpoint of the model in $\xi \times V$ space, there is a corresponding steady-state point in $\bar{x} \times H$ space: the sustainable yield space Ω_{SY} . In figure 5, the rent contours are shown in the $\bar{x} \times H$ space. The sustainable yield space Ω_{SY} is clearly restricted to a subset in the lower left part of the total output space. The mapping of (ξ, V) on $\bar{x} \times H$ space by the model is unambiguous, and there is therefore a reverse mapping for all $(\bar{x}, H) \in \Omega_{SY}$ on $\xi \times V$ space.

To analyze the consequences of moving to another point in the sustainable yield space, the controls have to change. In the present analysis, the controls are changed once to the values corresponding to the desired point, this induces fluctuations in the harvest, sizes and the resulting rent.

Similar to the analyses of control space, this analysis is performed by first aiming for a fixed H and a variable \bar{x} , and the internal rate of return is found for all relevant grid points. From these points, the lines $l_w^{\bar{x}}$ are found by interpolation, giving the points in the sustainable yield space where a change in the controls with the aim of increasing mean size is an investment and an internal rent of return of w is generated by the ecosystem.

Next, the aim for \bar{x} is fixed while the aim for H is variable. The internal rate of

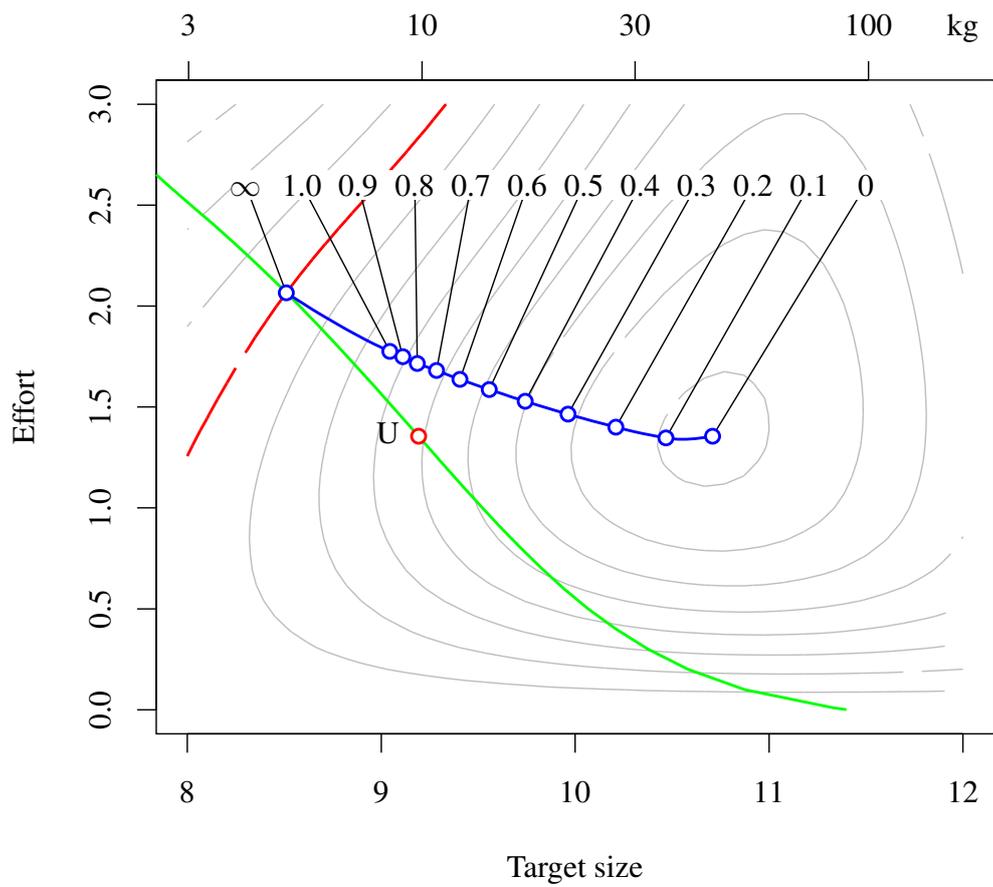


Figure 8: Optimal points in control space. The blue lines and points are optimum points given the discount rate as indicated by the numbers. The grey line is rent contours, the red line is l_∞^V or the zero rent contour, and the green line is l_∞^ξ .

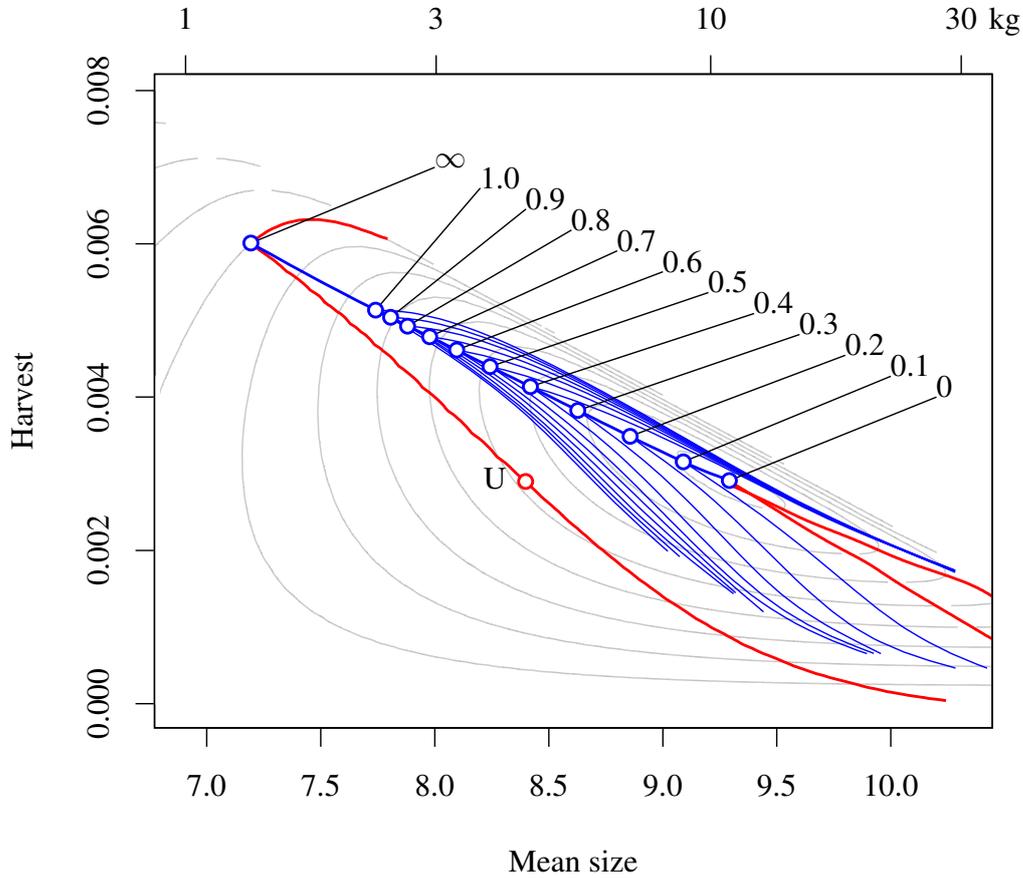


Figure 9: Optimal points in sustainable yield space. The diagram shows the optimal points given a discount rate ρ . The points are found at the cross of the $l_w^{\bar{x}}$ and the l_w^H lines where $\rho = w$. The $l_w^{\bar{x}}$ is shown below the optimal line, and the l_w^H is shown above. The sustainable yield rent contours are shown in grey.

return is found for all relevant grid points. From these points, the lines l_w^H are found by interpolation. These lines give the points in the sustainable yield space where a change in controls with the aim of decreasing the harvest is an investment and where an internal rent of return of w is generated by the ecosystem. The optimal points given a discount rate $\rho = w$ are found where the $l_w^{\bar{x}}$ and l_w^H lines cross, see figure 9. These points correspond to the optimal points found in the control space.

3 Discussion

The present model is the simplest possible using the principles of size as the determinant of the predator–prey interaction, mass balance in the predator–prey allocation and mortality and somatic growth as consequences of the predator–prey allocation. The fish has therefore only one attribute, its size. The model can be enhanced by applying other dimensions important in an ecosystem context, such as space or a life story. A discussion related to details in the model, parameterization, numerical implementation and possibly enhancements are, together with the details of the model and its numerical implementation, found in a separate paper that will be supplied by the author upon request.

This paper applies the “Bodenreinertragslehre”—the ground rent theory—of Faustmann (1849) and Pressler (1860) to the marine ecosystem. The analysis is performed as a normal ecosystem analysis with the underlying management objectives of a relatively stable production system, a continuous flow of rent, and the maximization of the capital value of the production system. The present analysis is therefore performed on a near steady-state system. The concept of an ecosystem in a steady-state situation is, however, an idealization not found in the real world. The present model and the method of economic analysis can handle dynamic situations as well. The dimensions of the controls and outputs will, however, increase as a result of allowing dynamics in the system, and results will be much less transparent. Therefore, as a useful simplification, the normal ecosystem approach is applied, and as the strategic goal can be defined as a steady state, the platonic world of a steady-state ecosystem are useful for strategic planning. The presented experiments show that the size-based ecosystem model has a structure suitable for evaluating the intertemporal choices raised by ground rent theory. The model is therefore suitable as a tool for the strategic planning of marine ecosystems.

The analysis of the size-based ecosystem model determines the optimum given a discount rate in figure 4 and 5. The specific path for optimum points given specific discount rates may seem of little interest because the object for the model is the long-term planning of an ecosystem. Here, discount rates in the interval of 1%–6% is probably appropriated. The whole analysis gives, however, additional knowledge about the economics related to the exploitation of the ecosystem. In figure 9, the lines above the optimum point line give the indicator rates w_H of changing the controls with the aim of reducing the harvest and keeping the mean size constant. Below the optimum point line, the lines give the indicator rates $w_{\bar{x}}$ of changing the controls with the aim of increasing the mean size with the aim for the harvest unchanged. If a investment is made by decreasing present rent, these lines represents the best of the two alternatives

of reducing harvest volume or increasing size. Other combinations of changes in the controls may give higher returns than those indicated by the lines, and so the lines indicate a minimum for the best achievable rate of return the ecosystem will generate in response to an investment made by a temporary decrease in rent. As seen in the figure, there can be expected an extremely high return if management is able to move the exploitation from an inferior point towards the optimum.

The present analysis is performed as a solo owner approach. In the real world, the marine ecosystem is managed as a common pool with different states and many fishermen exploiting the common resource. When there is an intertemporal choice to exploit the resource, the management of a common pool resource faces the problem that the return on an investment made by a single fisher refraining from harvesting has to be shared with all other users. The fisher has thus incentives not to invest but to exploit. Only the point where there is no short term gain from changing the controls will be stable in a common pool resource managed with open access. This point corresponds to an infinite internal rate of return, and fishery economists (Clark, 1985, 1990) traditionally find that the point of an infinite discount rate for a solo owner coincides with the open access equilibrium point. The same is true in this analysis. The infinity point in figure 8 and 9 is the expected point of equilibrium if fishing is conducted under open access and adaptive agents.³ The indicator rates with values lower than infinity in figure 9 then indicate where the management of a common pool will be faced with challenges because the agents, fishers and states, are participants in a game where the returns on investments are not assigned to the investors. This will, as discussed below, apply under management controls where only one of the control variables or output variables is controlled.

The line l_{∞}^{ξ} , the green line in figure 8, indicates the equilibrium points for adaptive agents free to set their target sizes but restricted by a effective effort control. This is an equilibrium with respect to both the ecosystem and the agents' choice of target. If a fishery with many agents is managed by a well-functioning effort control, the controls will in equilibrium end up on the l_{∞}^{ξ} line if this equilibrium is stable. If, for example, the total effort is controlled as the maximum-rent effort, the fishery will end up at point U in figure 8. Effort control will therefore be inefficient.

The line $l_{\infty}^{\bar{x}}$, the lower red line in figure 9, indicates the equilibrium points for adaptive agents free in their choice of mean size of harvest but restricted by a individual control on harvest volume. For example, if there is a total restriction on harvest volume corresponding to the maximum rent harvest, the fishery will end up with a sustainable harvest at point U in figure 9 if this equilibrium is stable. Control on harvest volume only is therefore inefficient.

To get a deeper insight into the results, it is of interest to discuss in more detail

what drives the model. There are two details in the model, namely how production is formulated in the model, and the price and cost functions. As discussed below, these two details together result in an economic force related to a trophic cascade that drives the result.

The model is built to reflect the trophic system and is therefore explicitly driven by the predator–prey interaction. The alternative to catching a fish, leaving it as food for predators or somatic growth, is therefore reflected in the economic analysis in a realistic manner for ecosystem production. The model is simple. The only attribute of the fish is its size, reflecting its position in the trophic system. It is of no surprise that the economic analysis shows that size is important for management, this is, after all, the idea and aim for the model.

The sigmoid price function in figure 3 reflects the idea that larger fish are more valuable than small ones. The exact shape is hypothetical, however, if the price is not increasing with size, the consequence is easy to predict: the catch has to be plankton only. There are not many commercial landings of plankton, so somewhere in the size spectrum the prices are increasing with size. The cost of fishing in the model is proportional to the effort, and the catch is proportional to the effort and the density of fish. This is a very simple functional description, preferred by fishery economics for its simplicity, but it can probably be improved.

The trophic system reaction, together with the price and cost functions, drives the need for intertemporal balancing. This is illustrated in figure 10. The data in the figure represents the maximum rent point, or the optimum point with a discount rate of zero. In the upper panel, the population density relative to the population density in a pristine ecosystem without a fishery is shown in black. The curve shows the population density in a steady state as a result of a fishery with controls corresponding to the maximum rent point. The dashed vertical line indicates the target size, and the pink curve illustrates the shape and breadth of the fishing mortality rate. The population shows what is known as a trophic cascade; the fishery causes some part of the spectrum to increase in density while others diminish. The population decreases at and above the target size, and there is an enhancement below the target size. A change in the target size to a lower target will, because of the increased population density, have an especially high harvest before the population reacts on the change in the controls. The lower diagram shows the instant rent $\overleftarrow{\pi}$, that is, the rent at time $t = 0$ for a change in target size.

In the situation in figure 10, there is an instant gain from lowering the target size that is represented by the negative slope of the instant rent curve where it crosses the dashed line. There is thus a short-term gain from lowering the target, while the sustainable yield rent will be lower after the controls have changed. It is this trophic

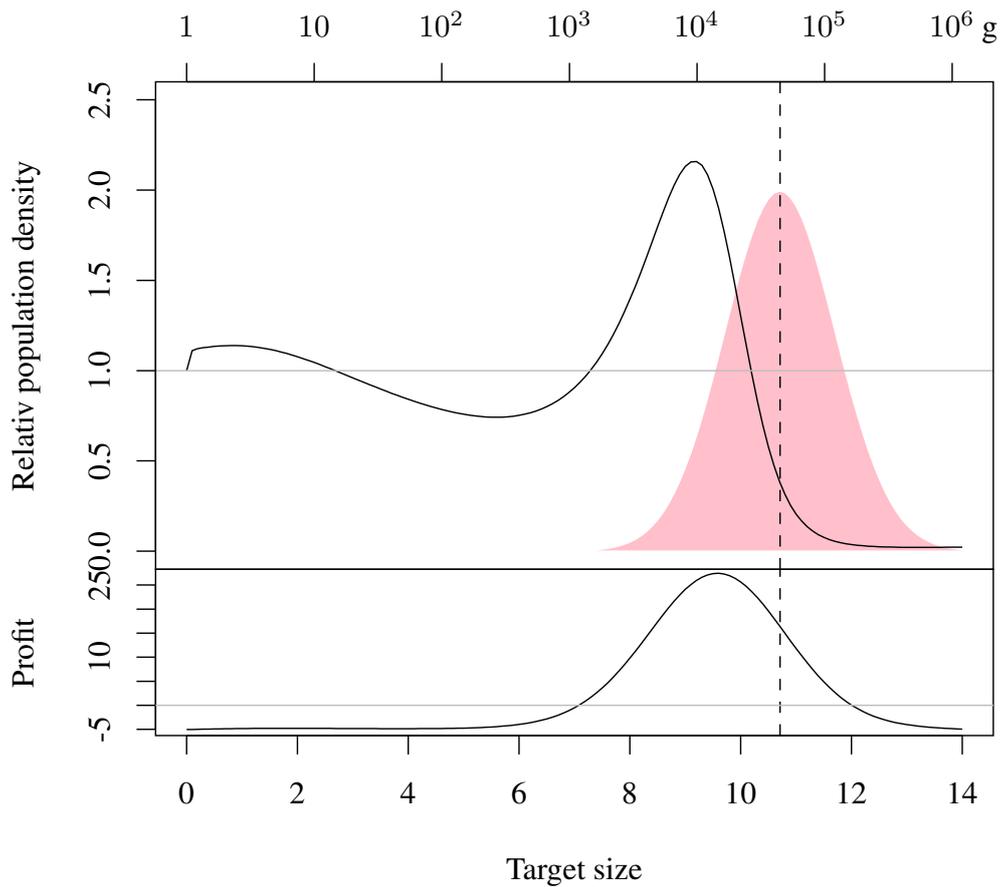


Figure 10: Population density and short term rent. In the upper diagram the population density relative to the population density in a pristine ecosystem without a fishery is shown in black. The diagram represents a steady state with a target size as shown by the dashed vertical line. The pink curve illustrates the shape and breadth of the fishing mortality in the population. The lower diagram shows the rent at time $t = 0$ (the instant rent $\overleftarrow{\pi}$) given the target size is changed.

cascade that, even under perfect effort control or output volume control, will drive the fishery towards smaller sizes lower in the trophic system.

Fishery economics traditional find that under inferior regulation systems the fisher will apply more effort than is optimal. The present analysis agrees, but this model reveals that the fishery not only will apply an excess of effort, but also will target smaller fish or lower in the trophic system than is optimal. In a compartment predator–prey model (as e.g. Hannesson, 2002, a two compartment model), a similar trophic cascade will drive the fishery towards other species. The present model shows that the economic drive caused by the trophic cascade is related with *size*, that is, the fishery can be expected gradually to target fish of smaller sizes, also within the same species.

Conclusion

The present model is the simplest possible given the principles of size as the determinant of the predator–prey interaction, mass balance in the predator–prey allocation and mortality and somatic growth as consequences of the predator–prey allocation. The fish has therefore only one attribute, its size. The presented model creates a link from ecosystem level to operation level, and, as shown in this article, it is suitable for applying ground rent theory. The model is then suitable for strategic planning in an ecosystem management context.

The model needs to be parameterized for the specific ecosystem, and the price and cost functions have to be established empirically, before a conclusion as to whether Fishing Down Marine Food Webs is detrimental from an economic point of view can be drawn directly. Nevertheless, the model does reveal that there is a need for an intertemporal balancing with respect to size. This aspect is not addressed in any systematic way on an ecosystem level under present management. Therefore, the economic prediction for an ecosystem managed as a common pool is that the exploited fish are too small in size.

The model also shows a need for an intertemporal balance in the volume of total harvest on an ecosystem level. This aspect is traditionally found in population models as well, but the present model is the first model that can address the problem at an ecosystem level in a capital theoretical context. There is therefore reason to suspect that the present management, using a population approach where each population is managed separately, overlooks the ability of the ecosystem to sustain the total volume of harvest or at best target an inferior point in the sustainable yield space.

With the two aspects of intertemporal choice revealed by the model, the conclusion must be that Fishing Down Marine Food Webs is probably driven by the inability of common pool management to conduct proper intertemporal balancing, and that it is

therefore probably detrimental from an economic point of view. The marine ecosystem therefore needs ecosystem management also for economic reasons. In this context, models like the present one can serve as planning tools.

Perspective

When, as shown above, the Fishing Down Marine Food Webs is detrimental and is not properly addressed in present management, it may seem appropriate to suggest that management use an instrument to remedy common pool management, such as a size-dependent tax on landings. This would, however, be premature. The ecosystem has other important properties, such as species and space, that have to be considered in an ecosystem management context as well. A one-dimensional remedy of problems may result in problems in other dimensions. Management regulation recommendations have to await further research.

The model is, however, meant as a strategic planning tool; that is, it is meant to be applied in a hierarchical planning context. One use of the model can then be to set limits for the total harvest volume, properly weighted according to size, from a marine ecosystem. As the model does not distinguish between species, other models must handle this aspect. The total harvest determined by the model will have to be divided in a hierarchical style, from the international level to the national level, the regional level, and finally to the fisher. One possible indicator of the landings being in accordance with the model could be the mean size of landings. If the mean size is decreasing over time, it is an indicator of disinvestment of the natural capital.

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A Technical details

This appendix gives some additional details related to the calculation of the optimum points

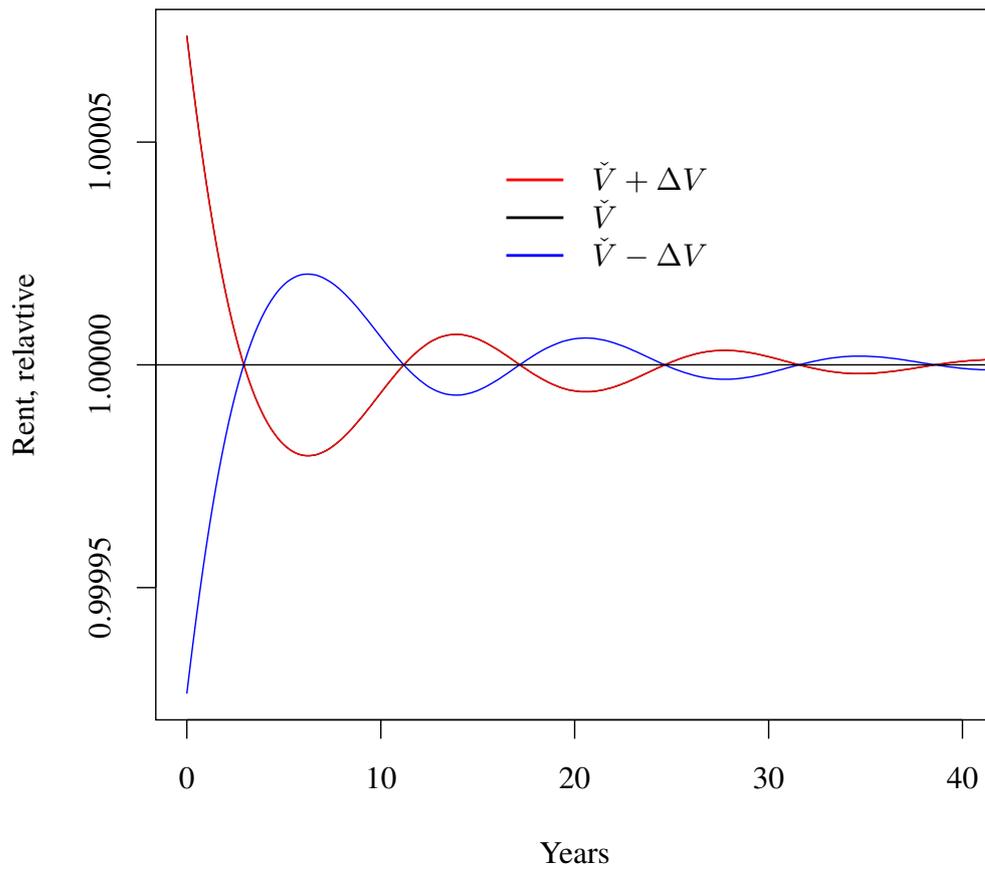


Figure 11: Rent flow. The diagram shows the rent flow following a small increase in effort with red, a small decrease in effort with blue, and sustainable yield rent with no change with black. The point is close to the optimal rent point

A.1 Details of the Control space

The lines l_0^ξ , l_∞^ξ , l_0^V and l_∞^V are not found with internal rate of return in the same way as the other l_w^ξ and l_w^V . A normal rent flow only crosses the alternative rent flow once, that is, the contribution margin from a management action is either first positive and then negative or the opposite. Most of the interesting part of the control space has a normal rent flow, but, near the lines l_0^ξ and l_0^V , the long term change in rent is close to zero. This implies that the rent flow will cross the sustainable yield rent several times, as in figure 11. The internal rate of return will be ambiguous, that is, there is more than one root that solves equation (9) (or actual equation (27) later in appendix). In the case of figure 11, both -0.078 and 0.008 satisfy the equation. The sustainable yield is, in this case, unchanged, and the internal rate of return is solely caused by the fluctuation of the system. In this way, the concept of internal rate of return is losing its meaning near the l_0^ξ and l_0^V lines. This is especially the case near the l_0^V , as can be seen in figure 7, where the left end of the l_w^V lines indicates the point below which the methods for finding w can no longer be used. As a consequence, both the l_0^ξ and l_0^V lines are found by maximizing rent:

$$l_0^\xi = \left\{ (\check{\xi}, \check{V}) \left| \check{\xi} = \operatorname{argmax}_\xi \check{\pi}(\xi, \check{V}) \right. \right\} \quad (10)$$

$$l_0^V = \left\{ (\check{\xi}, \check{V}) \left| \check{V} = \operatorname{argmax}_V \check{\pi}(\check{\xi}, V) \right. \right\} \quad (11)$$

Where $\check{\pi}$ indicates the sustainable yield rent, the rent given as a surface in figure 4. These points are found by numeric optimization using a Golden Section search in one dimension (see Press et al., 2007, p. 10.1) with effort and target size constant, respectively. The lines as defined in (10) and (11) correspond to the dashed lines in figure 4.

The short term change of rent near the lines l_∞^ξ and l_∞^V is very small, and the lines can only be found as the point where the short term gain from a small change in the respective controls is zero:

$$l_\infty^V = \left\{ (\check{\xi}, \check{V}) \left| \frac{\partial \overleftarrow{\pi}}{\partial V} \Big|_{(\check{\xi}, \check{V})} = 0 \right. \right\} \quad (12)$$

$$l_\infty^\xi = \left\{ (\check{\xi}, \check{V}) \left| \frac{\partial \overleftarrow{\pi}}{\partial \xi} \Big|_{(\check{\xi}, \check{V})} = 0 \right. \right\} \quad (13)$$

Where $\overleftarrow{\pi}$ is the instant rent if the control is changed, that is, the rent at time $t = 0$:

$$\overleftarrow{\pi}|_{(\check{\xi}, \check{V})}(\xi, V) = \pi|_{(\check{\xi}, \check{V})}(0, \xi, V)$$

Where $(\check{\xi}, \check{V})$ indicates the controls of the steady-state situation. The controls are at time $t = 0$ changed to (ξ, V) . At the time of a control change, the fishery will harvest in the steady-state population density of $(\check{\xi}, \check{V})$, so the instant rent per effort will be equal to the sustainable yield rent per effort

$$\left. \frac{\partial \overleftarrow{\pi}}{\partial V} \right|_{(\check{\xi}, \check{V})} = \frac{\tilde{\pi}(\check{\xi}, \check{V})}{\check{V}} \quad (14)$$

and the $\left. \frac{\partial \overleftarrow{\pi}}{\partial V} \right|_{(\check{\xi}, \check{V})} = 0$ line coincide with the zero rent contour. That is

$$\left\{ V \left| \left. \frac{\partial \overleftarrow{\pi}}{\partial V} \right|_{(\check{\xi}, \check{V})} = 0 \right. \right\} = \{ \check{V} \mid \tilde{\pi}(\check{\xi}, \check{V}) = 0 \} \quad (15)$$

the l_{∞}^V is then found as the red line in figure 4.

To find l_{∞}^{ξ} , there is a need to find the instant rent with a change in ξ , the curve in the lower diagram of 10. The instant rent is found by applying the fishing selection function θ to other ξ and finding the instant rent:

$$\overleftarrow{\pi}(\xi, \check{V}) = \check{V} \left(\int_{-\infty}^{\infty} P \theta(x, \xi) N (e^x)^2 dx - c \right) \quad (16)$$

where P is the price and c is the cost per unit effort. The partial derivatives $\overleftarrow{\pi}'_{\xi}$ are found for all grid points, and the l_{∞}^{ξ} are found as the line where $\overleftarrow{\pi}'_{\xi} = 0$. These points are found by interpolation (Akima et al., 2006) between the grid points.

A.2 Details related to output space

Although the curves in diagrams regarding the control space are drawn directly through the values found as described, it has been necessary to smooth the lines in diagrams regarding the output space by using cubic spline (with the `smooth.spline()` function, R Development Core Team, 2006), or the curves would be jagged. The reason for this is the accumulation of noise in the numeric estimations: The analysis in this section relies on the numeric calculations of partial deviates. The inputs for these calculations, as in others, are not the exact steady states but a convergence to the steady states. In this way, some errors build up through the

numerical process. More accurate calculations can be made, but the present work illustrates the economics of the output space as it is.

The lines $l_0^{\bar{x}}$ and l_0^H indicates where rent is maximized given fixed H and fixed \bar{x} , respectively. To find the lines $l_0^{\bar{x}}$ and l_0^H , the rent $\tilde{\pi}$ is estimated by interpolation in a 1000×1000 regular grid laid down over the $\bar{x} \times H$ space. The $l_0^{\bar{x}}$ and l_0^H lines are found as maxima in rows and columns, respectively.

$$l_0^{\bar{x}} = \left\{ (\check{\bar{x}}, \check{H}) \left| \check{\bar{x}} = \operatorname{argmax}_{\bar{x}} \tilde{\pi}(\bar{x}, \check{H}) \right. \right\} \quad (17)$$

$$l_0^H = \left\{ (\check{\bar{x}}, \check{H}) \left| \check{H} = \operatorname{argmax}_H \tilde{\pi}(\check{\bar{x}}, H) \right. \right\} \quad (18)$$

The line as defined in (17) and (18) correspond to the dashed lines in figure 5.

The line l_∞^H and $l_\infty^{\bar{x}}$ indicates where there is no short term gain from changing H and \bar{x} respectively given that the other parameter is fixed.

$$l_\infty^H = \left\{ (\check{\bar{x}}, \check{H}) \left| \frac{\partial \overleftarrow{\pi}}{\partial H} \Big|_{(\check{\bar{x}}, \check{H})} = 0 \right. \right\} \quad (19)$$

$$l_\infty^{\bar{x}} = \left\{ (\check{\bar{x}}, \check{H}) \left| \frac{\partial \overleftarrow{\pi}}{\partial \bar{x}} \Big|_{(\check{\bar{x}}, \check{H})} = 0 \right. \right\} \quad (20)$$

The first is simply approximated with the rent contour where $\tilde{\pi} = 0$:

$$\left\{ H \left| \frac{\partial \overleftarrow{\pi}}{\partial H} \Big|_{(\check{\bar{x}}, \check{H})} = 0 \right. \right\} \approx \left\{ \check{H} \mid \tilde{\pi}(\check{\bar{x}}, \check{H}) = 0 \right\} \quad (21)$$

The l_∞^H is then found as the red line in figure 5.

To find the $l_\infty^{\bar{x}}$ the net in sea price is defined as:

$$NS(\xi) = \frac{\int_{-\infty}^{\infty} P \theta(x, \xi) N (e^x)^2 dx - c}{\int_{-\infty}^{\infty} \theta(x, \xi) N (e^x)^2 dx} \quad (22)$$

The numerator is the rent per effort, and the denominator is the mass of the harvest per effort. The net in sea price then indicates what the immediate net income per harvest with a changed target size will be, that is, with a change of the mean size. The point where the NS , given the steady state population, does not change when target is changed around the target is used as an approximation to find the line $l_\infty^{\bar{x}}$:

$$\left\{ \bar{x} \left| \frac{\partial \overleftarrow{\pi}}{\partial \bar{x}} \Big|_{(\check{\bar{x}}, \check{H})} = 0 \right. \right\} \approx \left\{ \bar{x} \left| \frac{\partial NS}{\partial \xi} \Big|_{(\check{\bar{x}}, \check{H})} = 0 \right. \right\} \quad (23)$$

Item	Control	$\Delta \cdot$
$\frac{\partial \overleftarrow{\pi}}{\partial \xi}$	$\Delta \xi$	10^{-4}
$\frac{\partial \overleftarrow{\pi}}{\partial \bar{x}}$	$\Delta \xi$	10^{-4}
w_ξ	$\Delta \xi$	10^{-4}
$w_{\bar{x}}$	$\Delta \xi$	10^{-4}
$\frac{\partial \overleftarrow{\pi}}{\partial V}$	ΔV	10^{-4}
$\frac{\partial V}{\partial \pi}$	ΔV	10^{-4}
$\frac{\partial H}{\partial V}$	ΔV	10^{-4}
w_V	ΔV	10^{-4}
w_H	ΔV	10^{-4}

Table 1: Overview of the used $\Delta \cdot$.

A.3 Partial derivatives

As the implementation of the model is a numeric approximation, it follows that all analyses have to be done numerically too. All through partial derivatives are calculated as:

$$\left. \frac{\partial f(y, \dots)}{\partial y} \right|_{y'} \approx \frac{f(y' + \Delta y) - f(y' - \Delta y)}{2\Delta y}$$

The closer to the limit of $\Delta y \rightarrow 0$, the better the approximation is if the functional values are found analytically. When the values are found numerically, and are therefore only approximations, the error on the approximation will eventually be on the same scale as Δy . It is therefore a trade off between having a small Δy to give precise estimates and having a not too small Δy to avoid the influence of the numeric noise. To find the indicator rate w is to find the discount rate where the partial derivative of the capital value is zero. The discussion below of finding the proper $\Delta \cdot$ in the calculation of w is therefore the same problem. In table 1 is given the $\Delta \cdot$ used in the numeric calculus.

A.4 Capital value calculations

In the numeric implementation of the model, the rent flow is discrete and the rent π_t is the sum of the rent in period t to $t + \Delta t$. In order to transform this into a continuous

rent flow, it must be normalized to the time unit by dividing by the time step: The discrete version of the capital value (2) can be calculated as:

$$\mathbb{C} = \sum_{t=0, \Delta t, 2\Delta t, \dots} \left(e^{-\rho t} - e^{-\rho(t+\Delta t)} \right) \frac{\pi_t}{\rho \Delta t} \quad (24)$$

The numerical implementation has, however, a finite time period of T , and the rent is known for the periods $\tau = 0, \Delta t, 2\Delta t, \dots, T - \Delta t$. The equation (24) can then be written as

$$\mathbb{C} = \sum_{t \in \tau} \left(e^{-\rho t} - e^{-\rho(t+\Delta t)} \right) \frac{\pi_t}{\rho \Delta t} + e^{-\rho T} \mathbb{C}_T \quad (25)$$

where \mathbb{C}_T is the capital value at time T . At time T , fluctuation in the model system caused by the management action is almost gone. If the constant yearly rent flow from the resource in the steady state the system converges to after the management action is $\tilde{\pi}$, the capital value at time T is approximately:

$$\mathbb{C}_T \approx \frac{\tilde{\pi}}{\rho}$$

and the capital value will then be approximated by:

$$\mathbb{C} \approx \sum_{t \in \tau} \left(e^{-\rho t} - e^{-\rho(t+\Delta t)} \right) \frac{\pi_t}{\rho \Delta t} + e^{-\rho T} \frac{\tilde{\pi}}{\rho} \quad (26)$$

As a consequence of (26), the indicator rate in equation (9) is calculated as:

$$w = \rho \left| \begin{array}{l} \sum_{t \in \tau} \left(e^{-\rho t} - e^{-\rho(t+\Delta t)} \right) \frac{\pi_t(A)}{\rho \Delta t} + e^{-\rho T} \frac{\tilde{\pi}(A)}{\rho} \\ \sum_{t \in \tau} \left(e^{-\rho t} - e^{-\rho(t+\Delta t)} \right) \frac{\pi_t(B)}{\rho \Delta t} + e^{-\rho T} \frac{\tilde{\pi}(B)}{\rho} \end{array} \right. = 0 \quad (27)$$

where action A is a small change of control variables, and action B is the same change with opposite sign.

For the indicator rate to be a useful concept for evaluation of management options, the w has to be well-defined. Well-defined means that, for a small change, the w has to be independent of the chosen Δ . Preferably, the w would be defined in the limit for $\Delta \rightarrow 0$. Equivalent to the discussion of the size of Δ when finding the partials, there is, however, a tradeoff between having a smaller change in the controls for finding a

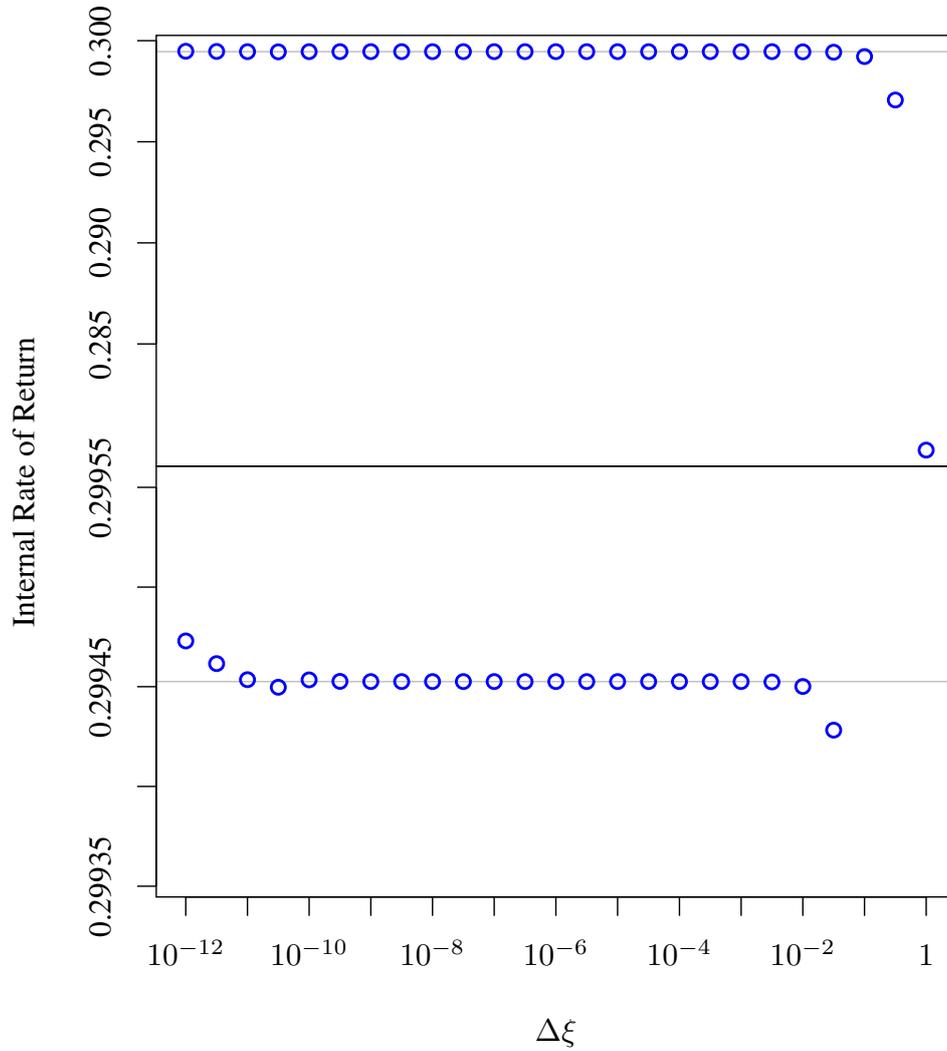


Figure 12: The stability of the w with different $\Delta\xi$. In the upper diagram, the w is given on a scale from 0.28 to 0.3. In the lower diagram, the observations are magnified so that the scale is ± 0.0001 around w . The investigated point is the optimum point given $\rho = 0.3$ found by interpolation, and the grey line indicates $w = 0.299452621$.

precise w and having a larger change to avoid the influence of the numeric noise. In figure 12 is shown the w found with different $\Delta\xi$. In the upper diagram, the found w is plotted on a scale from 0.28 to 0.3. In the lower diagram, the observations are magnified so the scale is ± 0.0001 around $w = 0.29945$. With a $\Delta\xi < 10^{-2}$, the w seems to be well-defined, when $\Delta\xi < 10^{-9}$, the numeric noise begins to influence the result. The system can also be said to be homogeneous with respect to small changes in controls. If the change is doubled, the consequences are doubled, resulting in the same w .

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