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Abstract: We analyze voter preferences for seven General Elections for the Danish parliament by using survey data to simulate alternative aggregations and with regard to the possible presence of five types of social choice paradoxes that may occur in list systems of proportional representation. Two serious paradoxes fail to manifest themselves, while three others occur. One paradox always occurs, namely for the social ordering of political parties based on pair-wise comparisons to be significantly different from that of the allocation of seats according to a principle of proportional representation. This result challenges the common view that a party that receives more votes than another must be preferred to it. Further simulations demonstrate that alternative aggregation methods produce radically different seat allocations, even when done in accordance with a requirement of proportionality. Elections seem to have a considerable degree of randomness to them, at least relative to the voter preferences, making it difficult to maintain that there is an unequivocal “will of the people.”

Keywords: Social choice; voting paradoxes; voting procedures; electoral systems.

JEL-codes: D71; D72.

Introduction

When we think of democracy, we usually associate it with the general idea of majority rule in a system, where all individuals have an equal weight in the voting process—in particular the notion that if one alternative is supported by a majority of the voters, then it should not be another alternative which wins the day.

However, over the last half century social scientists have increasingly become aware of the possibility that individual preferences may not necessarily translate easily into meaningful collective choices, and that the methods by which preferences are aggregated may, at least in theory, exert significant influence upon the outcomes (Black 1948; Black [1958] 1998; Arrow [1951]

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1963; Riker 1982; Riker 1986; Nurmi 1987; cf. Nurmi 1999). The present study attempts to go beyond the formal theorizing and examine the actual preferences of real-world voters with regard to the occurrence of a number of theoretical paradoxes that may manifest themselves in a particular type of electoral systems, namely list systems of proportional representation, such as found, first and foremost, in many European democracies.

The paper does so by, first, briefly introducing five types of relevant social choice paradoxes (section 1) and then by investigating survey data relating to seven General Elections for the national parliament of Denmark, the *Folketing*, for which survey data exist that will allow us to construct the social orderings of the voters,¹ and these findings in turn are interpreted and discussed (sections 2 and 3). Finally, an attempt is made at considering how different social choice mechanisms may assign parliamentary seats given the same requirement of proportionality used in actual elections (section 4).

1. Paradoxes of voting

Whenever more than two persons are to choose between more than two alternatives certain paradoxical results may occur. The classical illustration is the so-called Condorcet Paradox, first explicitly identified by the Marquis de Condorcet (1743-93) (Condorcet [1785] 1994; Condorcet [1789] 1994). This paradox consists in the observation that situations of social choice may exist, where decision makers are to choose between alternatives, but where no unique majority winner exists. More technically, there is no “Condorcet Winner,” i.e., an alternative that can beat all other alternatives when compared in pair-wise majority contests.

The paradox is usually illustrated by considering a simple hypothetical example, where it is assumed that we have three voters contained in the set $N = \{i_1, i_2, i_3\}$, who are faced with the three alternatives contained in the set $X = \{x_1, x_2, x_3\}$. Let us further assume that each voter $i \in N$ has a preference ordering, P_i , over the alternatives in X , which follows the standard formal assumptions about preferences,² and where the relation “ \succeq ” means “preferred

¹ These elections (1973, 1975, 1977, 1979, 1994, 1998, 2001) are the only national elections where survey data exists that allows such analysis.

² These assumptions are that preference orderings relative to $x_1, x_2, x_3 \in X$ must be: Complete (if $x_1 \bullet x_2$, then either $x_1 \succeq x_2$, or $x_2 \succeq x_1$); asymmetric (if $x_1 \succ x_2$, then not $x_2 \succ x_1$); transitive (if $x_1 \succeq x_2$ and $x_2 \succeq x_3$, then $x_1 \succeq x_3$); irreflexive (not $x \succeq x$). For the present purposes, when aggregating individual preference orderings to collective choices, we will also treat the preference orderings of the voters as if they were sincere, i.e., that the respondents have answered non-strategically in the surveys used here.

at least as much as,” and “ \succ ” means “preferred to,” so that, e.g., “ $x_1 \succeq_i x_2$ ” means that x_1 is preferred at least as much as x_2 by individual i .³ We may then hypothesize a situation, where the preferences are such, that they can be represented by a profile of individual preference orderings such as this example:

$$\begin{aligned} P_1: & \quad x_1 \succ_1 x_2 \succ_1 x_3 \\ P_2: & \quad x_2 \succ_2 x_3 \succ_2 x_1 \\ P_3: & \quad x_3 \succ_3 x_1 \succ_3 x_2 \end{aligned}$$

Given such preferences and pair-wise comparisons, the collective preference ordering (or social ordering) of the group N may be said to be this “cyclical” ordering:

$$P_N: \quad x_1 \succeq_N x_2 \succeq_N x_3 \succeq_N x_1$$

In this example it is impossible to construct a transitive collective preference ordering, and there is no Condorcet Winner, i.e., no stable equilibrium outcome exists: No matter which one of the three alternatives is selected, another can beat it in a pair-wise comparison. One majority (i_1 and i_3) prefers x_1 to x_2 , while another (i_1 and i_2) prefers x_2 to x_3 , and a third (i_2 and i_3) prefers x_3 to x_1 .

This is a non-trivial paradox, because there would seem to be an obvious discrepancy between what is the theoretical observation and what would usually be the intuitive or common sense expectation: That if a democratic decision takes place, it is reasonable to assume that—absent ties—one alternative would be clearly preferred by a majority of those voting, and that this alternative should win.

In applying this insight, social choice scholars have traditionally, in theoretical and empirical studies alike, tended to focus on choice-settings involving the choice between, say, competing policies or candidates, and where the aggregation method is one of plurality, i.e., where the winner is the alternative with a plurality of the votes (typically within a single member district). The late William H. Riker, for example, demonstrated how in several US presidential elections the actual winner may not have been the Condorcet Winner (Riker 1982).

However, while less studied it should also deserve attention that paradoxes of social choice also may occur in electoral systems with party lists and

³ This notation corresponds to the frequent use in the social choice literature of P meaning “strictly preferred to” (\succ) and R meaning “weakly preferred to” (\succeq), cf., e.g., Nurmi 1983: 181f; Johnson 1998: 9f.

proportional representation, i.e., as found in most Western democracies. In such systems votes are not necessarily cast solely for individual candidates as with the first-past-the-post systems of, e.g., the United Kingdom and the United States. Instead they are usually cast for candidates appearing on party lists and/or the party itself, with the votes subsequently being converted into seats allocated to parties through the use of some vote-seat conversion method aiming at proportionality (e.g., the d'Hondt or Saint Laguë formulas).

To illustrate how voting paradoxes may occur in list systems of proportional representation, let us assume that we identify the voters as those n voters contained in the finite set $N = \{i_1, \dots, i_n\}$, that these are confronted with the choice between m political parties included in the finite set $X = \{x_1, \dots, x_m\}$. Furthermore let $s(x_i)$ be the number of seats s allocated in an electoral system to a political party x_i . We may then identify the following five paradoxes of voting, which in theory can occur in democracies with list-systems of proportional representation (as well as in some other types of collective choices):⁴

1. *The Condorcet Paradox:* Where a majority of the voters in N prefer party x_1 to a party x_2 ($x_1 \succ_N x_2$), and where a majority in turn prefer party x_2 to party x_3 ($x_2 \succ_N x_3$), but where it is also the case that a majority prefers party x_3 to party x_1 ($x_3 \succ_N x_1$). In such a case, the social ordering is the intransitive ordering $x_1 \succ_N x_2 \succ_N x_3 \succ_N x_1$.
2. *The Majority-Reversal Paradox:* Where a majority relation for an election (e.g., $x_1 \succ_N x_2 \succ_N x_3$) may be exactly the reversal of the ranking of the parties if based on their number of seats as assigned by the system of proportional representation, i.e., in this case $s(x_3) > s(x_2) > s(x_1)$.
3. *The Condorcet-Winner-Turns-Loser Paradox:* Where a party x_1 is the Condorcet Winner (and thus can beat any party in pair-wise comparisons, $x_1 \succ_N (x_2, \dots, x_m)$) but where the party receives less seats than a party x_2 , i.e., $s(x_1) < s(x_2)$, or even no seats at all.
4. *The Condorcet-Loser-Turns-Winner Paradox:* Where a party x_1 is the Condorcet Loser (i.e. can be beaten by any other party in pair-wise majority comparisons, $x_1 \prec_N (x_2, \dots, x_m)$) but where the party receives more

⁴ We are here closely following Van Deemen 1993 and Van Deemen and Vergunst 1998: 239ff. These do, however, only identify four paradoxes relevant for list-systems of proportional representation, not including what we have here termed the Condorcet-Loser-Turns-Winner Paradox. These paradoxes may be seen as being somewhat similar to some of those considered by Hannu Nurmi as “paradoxes of representation, cf. Nurmi 1999: 109-19.

seats than a party x_2 , i.e., $s(x_1) > s(x_2)$, or even more seats than every other party.

5. *The More-Preferred-Less-Seats Paradox*: Where a majority of the voters prefer a party x_1 to party x_2 ($x_1 \succ_N x_2$), but where party x_1 receives less seats than party x_2 , i.e. $s(x_1) < s(x_2)$.

Obviously, these paradoxes are connected. Paradoxes no. 3 and no. 4 are versions of no. 2, while no. 5 in turn implies nos. 2, 3 and 4.

In principle such paradoxes may occur in any proportional system (cf. Van Deemen 1993), and it has been demonstrated that at least some of these paradoxes occur in practice (e.g., Van Deemen and Vergunst 1998). The purpose of the present study is to extend these studies and investigate whether the paradoxes occur in practice in Danish politics, and if so whether the results are different from those previously found. Yet furthermore, we will also consider whether the same set of voter preferences and a principle of proportionality may result in vastly different seat allocations among the parties given alternative ways of aggregating individual votes.

2. Research design

The present research is based on data derived from the Danish Election Survey Project, which over three decades of existence has surveyed representative samples of voters, typically about 1,000-2,000, around the time of general elections for the Danish parliament, the *Folketing*.⁵

In general, election surveys do seldom allow for investigations of the occurrence of intransitivity in voter preferences; only very few of the many questions typically asked make it possible to reconstruct the, more or less, full preference orderings from the individual respondents. In the Danish election surveys made over the years, there are seven occasions where this was possible in the case of preferences relating to political parties, namely where the respondents were asked to evaluate over-all the political parties standing in the

⁵ The seven sets of survey data used are obtainable from Dansk Data Arkiv, Odense, Denmark. The Danish Election Survey Project has undertaken systematic surveys of Danish voters since 1968, and results from the project have been published in numerous publications (including, e.g., Borre and Andersen 1997; Andersen et al. 1999). The participating researchers for these particular surveys were Johannes Andersen (1998), Jørgen Goul Andersen (1994, 1998, 2001), Ole Borre (1973, 1975, 1977, 1979, 1994, 1998), Erik Damgaard (1973), Hans Jørgen Nielsen (1973, 1975, 1977, 1979, 1998, 2001), Steen Sauerberg (1973, 1975, 1977, 1979), Søren Risbjerg Thomsen (2001), Ole Tonsgaard (1973), and Torben Worre (1973, 1975, 1977, 1979).

general election using a “thermometer” to assign values to the parties (Danish Election Survey 1973, 1975, 1977, 1979, 1994, 1998, 2001).⁶

In order to turn the “thermometer” values into Condorcet comparisons, we let $\psi_i(x_1)$ stand for the points assigned by individual i to alternative x_1 . We may assume that if a respondent i assigns more points to x_1 than to x_2 , then he strictly prefers x_1 to x_2 , i.e., if $\psi_i(x_1) > \psi_i(x_2) \leftrightarrow x_1 \succ_i x_2$. We furthermore assume that if an individual assigns the same number of points to x_1 and x_2 , then he is indifferent between the two, i.e., if $\psi_i(x_1) = \psi_i(x_2) \leftrightarrow x_1 \sim_i x_2$. When the voter preferences over the alternatives are constructed as such, they may be aggregated by majority rule, so that alternative x_1 may be said to be majority preferred to alternative x_2 , if the number of voters who prefer x_1 to x_2 is larger than the number of voters who prefer x_2 to x_1 . With S being the group of voters in question, we may express this as $(x_1 \succ_i x_2) > (x_2 \succ_i x_1) \leftrightarrow x_1 \succ x_2$, and $(x_1 \succ_i x_2) = (x_2 \succ_i x_1) \leftrightarrow x_1 \sim x_2$. We may thus also say that an alternative x_1 is a Condorcet Winner if and only if it is the case that for any given alternative x_k from the set X , it is the case that $x_1 \succ x_k$.⁷

3. Empirical analysis of voting paradoxes

In the present section we will use the comparisons described in the previous for an analysis of the data derived from the seven election surveys. We will not review the existing literature on the subject of empirical social choice analysis, since this has been done on several other, recent occasions (e.g. Rasch 1995; Rasch 2000; Van Deemen and Vergunst 1998; Van Deemen 1998; Kurrild-Klitgaard 2001a; Kurrild-Klitgaard 2001b; Kurrild-Klitgaard 2004).

⁶ The respondents were asked of their evaluation of the political parties according to how much they sympathize with them, by assigning values to the alternatives—usually with positive values to favoured parties, negative to disfavoured and neutral (zero) to parties to which the voter is indifferent. The 1994 question also included the alternative “the present government as a whole,” which has been excluded here.

⁷ For more or less similar approaches to the use of survey data for the study of the occurrence of cycles, see, e.g., Van Deemen and Vergunst 1998; Regenwetter and Grofman 1998; Regenwetter, Adams and Grofman 2002. In some ways the data considered here are superior to those considered by Van Deemen and Vergunst in their analysis, or at least more meaningful. Their data, also derived from election surveys (The Dutch parliamentary election study, NKO, of 1982, 1986, 1989, 1994), is based upon questions of how probable it is that voters will vote for one or another of the political parties. But strictly speaking the probability that a voter will vote for a party and the intensity of the preference for that party are not identical measures, since it is possible for a voter to vote strategically, and it would thus seem to possibly confuse data suited for behavioural research and those suited for aggregation of preferences.

The Condorcet Paradox

The results of the Condorcet comparisons are contained in Tables A-1 – A-7 in the Appendix. Based on these comparisons, we may now construct the majority relation for the voters in each of the seven elections, and Table 1 gives these in the columns labelled MR.

It is evident that there were no examples in any of the seven elections of the infamous Condorcet Paradox (i.e. of a cycle involving all the alternatives) or of intransitivity more generally speaking (e.g., with a cycle among a sub-set of alternatives). In all seven cases the samples of Danish voters had individual preferences, which, when aggregated, resulted in a transitive collective preference ordering.

Compared to the theoretical literature, these findings are somewhat surprising, i.e., much of social choice theory has predicted that intransitivity should be widespread in collective preferences. But compared to previous empirical investigations, the present results are less surprising: With a very few notable exceptions (Niemi 1970; Kurrild-Klitgaard 2001a; cf. Kurrild-Klitgaard 1999), social choice theorists have, so far, not been able to detect any examples of the full-fledged Condorcet Paradox in larger electorates (or samples thereof) and only some examples of intransitivity (cf., e.g., Regenwetter, Adams and Grofman 2002). Specifically, this result is similar to the findings in the study of four Dutch election surveys, which found no examples of intransitivity in the preferences of voters over parties (Van Deemen and Vergunst 1998).

Table 1. Social orderings and seats of political parties, Danish election surveys and parliamentary elections, 1973, 1975, 1977, 1979, 1994, 1998, 2001. Majority relations (MR) and seats according to proportional representation (PR).

Rank order	1973		1975		1977		1979		1994		1998		2001	
	MR	PR	MR	PR	MR	PR	MR	PR	MR	PR	MR	PR	MR	PR
1	A	A (46)	V	A (53)	A	A (65)	A	A (68)	A	A (62)	A	A (63)	V	V (56)
2	B	Z (28)	A	V (42)	M	Z (26)	B	C (22), V (22)	V	V (42)	V	V (42)	A	A (52)
3	V	V (22)	Q	Z (24)	B	V (21)	V		C	C (27)	C	C (16)	C	O (22)
4	Q	B (20)	B	B (13)	C	C (15)	C	Z (20)	B	F (13)	D	F (13), O (13)	B	C (16)
5	M	C (16)	C	C (10)	E	M (11)	F	F (11)	D	Z (11)	B		F	F (12)
6	C	M (14)	Z	F (9), Q (9)	V	F (7), K (7)	E	B (10)	F	B (8)	F	D (8)	Q	B (9)
7	E	F (11)	M		Q		Q	M (6), Y (6)	Q	∅ (6)	Q	B (7)	O	∅ (4), Q (4)
8	Z	Q (7)	E	K (7)	F	B (6), E (6), Q (6)	M		Z	D (5)	O	∅ (5)	∅	
9	F	K (6)	F	M (4), Y (4)	Y		Y	E (5), Q (5)	∅	Indp (1) [§]	Z	Q (4), Z (4)		
10	Y	E (5)	Y		K		K			Q (0)	∅			
11	K	Y (0)	K	E (0)	Z	Y (5)	Z	K (0)			U	U (0)		

Sources: Own calculations on the basis of election surveys (Dansk Data Arkiv); actual election results, with the number of seats received by the party given in brackets.

Notes: [§] Jacob Haugaard, elected as an independent, was not included in the election survey. [£] Due to an error made by the Danish Election Project, two parties which stood for the election but did not receive any seats (D and Z) were omitted in the survey. The data thus only includes preferences over represented parties.

Abbreviations: MR: Majority Relation (i.e. ranking according to results of pair-wise Condorcet comparisons); PR: Proportional Representation (i.e. ranking according to proportions of votes in the election); A: *Socialdemokratiet* (Social Democratic Party); B: *Radikale Venstre* (Social Liberal Party); C: *Det Konservative Folkeparti* (Conservative People's Party); D/M: *Centrum-Demokraterne* (Center-Democrats); E: *Retsforbundet* (Justice Party); F: *Socialistisk Folkeparti* (Socialist People's Party); Indp: Jacob Haugaard, elected as an independent; K: *Danmarks Kommunistiske Parti* (Danish Communist Party); Q: *Kristeligt Folkeparti* (Christian People's Party); O: *Dansk Folkeparti* (Danish People's Party); U: *Demokratisk Fornyelse* (Democratic Renewal); V: *Venstre* (Danish Liberal Party); Y: *Venstre Socialisterne* (Left Socialists); Z: *Fremskridtspartiet* (Progress Party); ∅: *Enhedslisten* (Socialist Unity List).

However, one must add a few reservations to this conclusion, and the present results do in fact raise a serious methodological issue. For a problem is present in interpreting data such as those considered here, and one which has gone unnoticed by most previous attempts at studying voting paradoxes by using election surveys, namely the question of the statistical significance of the results.⁸ Since the election surveys are based only on samples of voters, we cannot be sure that the collective preference orderings found necessarily can be generalized to be representative of all the voters participating in the election.

This has some specific consequences. In the case of at least five of the seven elections, there are some pair-wise comparisons for which the alternative preferred most by the population as a whole cannot be inferred with certainty, i.e., we cannot predict with safety, which of the parties in fact were majority-preferred to each other by the Danish voters as a whole. In the relevant tables in the Appendix a requirement of a 95 pct. confidence interval has been applied to the results of the pair-wise comparisons, and those cases where the comparisons gave results at this level of significance have been indicated with an asterisk.

In this case, the calculations do not force us to make many reservations, although in some cases it does. In at least three elections there are instances of rankings where the differences between two alternatives are so small that we cannot say at a 95 pct. level of confidence that the ordering in the population as a whole was the same as in the sample. In at least two elections this includes some top-ranked alternatives (1973, 1994). The same problem was present in the pair-wise comparisons of several parties in 1977, most importantly in the case of the differences between the Justice Party and the Christian People's Party and between the latter and the Liberal Party. This was the case again in the 1994 election with the differences between several, including the two top-ranked parties, the Social Democratic Party and the Liberal Party, and between the latter and the Conservative People's Party.

⁸ Van Deemen and Vergunst 1998, for example, fail to consider the statistical significance of their results despite the fact that in several cases the margins of voters preferring x_1 to x_2 and reverse is so small as to be insignificant. I am grateful to Dorthe Lund Nielsen for initially pointing this out to me. For a discussion (and many examples) of the importance of paying attention to the question of samples vis-à-vis electorates, when it comes to detecting examples of intransitivity, see, e.g., Tsetlin and Regenwetter 2001; Regenwetter, Adams and Grofman 2002.

Table 2. Summary of finding of paradoxes from seven Danish election surveys.

Paradox	No. of elections with occurrences	Remarks regarding generalization of sample results to electorate as a whole
Majority-Reversal Paradox	None	
Condorcet Paradox	None	In two instances top-cycles may have been present in preferences of the electorate (1973, 1994); in one instance other intransitivities may have been present in preferences of the electorate (1994).
Condorcet-Winner-Turns-Loser Paradox	One (1975)	In at least three other instances the CW in the sample may not have been so in the preferences of the electorate (1973, 1994, 1998).
Condorcet-Loser-Turns-Winner Paradox	Five (1973, 1975, 1977, 1979, 1994)	In one further instance a party was the CL, but shared last place in seats (2001).
More-Preferred-Less-Seats Paradox	All seven	

Other paradoxes

In order to establish whether any of the other social choice paradoxes identified here were present in the preferences of the Danish voters, we must compare the social ordering according to the majority-relation with the actual results of the seven elections. Table 1 thus also summarizes the actual results of the seven elections, measured in terms of the number of seats won by each of the parties under proportional representation (PR) and comparing this to the social ordering given pair-wise comparisons of the alternatives (MR), while Table 2 summarizes the occurrences of the paradoxes in the seven elections.

The Majority-Reversal Paradox—which was not found in the investigation of the four Dutch election surveys—was not present in any of the Danish elections, and this result stands for the sample as well as for the electorate as a whole. This should come as no big surprise, since it should *a priori* seem an exceedingly unlikely phenomenon.⁹

The Condorcet-Winner-Turns-Loser Paradox seems relatively rare and does not occur in six of the seven elections; however, in 1975 the Liberal Party was the Condorcet Winner and could beat all parties in pair-wise comparisons, including the Social Democrats, but it was the latter party which received most votes. Furthermore, we cannot say at a 95 pct. confidence level that the Social Democratic Party in fact was the Condorcet Winner for the electorate in 1998, and so the Condorcet-Winner-Turns-Loser Paradox may have been present in the preferences of the electorate as a whole. In comparison, the investigation of four Dutch election surveys found two examples of this paradox (Van Deemen and Vergunst 1998).

While the three first paradoxes thus were less than characteristic, two paradoxes are manifest: The Condorcet-Loser-Turns-Winner Paradox and the More-Preferred-Less-Seats Paradox are abundantly present in Danish elections, or at least in respectively five and all seven of the seven elections for which we have survey data.

In three elections (1973, 1975, 1994) the Communists and their contemporary successors were the Condorcet Losers but received more seats than two or three other

⁹ If we assume only strict preferences, then with 11 parties, there would be no less than 11! possible social orderings, i.e. $11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 39,916,800$ possibilities. For two aggregation methods to produce two exactly opposite orderings would thus seem highly unlikely.

parties; in two elections (1977, 1979) the Progress Party was the Condorcet Loser but received more seats than most other parties. In one election (2001) the Socialist Unity List was the Condorcet Loser and also received the least number of seats, but shared this result with the Christian People's Party, whom they narrowly edged out in actual votes.

As for seats relative to place in the collective preference given majority comparisons, there were only two of the eleven parties in the 1973 election, the Social Democrats and the Liberals, who had the same place in the social ordering and in terms of the allocation of seats. Most significantly, the Progress Party received more seats than no less than six other parties (the Liberals, the Social Liberals, the Conservatives, the Center-Democrats, the Justice Party and the Christian People's Party), despite being lower in the social ordering than all of these. Other parties, which did better in the actual allocation of seats than their rank in the social ordering, included the Conservatives, the Socialist People's Party and the Communists. A party which received a significant lower place in terms of number of seats than in the social ordering was the Justice Party, which was out-sized by not only the Progress Party, but also the Socialist People's Party and the Communists—which in fact was the Condorcet Loser.

The picture was very similar in the next three elections. In 1975 only two parties had the same rank in terms of the majority-relation and the number of seats—and none of them were in the top-three. Again, the Progress Party, the Socialist People's Party and the Communists, as well as the Left Socialists, received more seats than a number of parties preferred by a majority. Most notably, the Justice Party won no seats, while three parties ranked lower did—including the Condorcet Loser. In the 1977 only two parties, the one of them the Condorcet Winner, had the same place in the orderings. In contrast, one party, the Progress Party, jumped no less than ten places—from being the Condorcet Loser to coming in no. two in terms of number of seats—while the Communists jumped four places. Again in 1979 only two parties had the same rank, and the Progress Party again jumped from being the Condorcet Loser, now to no. four in terms of seats.

In the 1994 election the same three parties were ranked at the top irrespective of method (although the differences between them were statistically insignificant), but beyond this the orderings were different. The Socialist People's Party and the Progress Party both received more seats than the Social Liberals, although the latter party was ranked higher in the social ordering. Furthermore, the Progress Party, the Social Liberals, and the Socialist Unity List all received more seats than the Center-Democrats, which however were ranked higher in the social ordering. Finally, the Progress Party and the Socialist Unity List both received more seats than the Christian People's Party, which did not receive any seats at all despite beating both these parties in pair-wise comparisons.

In 1998 there was again consistency between the two methods concerning the top-three alternatives (even if the differences between the two top-ranker alternatives were so small that they were statistically insignificant), and a fourth party was ranked last in both. But beyond this the Socialist People's Party and the Danish People's Party both received more seats than the Center-Democrats and the Social Liberals, despite that the latter two were ranked higher in the social ordering. The Danish People's Party and the Socialist Unity List also both received more seats than the Christian People's Party despite the latter being ranked higher in the social ordering. Finally, the Socialist Unity

List also received more seats than the Progress Party despite the latter being ranked higher in the social ordering. In 2001, the top-two parties were the same, irrespective of the methods, but beyond this the orderings were far from identical. Again, the Danish People's Party jumped from seventh in the MR-ordering to third in size under PR.

We can thus with some safety conclude that the More-Preferred-Less-Seats Paradox occurs frequently in Danish politics—but how often and how “much” does it occur? What is initially noteworthy is that in six of seven elections the majority-winner and the party receiving the most seats/votes indeed was the same (at least in the preferences of the voters in the sample, if not necessarily in the electorate as a whole). However, this observation may cloud the fact that the lower ranked alternatives seem to change places depending on the aggregation method. Following Van Deemen and Vergunst we will calculate the robustness of the orderings by using Kendall's Tau (τ), which may be seen as being a good indicator of the number of “reversals” (Van Deemen and Vergunst 1998: 484f). It measures correlation for ranked data that relies on the number of inversions in the rank order of one variable when the other variable is ranked in order. The values range from -1 (inversely related) over 0 (no relation) to +1 (directly related). Table 3 gives the results of the calculations of the coefficients for the respective elections. Another relevant measure is Spearman's coefficient of rank order correlation, known as Spearman's Rho (ρ), which is a measure of the correlation between two orderings, ranging from -1 (perfect negative correlation) over 0 (no correlation) to +1 (perfect positive correlation). Table 3 also gives these results.¹⁰

The calculations demonstrate that the More-Preferred-Less-Seats Paradox seems to be a quite frequent phenomenon in the preferences of Danish voters, although not uniformly so. The average of the τ coefficients was 0.507, while the average of the coefficients was 0.642. In particular, the allocation of seats following the 1973, 1977 and 1979 election display a considerable number of reversals relative to the social ordering based on majority relations. For the 1977 election it is the case that almost any resemblance of consistency between the collective preferences based on majority-relations and the allocation of seats based on proportional allocation of seats breaks down and becomes close to random. Looking at the data, one way of interpreting them would be that in the seven elections considered, the number of seats allocated to a party relative to another was on average just about as much random as it was strictly straightforward.

In comparison, similar calculations for four Dutch elections had τ coefficients between 0.641 and 0.944 and with an average coefficient of 0.752 (cf. Van Deemen and Vergunst 1998: 484), and we may thus conclude that the More-Preferred-Less-Seats Paradox was even more markedly present in the seven Danish elections considered. Together the results from the two countries suggest that this particular paradox is likely to be one occurring frequently in democratic decision-making, when the proportional method is used.

¹⁰ In calculating these coefficients the higher rank order for two alternatives have been used if they scored an identical number of seats.

Table 3. Correspondence between social orderings and seats of political parties, Danish election surveys and parliamentary elections, 1973, 1975, 1977, 1979, 1994, 1998, 2001. Kendall's Tau coefficients (τ) and Spearman's Rho (ρ) rank order correlation coefficients.

Election	τ	ρ
1973	0.481 (0.042)	0.662 (0.026)
1975	0.537 (0.023)	0.753 (0.007)
1977	0.132 (0.580)	0.202 (0.551)
1979	0.449 (0.059)	0.590 (0.056)
1994	0.611 (0.022)	0.717 (0.030)
1998	0.722 (0.002)	0.849 (0.001)
2001	0.618 (0.034)	0.719 (0.045)
Average	0.507	0.642

p values in parenthesis.
See notes for Table 1.

Discussion

The present analysis, as well as the analysis of Dutch elections, suggests that one social choice paradox is present in virtually all elections using list-systems with proportional representation, albeit to different extents, namely the More-Preferred-Less-Seats Paradox, while two other paradoxes, the Condorcet-Winner-Turns-Loser Paradox and the Condorcet-Loser-Turns-Winner Paradox, seem to occur occasionally. The Majority-Reversal Paradox seems never to occur, while the fifth and most infamous, the Condorcet Paradox, is not found in any of the samples of voters, although we cannot completely rule out every form of intransitivity for the electorate as a whole in every single election. This finding is in line with both the theoretical expectation that the More-Preferred-Less-Seats Paradox will occur more often than the Condorcet-Winner-Turns-Loser Paradox, and that this will occur more often than the Majority-Reversal Paradox (Van Deemen 1993: 240).

The widespread presence of three of the paradoxes, i.e., that the proportional system entails a significant number of reversals vis-à-vis the majority relation, raises an interesting question, namely if a considerable discrepancy between the two may be seen as an indication of a more fundamental instability in the political system, or perhaps even as a part of the cause of it? It is particular worth considering whether the Danish political system would have experienced as much instability as it did in the 1970s and 1980s, had it not been the case that the electoral system allowed what was a frequent Condorcet Loser—the Progress Party—to almost consistently be among the top seat winners. The same may be said for, e.g., the Socialist People's Party and the Danish Communist Party (and the latter's contemporary representative, the Socialist Unity List), who generally receive more seats than their place in the social ordering would dictate, and which for long periods have made it difficult to construct stable government coalitions. Had any of these parties only received a number of seats consistent with their social rankings according to the majority-relation, it is very doubtful whether the parliamentary situation would have been the same as it was in decades, where it was virtually impossible for majority governments to be formed and with the result that general elections were called frequently.

These observations indicate that it would seem, at least in Denmark, often to be “fringe” parties, which receive more seats under proportional representation than is

consistent with their rank in the social ordering according to the majority-relation.¹¹ On the left wing it was the case with the Socialist People's Party in five of seven elections, as well as for the Danish Communist Party in three of four elections and the Socialist Unity List in two of two elections. On the right wing it was the case with the Progress Party in five of the six elections, and with its off-spring, the Danish People's Party in two of two. In contrast, the "losers" to these parties would seem almost consistently to be the smaller, "centrist" parties. This suggests an empirical tendency for proportional representation to allocate more seats to the "fringe" parties when compared to the majority-relation social ordering, and to do so at the cost of the centre.¹²

More generally the presence of the paradox would seem to contradict a premise underlying much of contemporary democratic theory, namely that if more people prefer one party than another, then it would be wrong for the latter party to receive more seats. It would in particular seem contrary to the views and justifications usually offered in favour of proportional representation, namely that this somehow is more in line with "what the people want." Proportional representation obviously adds pluralism to a party system compared to, e.g., the first-past-the-post system, but it would also seem not only to do so by benefiting some parties at the cost of others but also to do so in a way which may seem intuitively to be in direct opposition to the majority principle.

This in turn also suggests that observers should be somewhat more careful than typically is the case not to infer from the fact that one party receives more seats than another that it therefore also, in some unequivocal and meaningful sense is the "more preferred" of the two.

4. Arbitrariness under alternative forms of proportional representation

Critics might intervene that the previous results are somewhat contrived. After all, they compare two radically different ways of aggregation preferences: the one, the MR, is obtained by using information about preferences over all the relevant alternatives and produces a rank order by majority comparisons, where the other, PR, only contains information about first preferences and produces an allocation of seats according to a proportional formula (which here is represented as an ordering). Is it then very surprising that the two methods produce so different outcomes, and perhaps even some paradoxical results? Maybe not—and yet this does take away from the fact that there seems to be empirical occurrences of features of proportional representation which seem quite at odds with how democratic decision-making and representation is often presented.

¹¹ The adjectives "centrist" and "fringe" used here should be seen in the context of the usual uni-dimensional illustrations of the Danish political system. In almost all attempts at locating Danish parties along a uni-dimensional spectrum, the Danish Communist Party and the Socialist People's Party are on the extreme "left," while the Progress Party is on the extreme "right."

¹² Some might suggest that the problem is that PR gives small parties (rather than "fringe" parties) more seats than their place according to MR would dictate. However, this is contradicted by the fact that other, small centrist parties (e.g. the Social Liberals and the Christian People's Party) often receive less seats under PR than their ranking according to MR would dictate.

However, we may consider the potentially arbitrary nature of proportional representation by using the same survey data, as we have already utilized, to play around with alternative versions of proportional assignment of seats. That is, we may try to compare the actual election results (in terms of parliamentary seats for individual parties) with simulated election outcomes, where different aggregation methods are applied to the same sets of voter preferences derived from the survey data. Proportional representation with party-lists entails that seats are allocated to political parties according to some measure of representativity, but there is no *apriori* necessity that this only relies on first preferences; a number of other methods of aggregating preferences may result in vote tabulations which may be used as the basis for distributing seats proportionally among the parties. Specifically, we will assume that Denmark is one big electoral district, where parties are running against each other, and where 175 seats are distributed among those parties obtaining more than two percent of the total number of votes and following the so-called Modified Saint-Laguë method.¹³ This is as close to the “real” Danish system as we can possibly get, given the data at hand. Furthermore, we need specific aggregation methods whereby we may transform the individual voters’ “thermometer” values into vote scores. Social choice theory and real-world politics know a vast number of such methods,¹⁴ and without going into details we may briefly summarize five alternative aggregation methods as such:

- *Approval method.* Each voter potentially has as many votes as there are alternatives and can assign one vote to as many alternatives as he likes (in this case the “thermometer” scale of the voter surveys, which have been assigned a positive value); all alternatives are compared once and the values are added.
- *Borda method.* Each voter assigns a number of points on a pre-given scale to the alternatives, e.g., given m alternatives the most preferred alternative is given $m-1$ points, the second most preferred is assigned $m-2$ points, etc., until the least preferred which receives zero points. All alternatives are compared once and the points are added.
- *Bentham method.* Each voter assigns, within a pre-defined scale (in this case the “thermometer” scale of the voter surveys), a value to each alternative; all alternatives are compared once and the values are added.
- *Nash method.* Each voter assigns, within a pre-defined scale (in this case the “thermometer” scale of the voter surveys), a value to each alternative; all alternatives are compared once and the values are multiplied.¹⁵
- *Cumulative method.* Each voter is given an equal, specified number of votes to assign to the alternatives at will, e.g., all to one alternative or distributed among them (in this case 100 votes, assigned to alternatives given a positive value according to the

¹³ The Danish Parliament has a total of 179 seats, but four of these are reserved to the voters of Greenland and the Faroe Islands.

¹⁴ Cf., e.g., Riker 1982; Nurmi 1983; Nurmi 1987; Malkevitch 1990; Levin and Nalebuff 1995. What is done here is somewhat similar to the simulations in, e.g. Lewin 1998; Härd 1999.

¹⁵ For practical purposes—being the very large numbers—the Log10 values have been used as the standard of measure.

“thermometer” values in the voter surveys); all alternatives are compared once and the votes are added.

With this operationalization we may “replay” the seven elections under consideration as summarized in Table 4. Here we have the simulated assignments of seats to the political parties according to alternative aggregation methods of the voter preferences, but with all assignments of seats having been done in a way fundamentally similar to the rules existing in practice. The table also includes correlation coefficients (Pearson’s r) measuring the correlation between the actual assignment of seats in the elections and what would be the result given the individual simulations, as well as the respective standard deviations in the number of seats. What seems clear is that there is considerable variation in the individual assignments of seats to the political parties, depending on the methods used. The Cumulative method typically gives results closest to the actual election results, although they are never identical. The four other methods all produce significantly different seat allocations, with the Nash method always producing the most different.

In the table the parties have been listed in columns from left to right, roughly corresponding to the usual one-dimensional picture of the Danish ideological spectrum and done following analysis of how the parties actually vote in the Danish parliament.¹⁶ Doing so enables us to compare how different voting blocs would do under alternative methods for aggregating votes to be used for proportional assignment of seats. This highlights that the group of socialist parties in every single election would have received significantly less parliamentary seats if one of the five other aggregation methods had been used, than they did in the real elections.

¹⁶ See, e.g., Pedersen 1967; Pedersen, Damgaard and Nannestad Olsen 1971; Damgaard and Rusk 1976; cf. Hansen, Klemmensen and Kurrild-Klitgaard 2004. In this case, the rankings generated by Skjæveland have been used (Skjæveland 2003: 109; 112ff; 125), supplemented by the present author’s interpretation of the 2001-ordering. Skjæveland has placed the Georgist Justice Party (E) in a second dimension—something which we for practical purposes have omitted here and instead followed Skjæveland’s suggestion and located the part between the Conservatives and the Progress Party.

Table 4. Actual and simulated proportional seat allocations given alternative aggregation methods of votes. Danish election surveys and parliamentary elections. 175 parliamentary seats.

												Pearson correlation (r), relative to actual seat allocation	Standard deviation	Seats, left-wing parties
1973	Y	K	F	A	B	M	Q	V	C	E	Z			
Actual election result	0	6	11	46	20	14	7	22	16	5	28		12.97	63
Cumulative method	5	6	12	29	26	17	18	24	16	10	12	0.752	7.92	52
Approval method	6	6	11	24	25	18	19	24	18	12	12	0.637	6.95	47
Bentham method	11	10	13	20	20	17	18	20	17	15	14	0.605	3.59	54
Borda method	10	9	13	21	21	18	18	20	17	15	13	0.598	4.23	53
Nash method	12	10	14	19	20	17	18	19	17	16	13	0.481	3.24	55
1975	Y	K	F	A	B	M	Q	V	C	E	Z			
Actual election result	4	7	9	53	13	4	9	42	10	0	24		16.96	73
Cumulative method	5	5	10	30	21	11	24	29	16	8	16	0.811	9.06	50
Approval method	6	6	10	24	23	13	24	26	17	10	16	0.676	7.48	46
Bentham method	10	9	12	20	19	15	19	25	17	14	15	0.680	4.72	51
Borda method	9	8	12	21	19	14	20	26	17	14	15	0.695	5.38	50
Nash method	10	8	13	20	19	15	19	24	17	15	15	0.648	4.59	51
1977	Y	K	F	A	B	M	Q	V	C	E	Z			
Actual election result	5	7	7	65	6	11	6	21	15	6	26		17.71	84
Cumulative method	7	7	12	36	18	21	13	17	19	13	12	0.821	8.07	62
Approval method	8	7	12	30	19	22	16	18	20	13	10	0.667	6.83	57
Bentham method	12	11	14	24	18	18	16	17	18	15	12	0.670	3.73	61
Borda method	11	10	14	25	18	19	16	17	18	16	11	0.631	4.35	60
Nash method	12	12	15	22	18	18	16	17	18	16	11	0.528	3.27	61
1979	Y	K	F	A	B	M	Q	V	C	E	Z			
Actual election result	6	0	11	68	10	6	5	22	22	5	20		18.84	85
Cumulative method	10	5	16	38	23	11	9	25	20	10	8	0.854	9.80	69
Approval method	11	6	16	30	24	13	12	24	20	11	8	0.739	7.61	63
Bentham method	13	10	16	24	20	14	14	20	18	15	11	0.736	4.23	63
Borda method	12	9	16	25	20	14	14	21	19	15	10	0.728	4.91	62
Nash method	13	11	16	22	20	15	15	19	18	16	10	0.609	3.70	62
1994	∅		F	A	B	D	Q	V	C		Z			
Actual election result	6		13	62	8	5	0	42	27		11		20.58	81
Cumulative method	6		17	35	19	11	8	38	29		12	0.903	11.84	58
Approval method	7		18	31	22	15	11	30	29		12	0.838	8.99	56
Bentham method	12		19	24	21	19	17	24	24		15	0.721	4.28	55
Borda method	11		19	26	21	19	16	25	24		14	0.778	5.13	56
Nash method	12		19	24	21	20	18	23	23		15	0.678	3.97	55
1998	∅	U	F	A	B	D	Q	V	C	O	Z			
Actual election result	5	0	13	63	7	8	4	42	16	13	4		19.27	81
Cumulative method	8	0	19	36	16	14	8	38	21	10	5	0.905	12.10	63
Approval method	8	0	20	31	18	18	10	30	22	11	7	0.835	9.71	59
Bentham method	10	0	19	26	19	19	15	25	21	11	10	0.721	7.66	55
Borda method	9	0	19	27	19	19	14	26	22	11	9	0.745	8.17	55
Nash method	11	4	18	23	18	19	16	22	21	12	11	0.676	5.80	56
2001	∅		F	A	B		Q	V	C	O				
Actual election result	4		12	52	9		4	56	16	22			20.73	68
Cumulative method	7		18	35	20		15	37	26	17		0.903	10.20	60
Approval method	8		20	32	21		14	38	27	15		0.860	9.96	60
Bentham method	14		21	27	22		20	29	25	17		0.767	5.03	62
Borda method	12		21	29	22		19	31	25	16		0.801	6.38	62
Nash method	14		22	27	22		21	27	25	17		0.693	4.61	63

Conclusion

The present paper has investigated the possible empirical occurrence in seven Danish elections of some of the paradoxes and the types of arbitrariness identified by social choice theorists.

The findings of the analysis, in general, corroborate the results of the few previous attempts to investigate the empirical occurrence of the paradoxes in large elections. Specifically, we found no statistically significant examples of the Condorcet Paradox (or other forms of intransitivity), which so far mostly has escaped empirical detection by social choice theorists. Considering the majority-plurality paradoxes, we also did not find any examples of the Majority-Reversal Paradox in the samples. We found only one example of the Condorcet-Winner-Turns-Loser Paradox, but five of the Condorcet-Loser-Turns-Winner Paradox, and one paradox, the More-Preferred-Less-Seats Paradox, occurs in every single of the elections.

It was also demonstrated that when the same data are used to simulate how alternative aggregation methods would assign parliamentary seats to the political parties, the results become quite different—even when a principle of proportionality is still used to assign seats according to the votes.

These results seem somewhat paradoxical indeed. The *raison d'être* of one voter/one vote combined with proportional representation is quite often given as being that because it represents the (first) choices of the voters more proportionally than other methods, it is somehow more democratic and more in tune with a “will of the people.” Yet apparently, it is quite often the case that the preference of a majority of the voters is quite different from what the seat allocations are under proportional representation—and even proportional representation is nothing unequivocal.

Data sources

Danish Voter Survey 1973, 1975, 1977, 1979, 1994, 1998, 2001, Dansk Data Arkiv (Danish Data Archives), Odense.

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Appendix. Calculation of Condorcet scores

Table A-1. Evaluation of political parties, Danish Election Survey 1973. Condorcet comparisons.

	A	B	C	E	F	K	M	Q	V	Y	Z
A		222 * 84 199	289 * 46 168	310 * 90 94	390 * 61 53	421 * 56 28	261 * 95 148	275 * 59 168	230 59 217	403 * 60 33	324 * 60 124
B	199 * 84 222		283 * 159 61	327 * 86 82	341 * 61 101	395 * 43 64	282 * 101 119	269 * 114 117	184 142 177	389 * 58 49	346 * 65 94
C	168 * 46 289	61 * 159 283		229 * 121 145	276 * 67 159	327 * 64 111	187 114 199	149 * 148 202	61 * 158 284	311 * 82 103	272 * 103 128
E	94 * 90 310	82 * 86 327	145 * 121 229		233 * 123 139	313 * 108 73	135 * 116 242	118 * 132 243	102 * 87 305	287 * 144 59	226 * 93 175
F	53 * 61 390	101 * 61 341	159 * 67 276	139 * 123 233		267 * 198 39	126 * 77 298	141 * 57 303	125 * 50 328	239 * 174 84	204 72 228
K	28 * 56 421	64 * 43 395	111 * 64 327	73 * 108 313	39 * 198 267		79 * 75 347	86 * 64 351	88 * 38 378	115 * 221 160	148 * 94 263
M	148 * 95 261	119 * 101 282	199 114 187	242 * 116 135	298 * 77 126	347 * 75 79		155 * 162 185	128 * 103 272	341 * 78 77	264 * 125 115
Q	168 * 59 275	117 * 114 269	202 * 148 149	243 * 132 118	303 * 57 141	351 * 64 86	185 * 162 155		99 * 156 247	339 * 83 74	289 * 94 119
V	217 59 230	177 142 184	284 * 158 61	305 * 87 102	328 * 50 125	378 * 38 88	272 * 103 128	247 * 156 99		369 * 64 64	328 * 82 96
Y	33 * 60 403	49 * 58 389	103 * 82 311	59 * 144 287	84 * 174 239	160 * 221 115	77 * 78 341	74 * 83 339	64 * 64 369		157 * 99 240
Z	124 * 60 324	94 * 65 346	128 * 103 272	175 * 93 226	228 72 204	263 * 94 148	115 * 125 264	119 * 94 289	96 * 82 328	240 * 99 157	

N = 533

Notes: The first number in each cell is the number of votes in favor of the row variable. The second number refers to the number of respondents indifferent between the row and the column variable. The third number refers to the number of votes in favor of the column variable. Shaded cells are "victories" for the row variable. * p < 0.05, i.e., the majority relation between the two alternatives is statistically significant at a 95 pct. confidence level (2-tailed).

Table A-5. Evaluation of political parties, Danish Election Survey 1975. Condorcet comparisons.

	A	B	C	E	F	K	M	Q	V	Y	Z
A		610 * 241 429	688 * 143 438	797 * 161 237	1021 * 126 119	1114 * 89 80	765 * 171 336	598 123 555	521 * 126 644	1057 * 111 91	681 * 142 437
B	429 * 241 610		582 * 302 385	733 * 232 230	852 * 168 241	998 * 128 144	747 * 231 283	459 * 241 565	390 * 205 682	958 * 148 146	653 * 158 440
C	438 * 143 688	385 * 302 582		598 * 292 304	760 * 126 367	906 * 120 232	628 * 356 271	311 * 309 639	230 * 263 775	830 * 173 239	568 * 250 424
E	237 * 161 797	230 * 232 733	304 * 292 598		623 * 233 331	797 * 201 193	450 284 456	232 * 217 742	244 * 151 798	729 * 240 213	466 * 179 541
F	119 * 126 1021	241 * 168 852	367 * 126 760	331 * 233 626		706 * 477 85	421 * 170 667	300 * 148 800	282 * 130 853	600 * 484 167	443 * 145 657
K	80 * 89 1114	144 * 128 998	232 * 120 906	193 * 201 797	85 * 477 706		262 * 204 802	210 * 99 958	200 * 119 965	229 * 658 377	300 * 199 762
M	336 * 171 765	283 * 231 747	271 * 356 628	456 284 450	667 * 170 421	802 * 204 262		149 * 281 832	150 * 215 908	748 * 219 283	418 * 311 522
Q	555 123 598	565 * 241 459	639 * 309 311	742 * 217 232	800 * 148 300	958 * 99 210	832 * 281 149		321 * 357 600	893 * 160 197	699 * 215 340
V	644 * 126 521	682 * 205 390	775 * 263 230	798 * 151 244	853 * 130 282	965 * 119 200	908 * 215 150	600 * 357 321		919 * 132 209	778 * 210 277
Y	91 * 111 1057	146 * 148 958	239 * 173 830	213 * 240 729	167 * 484 600	377 * 658 229	283 * 219 748	197 * 160 893	209 * 132 919		318 * 210 717
Z	437 * 142 681	440 * 158 653	424 * 250 568	541 * 179 466	657 * 145 443	762 * 199 300	522 * 311 418	340 * 215 699	277 * 210 778	717 * 210 318	

N = 1,143
Note: See previous table.

Table A-6. Evaluation of political parties, Danish Election Survey 1977. Condorcet comparisons.

	A	B	C	E	F	K	M	Q	V	Y	Z
A		1001 * 247 122	889 * 201 284	1039 * 141 152	1128 * 110 147	1225 * 86 90	894 * 199 307	1024 * 161 206	939 * 158 306	1173 * 106 102	1061 * 101 239
B	122 * 247 1001		519 349 479	634 * 340 343	804 * 229 323	1004 * 154 204	489 * 284 591	671 * 285 408	652 * 248 467	914 * 203 237	876 * 127 359
C	284 * 201 889	479 349 519		652 * 253 413	797 * 159 402	927 * 158 282	471 * 382 513	658 * 392 310	599 * 343 428	891 * 165 301	917 * 179 268
E	152 * 141 1039	343 * 340 634	413 * 253 652		611 * 333 381	814 * 292 219	391 * 222 711	537 293 495	563 216 551	771 * 300 253	763 * 172 387
F	147 * 110 1128	323 * 229 804	402 * 159 797	381 * 333 611		748 * 507 128	379 * 178 819	474 * 223 675	482 * 224 670	656 * 512 204	710 * 199 465
K	90 * 86 1225	204 * 154 1004	282 * 158 927	219 * 292 814	128 * 507 748		282 * 149 958	336 * 225 820	354 * 200 837	320 * 644 416	554 300 535
M	307 * 199 894	591 * 284 489	513 * 382 471	711 * 222 391	819 * 178 379	958 * 149 282		671 * 441 273	638 * 331 424	927 * 186 262	875 * 265 251
Q	206 * 161 1024	408 * 285 671	310 * 392 658	495 293 537	675 * 223 474	820 * 225 336	273 * 441 671		398 * 442 549	776 * 259 337	765 * 292 328
V	306 * 158 939	467 * 248 652	428 * 343 599	551 216 563	670 * 224 482	837 * 200 354	424 * 331 638	549 * 442 398		785 * 252 340	839 * 256 300
Y	102 * 106 1173	237 * 203 914	301 * 165 891	253 * 300 771	204 * 512 656	416 * 644 320	262 * 186 927	337 * 259 776	340 * 252 785		552 * 328 494
Z	239 * 101 1061	359 * 127 876	268 * 179 917	387 * 172 763	465 * 199 710	535 300 554	251 * 265 875	328 * 292 765	300 * 256 839	494 * 328 552	

N = 1,602
Note: See previous table.

Table A-7. Evaluation of political parties, Danish Election Survey 1979. Condorcet comparisons.

	A	B	C	E	F	K	M	Q	V	Y	Z
A		1085 * 339 295	1079 * 176 493	1303 * 173 178	1320 * 153 256	1528 * 98 125	1286 * 162 291	1288 * 175 256	1011 * 202 538	1410 * 101 184	1399 * 103 242
B	295 * 339 1085		861 * 270 571	1063 * 345 223	998 * 289 394	1349 * 170 172	1088 * 249 349	1074 * 277 319	774 * 260 656	1161 * 225 263	1246 * 160 275
C	493 * 176 1079	571 * 270 861		818 * 316 515	879 * 178 659	1173 * 209 349	1032 * 404 286	995 * 398 316	490 * 487 756	1000 * 214 473	1281 * 239 198
E	178 * 173 1303	223 * 345 1063	515 * 316 818		586 * 394 661	1046 * 344 256	762 * 312 568	724 * 360 548	418 * 271 953	800 * 367 448	1017 * 240 376
F	256 * 153 1320	394 * 289 998	659 * 178 879	661 * 394 586		1187 * 445 91	828 * 225 659	817 * 219 661	585 * 208 920	885 * 524 273	1044 * 211 449
K	125 * 98 1528	172 * 170 1349	349 * 209 1173	256 * 344 1046	91 * 445 1187		452 * 346 935	433 * 321 963	292 * 185 1257	248 * 616 825	671 * 462 598
M	291 * 162 1286	349 * 249 1088	286 * 404 1032	568 * 312 762	659 * 225 828	935 * 346 452		473 755 485	216 * 352 1155	754 * 304 623	944 * 460 316
Q	256 * 175 1288	319 * 277 1074	316 * 398 995	548 * 360 724	661 * 219 817	963 * 321 433	485 755 473		202 * 349 1160	767 * 295 611	996 * 383 330
V	538 * 202 1011	656 * 260 774	756 * 487 490	953 * 271 418	920 * 208 585	1257 * 185 292	1155 * 352 216	1160 * 349 202		1056 * 244 391	1335 * 196 197
Y	184 * 101 1410	263 * 225 1161	473 * 214 1000	448 * 367 800	273 * 524 885	825 * 616 248	623 * 304 754	611 * 295 767	391 * 244 1056		859 * 345 479
Z	242 * 103 1399	275 * 160 1246	198 * 239 1281	376 * 240 1017	449 * 211 1044	598 * 462 671	316 * 460 944	330 * 383 996	197 * 196 1335	479 * 345 859	

N = 1,989

Note: See previous table.

Table A-5. Evaluation of political parties, Danish Election Survey 1994. Condorcet comparisons.

	A	B	C	D	F	Q	V	Z	Ø
A		564 * 299 180	505 * 110 430	639 * 197 204	675 * 190 172	695 * 171 178	487 101 455	715 * 116 212	842 * 120 73
B	180 * 299 564		383 * 180 480	456 * 329 254	506 * 242 288	589 * 240 214	417 * 120 506	638 * 138 266	740 * 174 121
C	430 * 110 505	480 * 180 383		560 * 223 256	572 * 155 311	689 * 183 173	345 330 369	776 * 149 119	765 * 105 166
D	204 * 197 639	254 * 329 456	256 * 223 560		424 * 254 358	508 * 287 246	328 * 173 539	603 * 172 264	710 * 174 148
F	172 * 190 675	288 * 242 506	311 * 155 572	358 * 254 424		477 * 211 349	355 * 130 551	537 * 178 322	711 * 223 96
Q	178 * 171 695	214 * 240 589	173 * 183 689	246 * 287 508	349 * 211 477		231 * 159 654	478 * 228 337	587 * 260 189
V	455 101 487	506 * 120 417	369 330 345	539 * 173 328	551 * 130 355	654 * 159 231		764 * 190 88	724 * 120 191
Z	212 * 116 715	266 * 138 638	119 * 149 776	264 * 172 603	322 * 178 537	337 * 228 478	88 * 190 764		506 * 243 286
Ø	73 * 120 842	121 * 174 740	166 * 105 765	148 * 174 710	96 * 223 711	189 * 260 587	191 * 120 724	286 * 243 506	

N = 2,021

Notes: See previous table.

Table A-6. Evaluation of political parties, Danish Election Survey 1998. Condorcet comparisons.

	A	B	C	D	F	O	Q	U	V	Z	Ø
A		1172 * 404 320	1048 * 172 710	1095 * 249 561	1279 * 236 392	1399 * 144 392	1233 * 234 402	1076 * 92 46	924 135 899	1446 * 152 332	1439 * 173 172
B	320 * 404 1172		760 * 257 869	732 * 342 789	823 * 352 688	1194 * 166 517	963 * 292 572	939 * 188 70	714 * 168 1013	1190 * 230 459	1170 * 294 290
C	710 * 172 1048	869 * 257 760		806 * 418 663	961 * 174 759	1376 * 196 342	1094 * 319 437	999 * 104 103	607 * 387 938	1438 * 225 252	1221 * 157 397
D	561 * 249 1095	789 * 342 732	663 * 418 806		868 * 301 707	1309 * 162 419	1003 * 384 450	972 * 147 83	665 * 248 993	1334 * 216 340	1187 * 220 348
F	392 * 236 1279	688 * 352 823	759 * 174 961	707 * 301 868		1109 * 265 522	889 * 283 661	911 * 236 57	724 * 145 1040	1158 * 223 516	1210 * 409 147
O	392 * 144 1399	517 * 166 1194	342 * 196 1376	419 * 162 1309	522 * 265 1109		492 * 332 1028	573 * 418 218	238 * 224 1475	573 * 896 448	733 * 368 675
Q	402 * 234 1233	572 * 292 963	437 * 319 1094	450 * 384 1003	661 * 283 889	1028 * 332 492		828 * 274 97	398 * 240 1227	1082 * 358 411	983 * 285 467
U	46 * 92 1076	70 * 188 939	103 * 104 999	83 * 147 972	57 * 236 911	218 * 418 573	97 * 274 828		104 * 121 987	219 * 422 570	132 * 570 488
V	899 135 924	1013 * 168 714	938 * 387 607	993 * 248 665	1040 * 145 724	1475 * 224 238	1227 * 240 398	987 * 121 104		1538 * 250 147	1231 * 148 406
Z	332 * 152 1446	459 * 230 1190	252 * 225 1438	340 * 216 1334	516 * 223 1158	448 * 896 573	411 * 358 1082	570 * 422 219	147 * 250 1538		697 * 443 636
Ø	172 * 173 1439	290 * 294 1170	397 * 157 1221	348 * 220 1187	147 * 409 1210	675 * 368 733	467 * 285 983	488 * 570 132	406 * 148 1231	636 * 443 697	

N = 2,001

Notes: See previous table.

Table A-7. Evaluation of political parties, Danish Election Survey 2001. Condorcet comparisons.

	A	B	C	F	O	Q	V	Ø
A		1113 * 379 401	956 * 172 794	1228 * 260 413	1325 * 130 517	1148 * 238 483	811 * 135 1031	1507 * 143 181
B	401 * 379 1113		708 * 253 909	824 * 355 673	1093 * 159 636	871 * 294 657	646 * 127 1119	1243 * 290 258
C	794 * 172 956	909 * 253 708		982 * 215 679	1335 * 194 392	1028 * 312 503	500 * 361 1065	1240 * 173 391
F	413 * 260 1228	673 * 355 824	679 * 215 982		1028 * 229 646	814 * 282 735	629 * 142 1137	1262 * 360 185
O	517 * 130 1325	636 * 159 1093	392 * 194 1335	646 * 229 1028		550 * 277 1045	253 * 205 1517	863 * 279 688
Q	483 * 238 1148	657 * 294 871	503 * 312 1028	735 * 282 814	1045 * 277 550		440 * 210 1226	1093 * 277 405
V	1031 * 135 811	1119 * 127 646	1065 * 361 500	1137 * 142 629	1517 * 205 253	1226 * 210 440		1313 * 147 376
Ø	181 * 143 1507	258 * 290 1243	391 * 173 1240	185 * 360 1262	688 * 279 863	405 * 277 1093	376 * 147 1313	

N = 2,026

Notes: See previous table.

Due to an error made by the Danish Election Project, two parties which stood for the election but did not receive any seats (D and Z) were omitted in the survey. The 2001 data thus only includes preferences over represented parties.

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