GEM: Game Theory – Economics and Mathematics

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Preface

Game theory is a branch of applied mathematics that is mainly used in economics, political science, psychology, logic, computer science, and evolutionary biology. The international GEM workshop gathers scientists predominantly from economics and mathematics departments for scientific presentations and discussions of mathematical foundations of game theory as well as of applications of game theory in economics, particularly in microeconomics.

Abstracts

Gustavo Bergantiños (Universidade de Vigo), Fr 10:00–10:45

How to apply penalties to avoid delays in projects

A planner wants to carry out a project involving several firms. In many cases the planner, for instance the Spanish Administration, includes in the contract a penalty clause that imposes a payment per day if the firms do not complete their activities or the project on time. We discuss two ways of including such penalty clauses in contracts. In the first the penalty applies only when the whole project is delayed. In the second the penalty applies to each firm that incurs a delay even if the project is completed on time. We compare the two penalty systems and find that the optimal penalty (for the planner) is larger in the second method, the utility of the planner is always at least as large or larger in the second case and the utility of the firms is always at least as large or larger in the first. Surprisingly, the final delay in the project is unrelated to which penalty system is chosen.

Joint work with L. Lorenzo.

Françoise Forges (CEREMADE and LEDa, Université Paris-Dauphine), Fr 9:15–10:00

Strategic information transmission with sender's approval

– We consider a sender-receiver game in which the sender has a finite set of types, the receiver makes a decision in a compact convex set X and the (typically type-dependent) utility functions are continuous. We assume that, after the cheap talk phase, the receiver makes a proposal to the sender, which the latter can reject in favor of an outside option, and that the sender's approval is crucial to the receiver. We ask whether the game has a perfect Bayesian equilibrium (PBE).
– We construct a counter-example (with three types for the sender and type-dependent affine utility functions) in which there is no PBE, but there is a communication equilibrium.
– We show that a PBE exists if either (i) the sender only has two types or (ii) the decision set X is a real interval and the sender's utility function is monotonic, whatever his type or (iii) the receiver's utility function does not depend on the sender's type.
– We show that a communication equilibrium always exists when the sender has three types and the utility functions are affine.

Joint ongoing work with Jérôme Renault.

Michel Grabisch (University Paris 1 Panthéon-Sorbonne, Paris School of Economics), Sa 9:15–10:00

A model of anonymous influence with anti-conformist agents

We study a stochastic model of anonymous influence with conformist and anti-conformist individuals. Each agent with a 'yes' or 'no' initial opinion on a certain issue can change his opinion due to social influence. We consider anonymous influence, which depends on the number of agents having a certain opinion, but not on their identity. An individual is conformist/anti-conformist if his probability of saying 'yes' increases/decreases with the number of 'yes'-agents. We focus on three classes of aggregation rules (pure conformism, pure anti-conformism, and mixed aggregation rules) and examine two types of society (without, and with mixed agents). For both types we provide a complete qualitative analysis of convergence, i.e., identify all absorbing classes and conditions for their occurrence. Also the pure case with infinitely many individuals is studied. We show that, as expected, the presence of anti-conformists in a society brings polarization and instability: polarization in two groups, fuzzy polarization (i.e., with blurred frontiers), cycles, periodic classes, as well as more or less chaotic situations where at any time step the set of 'yes'-agents can be any subset of the society. Surprisingly, the presence of anti-conformists may also lead to opinion reversal: a majority group of conformists with a stable opinion can evolve by a cascade phenomenon towards the opposite opinion, and remains in this state.

Joint work with Alexis Poindron and Agnieszka Rusinowska.

Jens Leth Hougaard (University of Copenhagen), Fr 17:00–17:45

Core stability in minimum cost connection networks

We consider a generalization of the classic Minimum Cost Spanning Tree Model dubbed the Minimum Cost Connection Networks Model, where agents no longer are identified by the set of nodes (all wanting to connect to the same source) but are allowed to demand connection between any pair of nodes. As in the MCST-model the problem of finding a cost minimal network satisfying all connection demands induces a cooperative game where the coalitional values are given by the minimal cost of satisfying the demands of member in the coalition only. For the MCST-model this game is balanced. For the more general MCCN-model this may no longer be the case. We characterize classes of MCCN-problems for which the induced (cost) game is balanced.

Ehud Lehrer (Tel Aviv University), Fr 11:00–11:45

<div align="center">Set-valued game</div>

We study allocation problems when agents negotiate across different agendas. Unlike existing papers on multi-agenda disputes, we consider environments in which resources are constrained and investing (say, time or effort) in one agenda reduces the ability to invest in other agendas. In order to analyze such a scenario, we introduce a class of cooperative games, referred to as set-valued - SVG. The value of each coalition in an SVG is a subset of payoff vectors. Each vector is associated with a specific distribution of the resources that the coalition may allocate across the different agendas. In this environment we introduce and analyze the notion of the core. We show that the core of an SVG allows for more cooperation opportunities and exchanging favors than existing cooperative multi-agenda models. Interestingly, proving this relies on a general notion of a comparative advantage of a player. In addition, it is shown that the classical core characterization, resorting to duality, does not hold in the current setup.

Joint work with Roee Teper.

Juan D. Moreno-Ternero (Universidad Pablo de Olavide), Fr 11:45–12:30

<div align="center">**Sharing the revenues from broadcasting sport events**</div>

We study the problem of sharing the revenue from broadcasting sport league events, among participating teams. We provide direct, axiomatic and game-theoretical foundations for two focal rules: the *equal-split* rule and *concede-and-divide*. The former allocates the revenues generated from broadcasting each game equally among the participating teams in the game. The latter concedes each team the revenues from its fan base and divides equally the residual. We also provide an application studying the case of sharing the revenue from broadcasting games in *La Liga*, the Spanish Football League. We show that hybrid schemes, combining our rules with lower bounds and performance measures, yield close outcomes to the current allocation being implemented by the Spanish National Professional Football League Association.

Joint work with Gustavo Bergantiños.

Trine Tornøe Platz (Copenhagen Business School), Fr 14:30–15:15

Equilibrium arrivals to a queue under last-come-first-serve preemptive-resume

We consider a queueing environment where a finite number of customers independently choose when to arrive at a queueing system that opens at a specific time and serves customers on a last-come first-serve preemptive-resume (LCFS-PR) basis. Each customer has a service time requirement that is identically and independently distributed, and customers want to complete service as early as possible while minimizing the time spent in the queue. We establish the existence of a symmetric (mixed) Nash equilibrium and show that there is at most one symmetric equilibrium. We provide a numerical method to compute this equilibrium and demonstrate it by an example in which the social efficiency is lower than that induced by a similar queueing system that serves customers on a first-come first-serve (FCFS) basis.

Joint work with Jesper Breinbjerg and Lars Peter Østerdal.

Agnieszka Rusinowska (University Paris 1 Panthéon-Sorbonne, Paris School of Economics), Fr 13:45–14:30

The degree ratio ranking method for directed networks

One of the most famous ranking methods for digraphs is the ranking by Copeland score. The Copeland score of a node in a digraph is the difference between its outdegree (i.e. its number of outgoing arcs) and its indegree (i.e. its number of ingoing arcs). In the ranking by Copeland score a node is ranked higher, the higher is its Copeland score. In this paper, we deal with an alternative to rank nodes according to their out- and indegree, namely ranking the nodes according to their degree ratio, i.e. the outdegree divided by the indegree. In order to avoid dividing by a zero indegree, we implicitly take the out- and indegree of the reflexive digraph. We provide an axiomatization of the ranking by degree ratio using a clone property which says that cloning a node (in a specific way) does not change the ranking among the original nodes. We also provide a new axiomatization of the ranking by Copeland score using the same axioms except that this ranking method satisfies a different cloning property. Finally, we modify the ranking by degree ratio by not considering the reflexive digraph, and by definition assume nodes with indegree zero to be ranked higher than nodes with a positive indegree. We provide an axiomatization of this ranking by modified degree ratio using another clone property and a maximal property.

Joint work with René van den Brink.

Andrés Salamanca (SDU), Sa 11:45–12:30

Optimal mediators in conflict resolution

We study the design of optimal mediation protocols in the framework of Mitusch and Strausz (2005). In this model, an agent possesses private information about a binary state of the world. This information is required by an uninformed principal in order to take an action in the real line. Individuals have quadratic preferences, with a difference in their bliss point parametrized by a state-dependent bias parameter. We consider mediated communication in which a neutral trustworthy mediator gathers information from the agent and makes non-binding recommendations to the principal. The agent's report is not verifiable either by the mediator or by the principal, which allows the agent to strategically manipulate his private information. We compare the scenarios in which every individual is given the right to control the communication channels by choosing a mediator. In both cases, we show that a necessary condition for mediation to be helpful is that the likelihood of a conflict of interests be not too high. This condition also turns out to be sufficient when the principal chooses the mediator. When the agent chooses the mediator, however, sufficiency further requires that the difference in the degree of conflict between both states be not too large. We also find that optimal mediation protocols coincide provided that mediation is helpful for the agent.

Karol Szwagrzak (Copenhagen Business School), Sa 10:00–10:45

Solidarity in the allocation of resources under a budget constraint

We model the use of a budget - a finite amount of money - to procure private consumption goods for a group of individuals. We describe the class of procurement mechanisms satisfying normatively desirable properties. These include: efficiency, solidarity, and consistency.

Ryan Tierney (SDU), Fr 15:30–16:15

Incentives in a job-market clearinghouse

We characterize the set of pairwise strategy-proof and non-discriminatory rules for allocating heterogeneous objects or positions, and monetary transfers, when there is unit-demand. We name the resulting class Endogenous Null Min-Price rules. Unlike previous studies, we do not require full distribution of the objects or any restriction on the transfer associated with the null. We thus provide novel solutions to the one-to-one matching with transfers problem: Endogenous Null Min-Price rules allow firms to demand reservation profits, and allow for unemployed workers

to receive subsidies. Moreover, these subsidies can increase in the number of agents allocated jobs (not all need be filled). The Endogenous Null Min-Price rules are a finite dimensional family of lattice-extremal rules. Each is given by a list of reserve prices, one for each real object, and possibly several for the null object. For each economy, the rule then selects a minimal price equilibrium allocation that respects these reserves, with the effective reserve of the null depending on the number of agents who get real objects. The family includes both min-price Walras and (for the one-object case) Sprumont's (2013) maxmed family. We also extend some existing results, from one to multidimensional preferences, for the case when full-distribution of the objects is required. Here we provide a characterization of Min-price Walras in terms of strategy-proofness, no-discrimination, and respect of the outside option.

José M. Zarzuelo (Basque University of Bilbao), Sa 11:00–11:45

A value for cost allocation under restricted cooperation

In the classical approach to cooperative games it is assumed that the worth of every coalition is known. However, in the real world problems there may be situations in which the amount of information is limited and consequently the worth of some coalitions is unknown. The games corresponding to those problems are called partially defined cooperative games and surprisingly have not received yet enough attention. Partially defined cooperative games were first studied by Wilson (1993). However, this author restricted the attention to partially defined games in which if the worth of a particular coalition is known, then it is also known the worth of all the coalitions with the same cardinality. Moreover, Wilson (1993) proposed and characterized an extension of the Shapley value for partially defined cooperative games. This extended Shapley value coincides with the ordinary Shapley value of a complete game (we say that a game is complete if the worth of all the coalitions are known). In this complete game the coalitions whose worth were known in the original game maintain the same worth, but otherwise they are assigned a worth zero, that seems to be not well justified. In this work we propose another extension of the Shapley value for general partially defined cooperative games by following the Harsanyi's approach. That is, it is assumed that each coalition guarantees certain payments, called the Harsanyi dividends (Harsanyi, 1963), to its members. We assume that coalitions whose worth is not known assign a dividend equal to zero. The final payoff will be the sum of these dividends. Moreover, an alternative definition of the proposed value is given using the marginal contributions of the players in certain orderings. Further, we characterize the proposed value using four axioms. Three of them are the well known axioms of carrier, additivity and positivity. The fourth one, called indispensable coalition axiom, is a weaker version of the anonymity axiom when referring to full defined games.

Joint work with M. Josune Albizuri and Satoshi Masuya.

Lars Peter Østerdal (Copenhagen Business School), Fr 16:15–17:00

Optimal management of evolving hierarchies

We study the optimal management of evolving hierarchies. An initiator invests into finding a subordinate, who will bring revenues to the joint venture and who will invest himself into finding another subordinate, and so on. The higher the individual investment (which is private information), the higher the probability of finding a subordinate (among a pool of equally valuable candidates). The initiator sets a transfer scheme specifying how revenues are reallocated, via upward transfers, as the hierarchy evolves. We show that the socially optimal investment profile (which is independent of the transfer scheme) is unique and constant among members of the hierarchy. Each transfer scheme induces a game in which agents decide their investment choices. We consider two decentralized optimality notions for schemes: initiator-optimal and socially-optimal schemes. We show that the former are schemes imposing to each member a full transfer to two recipients (the predecessor and the initiator) with a constant ratio among the transfers. We show that the latter are schemes with a full transfer to the predecessor.

Joint work in progress with Jens Leth Hougaard and Juan D. Moreno-Ternero.

Speakers

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