

GEM: Game Theory – Economics and Mathematics

6th International Workshop

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University of Southern Denmark at Odense

Hosted by Department of Business and Economics

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Preface

Game theory is a branch of applied mathematics that is mainly used in economics, political science, psychology, logic, computer science, and evolutionary biology. The international GEM workshop gathers scientists predominantly from economics and mathematics departments for scientific presentations and discussions of mathematical foundations of game theory as well as of applications of game theory in economics, particularly in microeconomics.

Program

All regular sessions take place in the DBE Seminar Room

- 9:15–9:25 Arrival
- 9:25–9:30 Welcome
- 9:30–11:00 *Session 1* (Chair: Karol Szwagrzak)
Agnieszka Rusinowska (Paris School of Economics)
A model of anonymous influence with anti-conformist agents
Michel Grabisch (Paris School of Economics)
Characterization of TU games with stable cores by nested balancedness
- 11:00–11:15 Coffee break
- 11:15–12:45 *Session 2* (Chair: Rafael Treibich)
Bernhard von Stengel (London School of Economics)
Index and uniqueness of symmetric equilibria
José M. Zarzuelo (Basque University of Bilbao and SDU)
The least square approach on discrete cost allocation problems
- 12:45–14:00 Lunch
- 14:00–15:15 *Session 3* (Chair: Agnieszka Rusinowska)
Karol Szwagrzak (SDU, Copenhagen Business School)
Additive adjudication of conflicting claims
Lone Grønbaek (SDU)
Co-management and cooperative self-enforcement
- 15:15–15:30 Coffee break
- 15:30–17:30 *Session 4* (Chair: Bernhard von Stengel)
Rafael Treibich (SDU)
(Over)weighting nations: Optimal decision rules under participation constraints
Trine Tornøe Platz (SDU)
On totally balanced, submodular and PMAS-admissible weighted minimum coloring games
Peter Sudhölter (SDU)
On the generic finiteness of equilibrium distributions for outcome game forms
A small reception
- 17:30–18:00 Piano concert in O77 by Michel Grabisch
- 19:00– Dinner

Abstracts

Michel Grabisch (University Paris 1 Panthéon-Sorbonne, Paris School of Economics), 10:15–11:00

Characterization of TU games with stable cores by nested balancedness

We study necessary and sufficient conditions for the core of a TU game to be stable in the sense of von Neumann-Morgenstern (no domination among elements of the core and any pre-imputation not in the core is dominated by a core element). We introduce a different form of dominance relation, called outvoting, which is transitive and happens to be equivalent to domination when comparing a core element with a pre-imputation. Moreover, we restrict our attention to strong vital-exact coalitions, which determine unambiguously the core. We begin by establishing a necessary and sufficient condition for an element to be outvoted by another one via some strongly vital-exact coalition S , similar to balancedness. This leads to a first necessary and sufficient condition for core stability, involving however a variable x , where x is a core element. The second step consists in getting rid of this variable, by introducing a second kind of balancedness condition. The final condition obtained involves then a finite number of tests.

Joint work with Peter Sudhölter

Lone Grønbaek (SDU), 14:45–15:15

Co-management and cooperative self-enforcement

The paper focuses on the combination of the cooperative fisheries management with the non-cooperative literature. It has important modeling contributions by creating a new framework on co-management. The structure of the model is repeated coalition game with the possibility to deviate combined with introduction of a variation of a club good model. The club good is defined as an information flow in the coalition implying acting as a singleton is more costly compared to joint production. This model provides a more optimistic view of coalition formation than is traditionally seen. The model can explain the real world cases of strong fishing community cooperation arising even with a large set of players.

Joint work with Marko Lindroos.

Trine Tornøe Platz (SDU), 16:10–16:40

On totally balanced, submodular and PMAS-admissible weighted minimum coloring games

In this paper, we introduce the weighted minimum coloring (WMC) games, which is a class of cooperative combinatorial optimization games. A graph $G = (N, E)$ and a positive integer weight vector w that assigns a weight to each vertex in N induce a WMC game. A graph G is said to be globally (respectively, locally) WMC totally balanced, submodular, or PMAS-admissible, if for all positive integer weight vectors w (respectively, for at least one positive integer weight vector w), the corresponding WMC game is totally balanced, submodular, or admits a PMAS. We show that a graph G is globally WMC totally balanced if and only if it is perfect, and that any graph G is locally WMC totally balanced. Furthermore, we show that G is globally (respectively, locally) WMC submodular if and only if it is complete r -partite (respectively, $(2K_2, P_4)$ -free). Finally, we show that G is globally PMAS-admissible if and only if it is $(2K_2, P_4)$ -free.

Joint work with Herbert Hamers, Nayat Horozoglu, and Henk Norde.

Agnieszka Rusinowska (University Paris 1 Panthéon-Sorbonne, Paris School of Economics), 9:30–10:15

A model of anonymous influence with anti-conformist agents

We study a stochastic model of anonymous influence with conformist and anti-conformist individuals. Each agent with a ‘yes’ or ‘no’ initial opinion on a certain issue can change his opinion due to social influence. We consider anonymous influence which depends on the number of agents having a certain opinion, but not on their identity. An individual is conformist/anti-conformist if his probability of saying ‘yes’ increases/decreases with the number of ‘yes’-agents. In order to consider both conformists and anti-conformists in a society, we investigate a generalized aggregation mechanism. It uses the ordered weighted averages which are the only anonymous aggregation functions. Additionally, every agent has a coefficient of conformism which is a real number from -1 till 1, with the two extreme values corresponding to a pure anti-conformist and a pure conformist, respectively. We assume that both pure conformists and anti-conformists are present in a society, and we deliver a qualitative analysis of convergence in the model, i.e., find all terminal classes and conditions for their occurrence.

Joint work with Michel Grabisch and Alexis Pointron.

Peter Sudhölter (SDU), 16:40–17:10

On the generic finiteness of equilibrium distributions for outcome game forms

We recall the generic finiteness of the number of probability distributions on outcomes induced by Nash equilibria for two-person game forms such that either (i) one of the players has no more than two strategies or (ii) both of the players have three strategies, and (iii) for outcome game forms with three players, each with at most two strategies. We also recall a counterexample of a game form with three outcomes and three players. Finally, we provide sufficient and necessary conditions for the generic finiteness of the number of equilibrium distributions associated to a bimatrix outcome game form. These equivalent conditions are stated in terms of the ranks of two matrices constructed from the original game form.

Joint work with Cristian Litan and Francisco Marhuenda

Karol Szwagrzak (SDU, Copenhagen Business School), 14:00–14:45

Additive adjudication of conflicting claims

In a "claims problem" (O'Neill, 1982), a group of individuals have claims on a resource but its endowment is not sufficient to honor all of the claims. We examine when a collection of claims problems can be consolidated into a single problem, adding the claims and endowments across the problems. Since an individual may be prioritized differently across problems, consolidation is only meaningful if all individuals are revealed to deserve compensation in each of the underlying problems. Requiring consolidation to be welfare neutral implies that each individual's award in the consolidated problem is equal to the sum of her awards in the original problems. The only consistent and resource monotonic adjudication rules satisfying this requirement are generalizations of the canonical "constrained equal losses rule" sorting claimants into priority classes and distributing the amount available to each class using a weighted constrained equal losses rule. The "constrained equal awards rule" is the only symmetric rule in this class.

Joint work with Jaume Garcia-Segarra and Miguel Gines-Vilar

Rafael Treibich (SDU), 15:30–16:10

(Over)weighting nations: Optimal decision rules under participation constraints

We study the design of voting rules for international unions when countries' participation is voluntary. While efficiency recommends weighting countries proportionally to their stakes, we show that accounting for participation constraints entails overweighting some countries, those for which the incentive to participate is the lowest. We consider two extensions. First, we argue that allowing a sub-union of countries to cooperate can mitigate the cost of inducing participation. Second, we show that when collective decisions cannot be properly enforced, the participation constraints intensify and require giving a veto power to certain countries. The model has important implications for the problem of apportionment, the allocation of voting weights to countries of differing populations, where it recommends setting a minimum representation for small countries.

Bernhard von Stengel (London School of Economics), 11:15–12:00

Index and uniqueness of symmetric equilibria

In a symmetric two-player game, a symmetric equilibrium can only be dynamically stable if it has positive index. The sum of the indices of all equilibria is 1, so a unique equilibrium has index 1. The index is a topological notion related to geometric orientation, and defined in terms of the sign of the determinant of the payoffs in the equilibrium support. We prove a simple strategic characterization of the index conjectured by Hofbauer: In a nondegenerate symmetric game, an equilibrium has index 1 if and only if it is the unique equilibrium in a larger symmetric game obtained by adding further strategies (it suffices to add a linear number of strategies).

Our elementary proof introduces “unit-vector games” where one player's payoff matrix consists of unit vectors, and applies in a novel way simplicial polytopes. In addition, we employ a very different known result that any matrix with positive determinant is the product of three P-matrices, a class of matrices important in linear complementarity. We conjecture that in our setting, even two P-matrices suffice, which would give the smallest possible number of strategies to be added.

Joint work with Anne Balthasar.

José M. Zarzuelo (Basque University of Bilbao and SDU), 12:00–12:45

The least square approach on discrete cost allocation problems

The discrete Aumann-Shapley cost-sharing method is a classical method for solving cost-sharing problems in which agents consume a quantity of possibly different goods and the consumption of each agent is represented by a non-negative integer. Unlike the Aumann-Shapley cost-sharing method for perfect divisible goods, the discrete method has not been given much attention in order to be characterized until the papers of Calvo and Santos (2000), Sprumont (2005) and Albizuri et al. (2015). In the last one, authors give an axiomatic characterization employing a monotonicity axiom and a merging axiom introduced by Sprumont (2005). In this monotonicity axiom marginal contributions of a player in different problems are compared. In this work we will consider a monotonicity axiom with marginal contributions of different agents in the same problem. The discrete Aumann-Shapley will be axiomatically characterized employing this new axiom. On the other hand, we define a new cost-sharing method, the least square prenucleolus for discrete problems, by means of the least square prenucleolus for transferable utility games (Ruiz et al., 1996). This new method will be characterized by several axioms, among them an average marginal monotonicity axiom.

Keywords: cost sharing, monotonicity, Aumann-Shapley method, least square prenucleolus.

Joint work with M. Josune Albizuri.

Speakers

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