Macro-finance and factor timing: Time-varying factor risk and price of risk premiums

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Abstract

This paper documents empirically that increases in the book-to-market spread predict larger market premiums in sample and larger size, value, and investment premiums (also) out of sample. In addition, increases in the investment (or profitability) spread exclusively predict larger investment (or profitability) premiums. This predictability generates "factor timing" strategies that deliver substantial economic gains out of sample. I argue theoretically that the book-to-market spread is a price of risk proxy, while the investment and profitability spreads are factor risk proxies. The evidence confirms standard theoretical predictions in the macro-finance literature and contradicts the hypothesis of constant factor risks.

JEL Codes: G11, G12, G14.

Keywords: Out of sample, factor timing, time-varying risk, macro-finance, Fama and French.

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1 Introduction

Factor premiums, such as the ones in Fama and French (2015), are only forecastable if there is predictable variation over time (i) in the price of risk or (ii) in the factor risks. On the one hand, several macro-finance theories claim that risk premiums for the entire economy vary over time due to changes in the price of risk.[1] Under this hypothesis, there should be evidence of (common) predictability of the factor premiums related to variation in the price of risk. On the other hand, the factors in Fama and French (2015) are often assumed to have constant risks. For example, Fama and French (1997) argue that one of the reasons for not using industry portfolios for risk adjustment purposes is precisely their time-varying risks. Under this hypothesis, there should be no evidence of (independent) predictability of the factor premiums due to changes in factor risks. The present paper empirically investigates these two types of predictability, thereby testing the two hypotheses associated with them.

In summary, the paper provides in-sample and out-of-sample evidence of predictable time variation in both the (common) price of risk and in the (idiosyncratic) factor risks. This confirms the macro-finance hypothesis and supports risk-based explanations of the factor premiums in general but contradicts the constant factor risk hypothesis. In doing so, the paper also delivers the main ingredients of a "factor timing" strategy.[2] For example, a strategy that applies the trading rule of Daniel and Moskowitz (2016) to the findings that I report generates significant economic gains out of sample.

Regarding (i) in the first paragraph, increases in the cross-sectional book-to-market (BM) spreads significantly forecast increases in one-month ahead premiums in sample for every factor portfolio in Fama and French (2015) except profitability.[3] The market premium

[1] Examples of these models include Rietz (1988), Epstein and Zin (1989), Constantinides and Duffie (1996), Campbell and Cochrane (1999), Hansen and Sargent (2001), Bansal and Yaron (2004), Barro (2006), Piazzesi et al. (2007), Brunnermeier (2009), Bansal et al. (2012), Shiller (2014), Garleanu and Panageas (2015), and de Oliveira Souza (2019b).

[2] These strategies are a type of "managed portfolio", described in Cochrane (2005), in which the wealth allocated to portfolio factors, such as the ones in Fama and French (2015), changes over time according to a given signal that forecasts the factor premiums.

[3] The evidence in Section 3.2 is consistent with the hypothesis that, from a portfolio perspective, the profitability factor becomes safer as the BM spread increases, which could explain the lack of predictability.

is only predictable in the full sample starting in 1926; not in the shorter sample in which the remaining factor premiums are available (from 1963). However, the BM spread recursively forecasts the other three factor premiums out of sample as well.

I explain in Section 2.1.1 that size-related variables, such as the BM spread, are theoretically related to the price of risk, based on the framework in de Oliveira Souza (2019a). This happens under the hypothesis that the book value of equity (BE) is a proxy for expected cash flows (Berk, 1995). Under this hypothesis, a firm's BM tends to be directly proportional to its risk premium: By definition, the premium is the difference between market value (which is the BM denominator) and expected cash flow (which is related to the BM numerator under this hypothesis). When the price of risk is zero, all cash flows are discounted at the same rate, regardless of their risks. In this case, the BM of all firms are similar (there is very little dispersion in BM). However, the spread in BM widens as the price of risk increases. When this happens, the cash flows of risky and safe firms are discounted differently. Thus, their BMs also become different. This relates the BM spread to the price of risk.[4]

Regarding (ii) in the first paragraph, increases in the investment and profitability spreads exclusively forecast (marginally) increases in the investment and profitability premiums, respectively. The changes in all these spreads are relatively orthogonal, suggesting that the risks of the factors, and not only the price of risk, also fluctuate over time. Indeed, including both the BM and the investment spreads in the predictive regression boosts the forecastability of the investment premium in sample and out of sample.

I explain that the investment and profitability spreads (and, in fact, also the BM spread) are theoretically related to the risk of their respective factors in Section 2.2. The intuition, based on the framework in de Oliveira Souza (2019a), is the following: Assume that the CAPM of Sharpe (1964) and Lintner (1965) holds and that we create a long/short "beta-factor" portfolio based on market betas. The risk of this factor is the difference between the betas of the long and short portfolios. But this difference (which is the factor risk) is

[4]In fact, the BM spread can also be related to expected returns using the ideas in Cohen et al. (2003) or Polk et al. (2006), for example. In addition, the assumption that BE is a proxy for cash flows finds support especially in de Oliveira Souza (2019c,e), Ball et al. (2018), and Gerakos and Linnainmaa (2017).

exactly a "beta spread". Hence, if investment and profitability are also risk proxies (like market betas), a similar channel links each spread to its respective factor risk.

Finally, I use this predictability to create factor timing strategies based on the dynamic trading rule of Daniel and Moskowitz (2016), for example. Unsurprisingly, the biggest gains in this case arise from timing the investment premium (Fig. 7, Section 6). In particular, the out-of-sample Sharpe ratio of the strategy is always positive, regardless of the breakpoint from which the performance is calculated. The contrast to the Sharpe ratio of the unconditional investment premium, which is often negative since 2003, becomes extreme towards the end of the sample.

Timing the value premium also generates out-of-sample gains, regardless of the date used to compare the strategy with the unconditional premium (Fig. 6, Section 6). The difference in performance is not as impressive as it is for the investment premium because there is no independent factor risk proxy for the value portfolio: The BM spread is the only return forecaster of the value premium. Nevertheless, the timing strategy often delivers significant economic gains. For example, the unconditional cumulative value premium is negative starting from any year since 2004; but the value timing strategy delivers positive (even if small) Sharpe ratios starting from almost any of those years.

On the other hand, the trading rule of Daniel and Moskowitz (2016) does not generate large gains for the size premium. Indeed, de Oliveira Souza (2019d,c) shows that the size premium only exists for the portfolios created in high price of risk states. Hence, the optimal strategy to time the size premium is to invest exclusively in high price of risk states. Section 6.1 shows that this other type of trading rule is another way to create a strategy from the predictability that I document. This works especially for the size premium.

1.1 Related literature and contribution

Essentially, the present paper contains a general test of standard macro-finance models, a specific test of the hypothesis that the factors in Fama and French (2015) have constant risks, and the main ingredient of factor timing strategies in the form of factor premium predictability. The paper relates to these three literatures.

Within the (empirical) macro-finance literature, the present paper relates especially to papers that document common variation in expected returns, going back to Fama and French (1989) at least. More broadly, it also relates to papers on equity premium predictability in general. Cochrane (2011, 2017) surveys this entire literature in detail. The paper fills a gap in this literature by providing detailed evidence of common variation in the expected returns of individual stocks compared to the existing evidence of variation across asset classes.

Within the (empirical) literature on the risks of the factors in Fama and French (1996, 2015), the closest related papers are de Oliveira Souza (2019d,c), Gerakos and Linnainmaa (2017), and Ball et al. (2018). The paper fills a gap in this literature by providing evidence of time variation in the risk of the investment and, to some extent, the profitability factors. It adds to similar evidence about the size and value factors in de Oliveira Souza (2019c,d), for example. This is important because risk adjustments based on portfolios with time-varying risks, such as industry portfolios, are problematic according to Fama and French (1997).

Finally, a few related papers within the literature on factor premium predictability are de Oliveira Souza (2019d,c), Haddad et al. (2018), Cohen et al. (2003), Cooper et al. (2001), and Pontiff and Schall (1998). The main contribution of the present paper to this literature is documenting predictability based on a strict theoretical framework that defines risks and price of risk and explains economically how each forecasting variable relates to the premiums being predicted. This is what makes the theoretical analysis that I perform in the paper possible, which is not straightforward in more data-driven methods, such as in Haddad et al. (2018), for example.

2 Theory

Let $\zeta = (\zeta_t)$ be the unique stochastic discount factor (SDF) that follows the continuous-time stochastic process

$$d\zeta_t = -\zeta_t \left[r_t^f \, dt + \boldsymbol{\lambda}_t^\top d\boldsymbol{z}_t \right], \tag{1}$$

where $\boldsymbol{z} = (\boldsymbol{z}_t)$ is a standard Brownian motion and $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_t)$ is a given market price of risk process, both with dimension d (boldfaces denote vectors), r_t^f is the short-term risk-free rate, and $^\top$ is the transposition sign.

Let $P_i = (P_{i,t})$ be the price of asset i, such that

$$dP_{i,t} = P_{i,t} \left[\mu_{i,t} \, dt + \boldsymbol{\sigma}_{i,t}^\top \, d\boldsymbol{z}_t \right], \tag{2}$$

where $\mu_{i,t}$ is the instantaneous expected rate of return on asset i (which pays no dividends for simplicity), with equilibrium risk premium given by

$$\mu_{i,t} - r_t^f = \boldsymbol{\sigma}_{i,t}^\top \boldsymbol{\lambda}_t. \tag{3}$$

As Munk (2013) explains, a conditional pricing factor is a stochastic process $\boldsymbol{x} = (\boldsymbol{x}_t)$ of dimension $K = 5$ (for the factor model in Fama and French (2015) in particular), with dynamics of the form[5]

$$d\boldsymbol{x}_t = \boldsymbol{\mu}_{xt} \, dt + \Sigma_{xt}^\top \, d\boldsymbol{z}_t, \tag{4}$$

in which the conditional variance-covariance matrix, $\Sigma_{xt}\Sigma_{xt}^\top$, is always non-singular (capital letters denote matrices). And, in the presence of a short-term risk-free asset, the risk premium of any asset is

$$\mu_{i,t} - r_t^f = \left(\boldsymbol{\beta}_t^{ix} \right)^\top \boldsymbol{\eta}_t, \tag{5}$$

[5] Pages 402-405 in Munk (2013) contain the details of the derivation.

where $\boldsymbol{\eta} = (\boldsymbol{\eta}_t)$ is the conditional risk premium associated with the factor, and the conditional factor-beta is

$$\boldsymbol{\beta}_t^{ix} = \left(\Sigma_{xt}\Sigma_{xt}^\top\right)^{-1}\Sigma_{xt}\boldsymbol{\sigma}_{i,t}. \tag{6}$$

Taken together, Eq. (3), Eq. (5), and Eq. (6) link the market price of risk and factor risk to the factor risk premium:

$$\boldsymbol{\lambda}_t = \Sigma_{xt}^\top\left(\Sigma_{xt}\Sigma_{xt}^\top\right)^{-1}\boldsymbol{\eta}_t. \tag{7}$$

2.1 Time-varying market price of risk and factor premiums

The theoretical framework above explicitly connects multifactor asset pricing models, such as Fama and French (1996, 2015) or Hou et al. (2015), and the several classes of macro-finance models discussed in Cochrane (2017), for example. Indeed the macro-finance models discussed are all equivalent within this framework, as Cochrane (2017) explains. Their main result is to generate large and, ideally, time-varying Sharpe ratios for the mean-variance frontier (MVF). These Sharpe ratios are given by the norm of the market price of risk, $\|\boldsymbol{\lambda}_t\|$.

According to Eq. (4), the risk of the factor depends on Σ_{xt}. The individual components of factor $\boldsymbol{x}_t$, which are the market, size, value, profitability, and investment factors for Fama and French (2015), do not have constant risks in general if their variance, given by Σ_{xt}, is time-varying. Empirically, this means that tests that look for comovement in risk premiums as evidence that the premiums arise because of risk, as in Fama and French (1989), are not completely conclusive in case there is no such evidence. However, Eq. (7) still shows that the factor premiums should increase with the market price of risk if all else is kept constant. Hence, proxies for the MVF Sharpe ratio, $\|\boldsymbol{\lambda}_t\|$, are good candidates to forecast the factor premiums in Fama and French (2015). The question, which I address next based on the framework in de Oliveira Souza (2019a), is how to obtain these proxies.

2.1.1 Two market price of risk proxies

Without loss of generality, let the risk-free rate be zero, $r_t^f = 0$, and $\boldsymbol{z} = (\boldsymbol{z}_t)$ be one-dimensional in Eq. (1); then the conditional MVF Sharpe ratio becomes simply $\|\boldsymbol{\lambda}_t\| = \lambda_t$. Asset i is expected to pay a unique final uncertain cash flow at time $t+1$ given by $\mathrm{E}_t[D_{i,t+1}]$. The dynamics in Eq. (2) implies that the price of this asset at time t is

$$P_{i,t} = \mathrm{E}_t[D_{i,t+1}]e^{-\sigma_{i,t}\lambda_t}. \tag{8}$$

Within the theoretical framework in Berk (1995) and de Oliveira Souza (2019c), the firm's book equity (BE) is a proxy for its expected cash flows. In a simplified way, this can be represented as, for example,

$$BE_{i,t} \approx a_t \, \mathrm{E}_t[D_{i,t+1}], \tag{9}$$

where a_t is a constant, which is possibly time-varying because the relation between BE and future cash flows can change over time. One reason for these changes could be steady state shifts (Lettau and Van Nieuwerburgh, 2008), for example. Eq. (8) and Eq. (9) imply that the BM characteristic of firm i is given by

$$BM_{i,t} \equiv \frac{BE_{i,t}}{P_{i,t}} \approx a_t e^{\sigma_{i,t}\lambda_t}. \tag{10}$$

Hence, if everything else remains constant in Eq. (10), BM ratios increase with the conditional MVF Sharpe ratio, λ_t. In particular, aggregate BM ratios or their cross-sectional averages or medians, for example, can be used as proxies for the price of risk. This explains why Pontiff and Schall (1998) is able to forecast the equity premium based on the Dow Jones' BM.

Another price of risk proxy is the BM spread. Eq. (10) implies that riskier firms, r, with $\sigma_{r,t} > \sigma_{s,t}$, tend to have higher BM than safer firms, s. This implies that the spread also tends to increase with the price of risk:

$$BM_{r,t} - BM_{s,t} \approx a_t \left(e^{\sigma_{r,t}} - e^{\sigma_{s,t}}\right) e^{\lambda_t}. \tag{11}$$

I consider logs of both the median BM and the BM spread as price of risk proxies due to the exponential forms in Eq. (10) and Eq. (11). But the empirical results are similar with these variables in levels.

2.2 Time-varying factor risks and factor premiums

The second type of predictability mentioned in the introduction arises from changes in factor risks. Indeed, Eq. (7) shows that even if the market price of risk, $\boldsymbol{\lambda}_t$, stays constant, the factor risk premiums, $\boldsymbol{\eta}_t$, can change as a consequence of changes in risks given by the covariance structure in Σ_{xt}. This section provides a theoretical link between characteristic spreads and the risk of the factors based on that characteristic, in line with the framework in de Oliveira Souza (2019a). It explains why these spreads can be used as factor risk proxies.

Let a given characteristic of stock i, $\theta_{i,t}$, be a good proxy for its risk, $\sigma_{i,t}$, for any given reason. Traditionally, such characteristics could be leverage or estimated market betas, for example. Without loss of generality, this can be expressed as

$$\theta_{i,t} \approx b_t \sigma_{i,t}, \tag{12}$$

where $b_t > 0$ is a positive constant that is possibly time-varying. Let us create a long-short factor portfolio based on sorts on this characteristic: We buy stocks with large characteristics, $\theta_{h,t}$ on average, and sell the ones with low characteristics, $\theta_{l,t}$ on average. The characteristic of this portfolio is (a spread) given by

$$\theta_{sp,t} = \theta_{h,t} - \theta_{l,t}, \tag{13}$$

implying, from Eq. (12), that its risk is approximately

$$\sigma_{sp,t} \approx \sigma_{h,t} - \sigma_{l,t}, \tag{14}$$

which finally shows that the risk of the long-short portfolio is proportional to the spread in the characteristic used to create the portfolio.

In particular, this shows that the investment spread is related to the risk of the investment portfolio under the assumption that investment is a risk proxy. The same is true for profitability.[6] I consider both spreads as factor risk proxies. I also consider their medians as possible alternatives to compare the results.

3 Data description and variables

All the return and breakpoint data (used to construct the factor risk and price of risk proxies) are from Kenneth French's data library, and every series ends in August 2018.

I collect realized monthly returns on the SMB, HML, RMW, and CMA portfolios of Fama and French (2015) starting in July 1963. These are the size, value, profitability, and investment premiums. The series with excess returns on the market portfolio (MP) is available from July 1926 instead.

I also collect the median, 30^{th}, and 70^{th} cross-sectional percentiles for the BM, profitability, and investment characteristics among all NYSE stocks. The values come from their respective breakpoint files in the data library. They are valid from the end of June 1963 (June 1926 for the BM) and change yearly.

[6]The size characteristic is different because the series is not stationary (even in real terms, due to economic growth). Thus, the size spread cannot be used efficiently as a proxy for the risk of the size factor.

3.1 Factor risk and market price of risk proxies

As explained in Section 2.1 and Section 2.2, I investigate six return forecasters: The cross-sectional median or spreads in BM, profitability (OP), and investment (Inv). In a given period, the spread is the difference between the 70^{th} and 30^{th} NYSE cross-sectional percentiles for each respective characteristic,

$$\theta_{sp,t} \equiv \theta_{0.7,t} - \theta_{0.3,t}, \quad \theta = \{BM, OP, Inv\}, \tag{15}$$

which is in logs for the BM spread, $\log\left(BM_{0.7,t} - BM_{0.3,t}\right)$.

Fig. 1 plots the six proxies in time series with their respective histograms. The figure shows that the sample related to BM starts earlier and all variables are in annual frequency. All the series seem to have trends in the sample period, especially the profitability spread. Indeed, the tests of Phillips and Perron (1988) in Table 1 also indicate that the hypotheses of unit roots in most series cannot be rejected. However, these variables are stationary by construction because they are essentially ratios that cannot increase forever. So the apparent trends should disappear in large samples instead of being a feature of the data. Table 1 also confirms that both variables related to BM are the least normally distributed over time among the regressors, as suggested by the histograms in Fig. 1. Finally, the similarity between the value spread and the average BM series in Fig. 1 is consistent with the theoretical prediction that both are price of risk proxies. This is much less apparent for investment and profitability.

3.2 The covariance structure of the premiums

At first, the results in Table 2 provide very little support for the hypothesis that the factor premiums of Fama and French (2015) move together – for example, as the premiums investigated in Fama and French (1989) do. In fact, the main table in Table 2 seems to suggest the opposite: Most premiums are negatively correlated with one another and especially with the market premium.

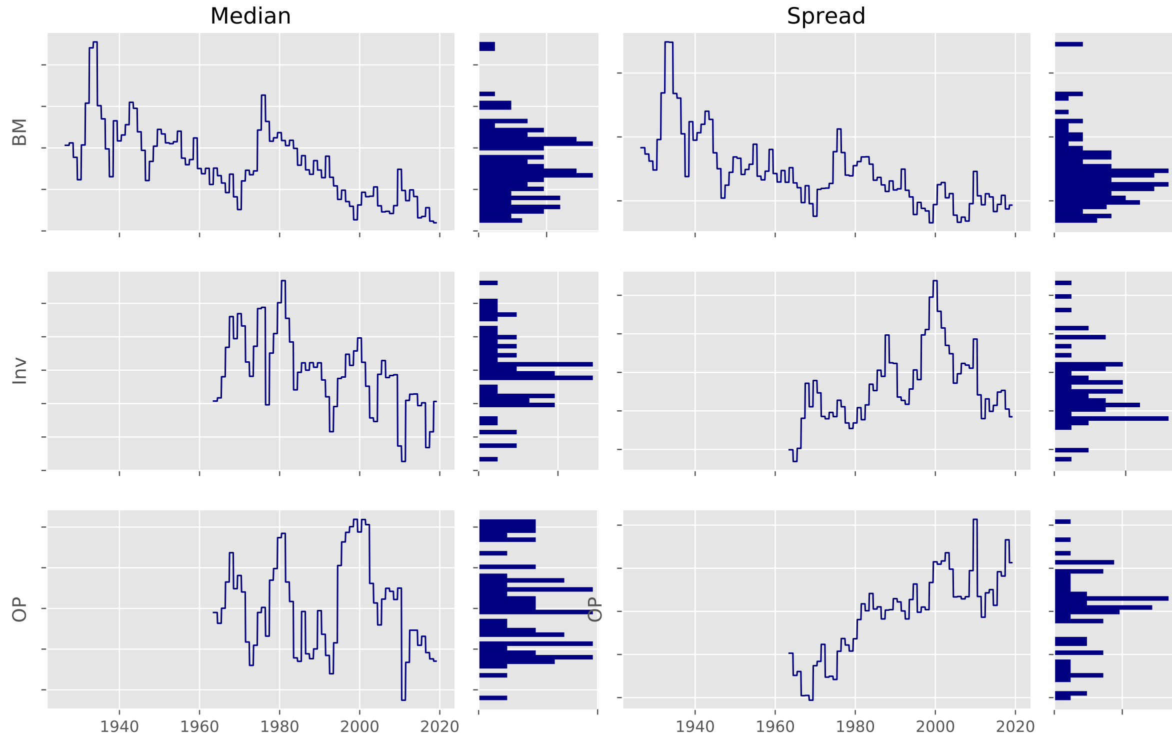

Figure 1: Time series and histogram for each regressor between June 1926 (1963) and 2018. The graphs on the left-hand side correspond to the cross-sectional median values of the BM, investment (Inv), and profitability (OP) among the NYSE stocks. The graphs on the right-hand side are the corresponding cross-sectional spreads given by the difference in the breakpoints between the 70^{th} and 30^{th} percentiles of these characteristics within the same universe of stocks.

Table 1: Descriptive statistics of the regressors in June 1926 (1963)–2018. The first columns show the mean, μ, standard deviation, σ, skewness, γ_1, and kurtosis, α_4, for each regressor. The regressors are the cross-sectional median BM, $BM_{1/2}$, profitability, $OP_{1/2}$, or investment, $Inv_{1/2}$, and their respective cross-sectional spreads, as in i_{SP} for regressor i, are the differences between the 70^{th} and 30^{th} percentiles each period. For the BM, I use the log of both quantities. The last four columns show the Phillips-Perron unit root test results: The number of lags, the test statistic Z_t, its p-value, and the number of observations.

	μ	σ	γ_1	α_4	PP(lags)	Z_t	p	Obs
$BM_{1/2}$	-0.19	0.43	0.75	3.99	6	-2.46	0.13	1111
BM_{SP}	-0.56	0.54	1.43	5.83	6	-2.62	0.09	1111
$OP_{1/2}$	26.10	2.25	0.30	2.19	6	-2.52	0.11	667
OP_{SP}	13.88	1.90	-0.26	2.60	6	-1.83	0.37	667
$Inv_{1/2}$	7.34	2.86	0.02	2.87	6	-2.97	0.04	667
Inv_{SP}	11.58	2.38	0.68	3.46	6	-2.37	0.15	667

However, the profitability premium is the only premium that does not seem to increase with the other premiums, as I will show later. One hypothesis for this lack of comovement is that the profitability factor becomes a hedge for the other factors as the market price of risk increases. This can happen if the returns on the RMW portfolio covary more negatively with the other factor returns when the BM spread increases, for example. Table 2 provides some evidence that supports this explanation.

Table 2a and Table 2c correspond to the months in which the (lagged) BM spread is above its average (high market price of risk months) considering the period starting in July of 1963. Table 2b and Table 2d correspond to the other months (with low market price of risk). Comparing the last rows in Table 2a and Table 2b shows that the correlation between the returns on the RMW and the HML or CMA portfolios indeed decreases in high price of risk states, although it increases for the market and size portfolios. Table 2c and Table 2d provide more evidence in terms of covariances. The tables show that the variance of the profitability premium is substantially lower when the BM spread is high. Indeed, the risk of the RMW portfolio measured by the total of the last rows in each table, goes from 1.2 to -2.1 in low and high price of risk states, respectively. This suggests that the risk of the RMW portfolio in fact decreases, from a (mean-variance efficient) portfolio perspective, in high price of risk states. This could explain why increases in the BM spread fail to predict increases in the profitability premium.

Table 2: Correlation and covariance matrices of the monthly factor returns in June 1963–2018. The main (centered) table shows the correlations among the market, size, value, investment, and profitability premiums, respectively the (excess) returns on the market (MP), SMB, HML, CMA, and RMW portfolios. Subtables (a) and (b), respectively, show the same information for the months in which the BM spread is above or below its average for this period, while (c) and (d) show the corresponding covariance matrices (in percentage points). The tables show the coefficients with significance levels * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

	MP	SMB	HML	CMA	RMW
MP	1.00				
SMB	0.27***	1.00			
HML	-0.26***	-0.07	1.00		
CMA	-0.38***	-0.10**	0.70***	1.00	
RMW	-0.23***	-0.35***	0.06	-0.04	1.00

(a) High BM spread correlations

	MP	SMB	HML	CMA	RMW
MP	1.00				
SMB	0.21*	1.00			
HML	-0.27**	0.23**	1.00		
CMA	-0.38***	0.09	0.78***	1.00	
RMW	0.14	-0.22*	-0.55***	-0.63***	1.00

(b) Low BM spread correlations

	MP	SMB	HML	CMA	RMW
MP	1.00				
SMB	0.29***	1.00			
HML	-0.26***	-0.16***	1.00		
CMA	-0.39***	-0.15***	0.68***	1.00	
RMW	-0.31***	-0.37***	0.19***	0.06	1.00

(c) High BM spread covariances

	MP	SMB	HML	CMA	RMW
MP	39.5	2.9	-4.1	-3.4	1.1
SMB	2.9	8.4	2.1	0.5	-1.0
HML	-4.1	2.1	9.9	4.5	-2.8
CMA	-3.4	0.5	4.5	3.4	-1.9
RMW	1.1	-1.0	-2.8	-1.9	2.6

(d) Low BM spread covariances

	MP	SMB	HML	CMA	RMW
MP	20.2	3.7	-3.0	-3.3	-3.0
SMB	3.7	9.3	-1.3	-0.9	-2.6
HML	-3.0	-1.3	7.3	3.7	1.2
CMA	-3.3	-0.9	3.7	4.1	0.3
RMW	-3.0	-2.6	1.2	0.3	5.3

4 In-sample predictability

As explained earlier, Eq. (7) suggests that the factor premiums in Fama and French (2015) should increase with both the price of risk and the factor risks. This section tests these hypotheses based on proxies for these variables. In summary, I run monthly predictive regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}, \tag{16}$$

where R_i is either the market, size, value, profitability, or investment premiums of Fama and French (2015); α_i and β_i are constants; and $\varepsilon_{i,t}$ is an error term. I investigate the forecasting power of six variables, f_t: The cross-sectional median or spread in BM, investment, or profitability. The spreads are given in Eq. (15).

Table 3a contains the results based on the (BM-related) price of risk proxies. It shows that especially the BM spread forecasts – with the same significantly positive sign – all factor premiums in Fama and French (2015) except the profitability premium. The results based on the median BM are less strong but qualitatively the same.

For the profitability premium, the slope coefficients, β_i, are insignificant based on both variables but the point estimates are negative. This suggests that, if anything, the risk of the profitability factor decreases with the market price of risk. Indeed, as de Oliveira Souza (2019c) shows, the link between risk and characteristic can vary systematically with the state of the economy. In terms of Eq. (12), it could be that b_t decreases as the price of risk increases (for reasons that I do not address in this paper).

Finally, the market premium is significantly forecastable in the full sample starting in 1926, but not in the sample starting in 1963 (Table 4). This result is in line with the lack of statistical forecastability of the equity premium in recent years, especially after 1975 as documented by Welch and Goyal (2008), for example. In fact, one of the variables that they show to have no forecasting power after 1975 is exactly the Dow Jones BM of Pontiff and Schall (1998), similar to the BM-related price of risk proxies that I consider.

The other two panels in Table 3 confirm, with marginally significant coefficients, that the cross-sectional spread in each characteristic forecasts the returns on the factor that is created from that characteristic. This confirms the factor risk proxy hypothesis in Section 2.2: A large investment (profitability) spread forecasts a large investment (profitability) premium in Table 3b (Table 3c). All other coefficients in both panels are insignificant.[7]

In summary, the evidence in Table 3a and the evidence in Table 3b and Table 3c, respectively, supports the macro-finance hypothesis and contradicts the constant factor risk hypothesis mentioned in the introduction.

[7]In fact, the median profitability also seems to forecast the size premium in Table 3c. But apart from the lack of theoretical support, the result is also not robust out of sample, as I show later.

Table 3: Predictive regressions for the market, size, value, profitability, and investment premiums in June 1926 (1963)–2018. The estimated equations are of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The respective premium, $R = \{MP, SMB, HML, RMW, CMA\}$, appears in the top row of each panel. They are the market, size, value, profitability, and investment premiums, in this order. The regressor, f_t, appears in the first column. The regressor is either the cross-sectional median or spread in (a) BM, (b) profitability, or (c) investment. The tables show the number of months in each sample, the R^2, and coefficients with t statistics in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

(a) BM-related price of risk proxies

	MP		SMB		HML		RMW		CMA	
Median BM	0.009*		0.008*		0.008*		-0.002		0.004	
	(2.46)		(2.22)		(2.45)		(-0.88)		(1.94)	
Spread		0.006*		0.010**		0.01**		-0.0007		0.007**
		(2.07)		(2.64)		(3.29)		(-0.27)		(2.96)
Intercept	0.008***	0.01***	0.006**	0.01**	0.006***	0.01***	0.002	0.002	0.004***	0.009***
	(4.81)	(4.40)	(3.11)	(3.25)	(3.82)	(4.15)	(1.35)	(0.83)	(3.81)	(4.03)
Months	1107	1107	662	662	662	662	662	662	662	662
R-squared	0.005	0.004	0.007	0.010	0.009	0.016	0.001	0.000	0.006	0.013

(b) Investment-related factor risk proxies

	MP		SMB		HML		RMW		CMA	
Median Inv	-0.0007		0.0003		-0.00005		-0.0002		0.0003	
	(-1.24)		(0.81)		(-0.12)		(-0.51)		(0.94)	
Spread		-0.0002		-0.0004		-0.0001		0.0005		0.0006
		(-0.29)		(-0.73)		(-0.22)		(1.43)		(1.86)
Intercept	0.01*	0.008	0.0001	0.007	0.004	0.004	0.004	-0.003	0.0009	-0.004
	(2.31)	(0.92)	(0.04)	(1.15)	(1.19)	(0.82)	(1.58)	(-0.79)	(0.41)	(-1.10)
Months	663	663	662	662	662	662	662	662	662	662
R-squared	0.002	0.000	0.001	0.001	0.000	0.000	0.000	0.003	0.001	0.005

(c) Profitability-related factor risk proxies

	MP		SMB		HML		RMW		CMA	
Median OP	-0.0004		0.001		-0.0002		0.0004		0.0003	
	(-0.46)		(1.94)		(-0.45)		(1.14)		(0.94)	
Spread		0.0006		-0.0003		-0.0006		0.0008		0.00008
		(0.65)		(-0.44)		(-0.97)		(1.87)		(0.19)
Intercept	0.01	-0.003	-0.02	0.006	0.009	0.01	-0.009	-0.009	-0.006	0.002
	(0.73)	(-0.22)	(-1.74)	(0.73)	(0.70)	(1.37)	(-0.88)	(-1.44)	(-0.63)	(0.29)
Months	663	663	662	662	662	662	662	662	662	662
R-squared	0.000	0.001	0.006	0.000	0.000	0.001	0.002	0.005	0.001	0.000

Table 4: Predictive regressions for the market premium with BM-related price of risk proxies in June 1963–2018. The estimated equations are of the form

$$R_{mp,t+1} = \alpha + \beta f_t + \varepsilon_{t+1},$$

where the regressor, f_t (in the first column), is either the cross-sectional median or spread in BM. The table shows the number of months in each sample, the R^2, and coefficients with t statistics in parentheses, $^{*}\ p < 0.05$, $^{**}\ p < 0.01$, $^{***}\ p < 0.001$.

	MP	MP
Median BM	0.002	
	(0.41)	
BM spread		0.001
		(0.24)
Intercept	0.006*	0.006
	(2.39)	(1.34)
Months	664	664
R-squared	0.000	0.000

Indeed, Table 5 confirms that both the factor risk and the price of risk contribute to forecasting the investment premium with highly significant coefficients associated with both the investment and the BM spreads. The two variables carry different information about the investment premium. The BM spread, however, does not forecast the profitability premium even in a multivariate sense. Table 5 shows the results of predictive regressions for the profitability and investment premiums, similar to the ones in Eq. (16),

$$R_{i,t+1} = \alpha_i + \boldsymbol{\beta}_i^{\top} \boldsymbol{f}_t + \varepsilon_{i,t+1}, \tag{17}$$

but where the two-dimensional regressors, $\boldsymbol{f}_t = \left(BM_{SP}, \theta_{sp}\right)^{\top}$, with $\theta = \{Inv, OP\}$ are the BM spread (the price of risk proxy) together with the spread in the characteristic used to create the respective factor (the factor risk proxy).

Table 5: Predictive regressions for the profitability and investment factor premiums in June 1963–2018 based on the BM spread and the spread in the characteristic used to form the factor. The estimated equations are of the form

$$R_{i,t+1} = \alpha_i + \boldsymbol{\beta}_i^\top \boldsymbol{f}_t + \varepsilon_{i,t+1},$$

where the two-dimensional regressors, $\boldsymbol{f}_t$, are the BM spread and the spread in the characteristic used to create the factor, both appearing in the first column. The table shows the number of months in each sample, the R^2, and coefficients with t statistics in parentheses, $^{*}\ p < 0.05$, $^{**}\ p < 0.01$, $^{***}\ p < 0.001$.

	RMW	CMA
BM	0.0006	0.01***
	(0.21)	(3.95)
Profitability	0.0009	
	(1.86)	
Investment		0.001**
		(3.20)
Intercept	-0.009	-0.002
	(-1.41)	(-0.40)
Months	662	662
R-squared	0.005	0.028

5 Out-of-sample predictability

There are well-documented reasons to prefer either in-sample (Inoue and Kilian, 2005; Cochrane, 2008; Campbell and Thompson, 2008) or out-of-sample (OOS) evidence (Welch and Goyal, 2008). In particular, the in-sample results in the previous section could be more efficient to test the two hypotheses laid out in the introduction. However, the fitness of the recursive out-of-sample implementation of the model can be important for actual investment purposes. This section analyzes this performance.

In order to evaluate the OOS forecasting performance of the model I calculate a predictive R^2, as in Welch and Goyal (2008), for example. The OOS R^2,

$$R^2 = 1 - \frac{MSFE_m}{MSFE_h}, \tag{18}$$

increases with the ratio between the mean squared forecasting error of the model, $MSFE_m$, calculated from the vector of rolling (expanding) OOS errors from the model and the one associated with the historical average, $MSFE_h$, calculated from the vector of rolling (expanding) OOS errors from the historical mean up until that point in time. The R^2 can take any value below 1, and negative values mean that the model's forecast is less accurate than simply using the historical mean return.

The return forecast in each period, t, used to calculate the $MSEF_m$ is based on the recursively estimated coefficients from Eq. (16) and Eq. (17), $\hat{\alpha}_{i,t}$ and $\hat{\beta}_{i,t}$,

$$\mathrm{E}_t[R_{i,t+1}] = \hat{\alpha}_{i,t} + \hat{\beta}_{i,t} f_t, \tag{19}$$

$$\mathrm{E}_t[R_{i,t+1}] = \hat{\alpha}_{i,t} + \hat{\boldsymbol{\beta}}_{i,t}^{\top} \boldsymbol{f}_t. \tag{20}$$

The coefficients estimated at time t use only the information available until time t, so they become more accurate as the estimation sample expands. The return forecast in each period, t, used to calculate the $MSEF_h$ is simply the recursively calculated historical mean of the premium:

$$\mathrm{E}_t[R_{i,t+1}] = \frac{1}{t} \sum_{h=1}^{t} R_{i,h}. \tag{21}$$

The forecasting errors in both cases are the differences between realized and expected returns:

$$e_{i,t} = R_{i,t} - \mathrm{E}_{t-1}[R_{i,t}]. \tag{22}$$

Fig. 2 shows that the in-sample evidence regarding the BM-related price of risk proxies in Table 3a also holds OOS for the size and especially for the value and investment premiums. I split the full 1963-2018 sample into training and evaluation subsamples each month between January 1975 and January 2015. The horizontal axis displays the date used to split the sample. The graphs show that the predictions of the value and investment premiums tend to improve over time, which is consistent with the hypothesis of model stability, as noted by Welch and Goyal (2008). On the other hand, the OOS forecasts of the size premium are less accurate but still usually better than the unconditional mean.

In line with the in-sample evidence, the spread in BM provides better OOS forecasts for every premium compared to the BM median. In addition, the variables do not forecast the profitability nor the market premium (after 1963).

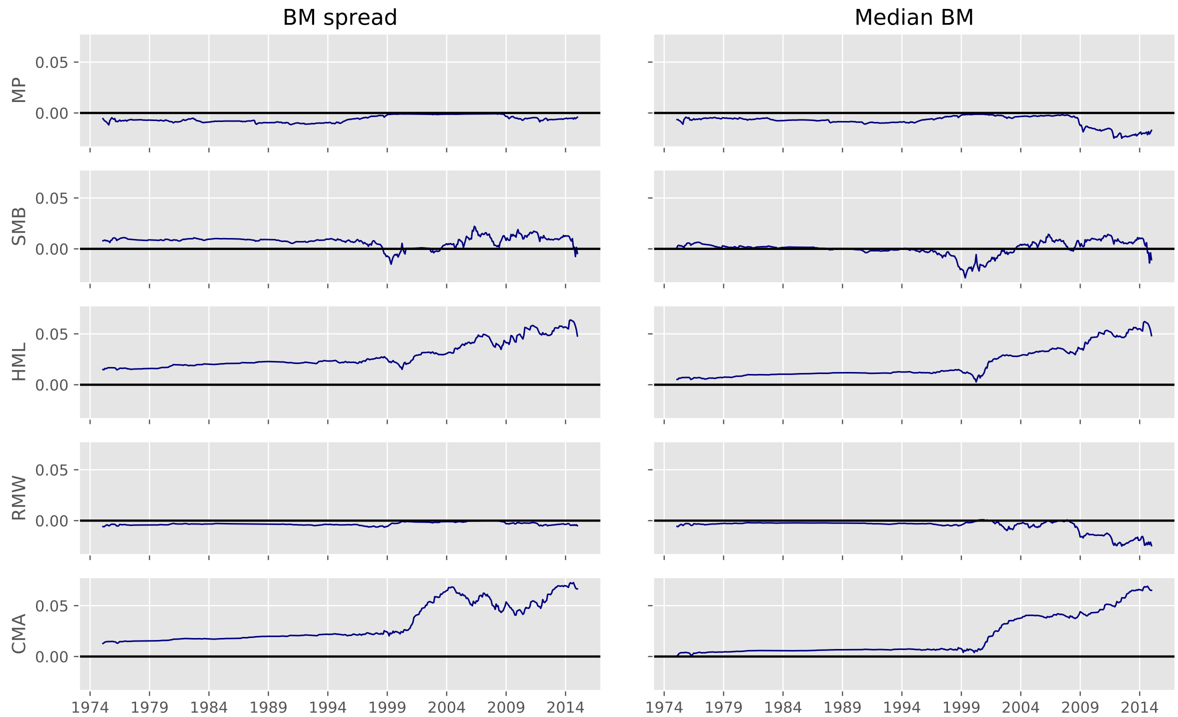

Figure 2: **Out-of-sample R^2 (until August 2018) based on the BM-related price of risk proxies.** The graphs on the left-/right-hand side use the BM spread/median, f_t, to recursively forecast $R_{i,t+1}$, which is the market premium (the top graph, MP), the size premium (SMB), the value premium (HML), the profitability premium (RMW), or the investment premium (CMA) in regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The graphs display the out-of-sample R^2 calculated from each point in time (the horizontal axis in the graphs) until 2018.

Fig. 3 also confirms the in-sample results in Table 3b: The investment spread forecasts the investment premium OOS, especially after 1999. Fig. 4, on the other hand, shows that the marginal ability of the profitability spread to forecast the profitability (and the size) premium in Table 3c is not reproduced OOS.[8] This adds to the evidence elsewhere in the paper suggesting that the profitability premium is somehow different from the others.

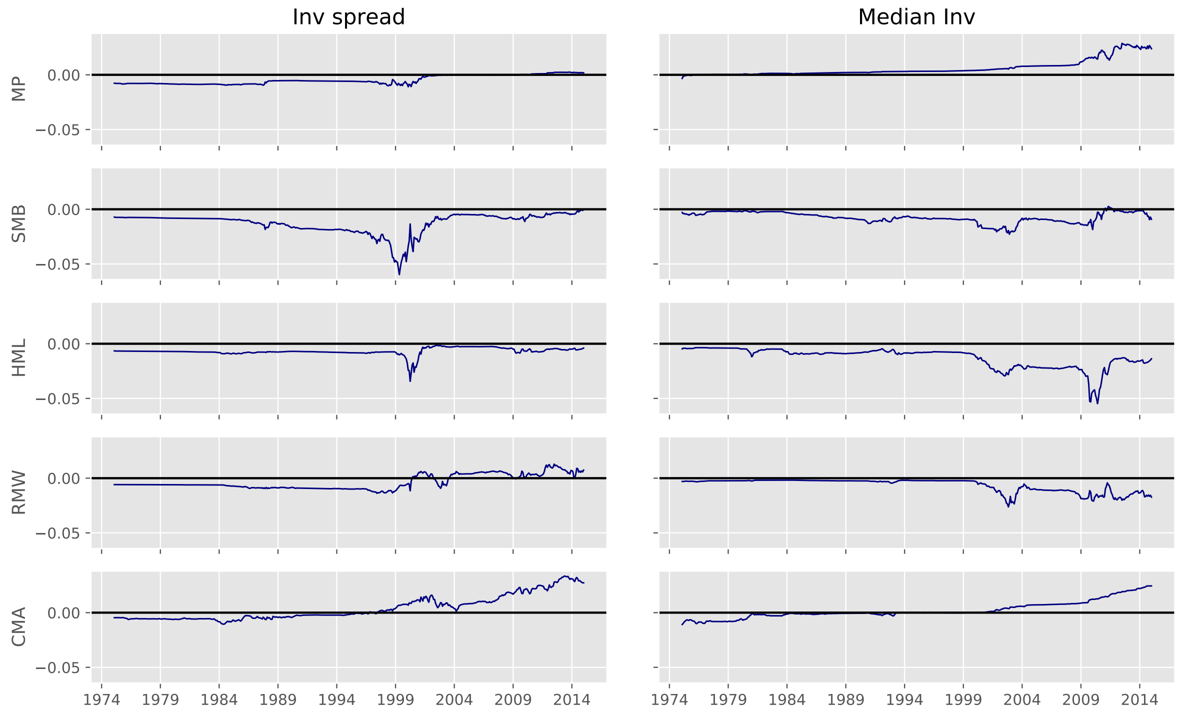

Figure 3: Out-of-sample R^2 (until August 2018) based on investment-related factor risk proxies. The graphs on the left-/right-hand side use the investment spread/median, f_t, to recursively forecast $R_{i,t+1}$, which is the market premium (the top graph, MP), the size premium (SMB), the value premium (HML), the profitability premium (RMW), or the investment premium (CMA) in regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The graphs display the out-of-sample R^2 calculated from each point in time (the horizontal axis in the graphs) until 2018.

Finally, Fig. 5 confirms the results above and the in-sample evidence in Table 5: The BM and the investment spreads together forecast the investment premium remarkably well, especially towards the end of the sample. On the other hand, the BM and the profitability spreads together still do not forecast the profitability premium.

[8]In fact, the median investment also seems to forecast the investment and the market premiums towards the end of the sample in Fig. 3, but without equivalent in-sample evidence in Table 3b. Something similar happens with the median profitability, which seems to forecast the investment spread towards the end of the sample in Fig. 4, with no equivalent in-sample evidence in Table 3c. However, as Welch and Goyal (2008) explain, OOS predictability is not a substitute for in-sample predictability, so these results are not very informative.

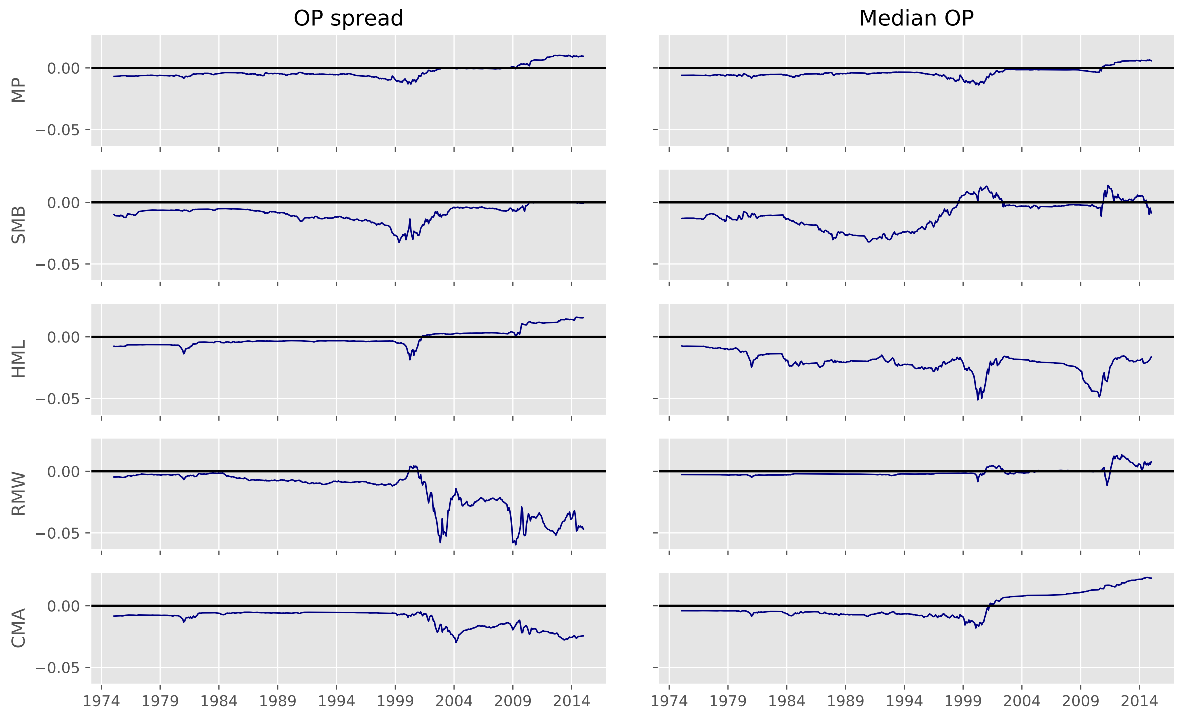

Figure 4: **Out-of-sample R^2 (until August 2018) based on profitability-related factor risk proxies.** The graphs on the left-/right-hand side use the profitability spread/median, f_t, to recursively forecast $R_{i,t+1}$, which is the market premium (the top graph, MP), the size premium (SMB), the value premium (HML), the profitability premium (RMW), or the investment premium (CMA) in regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The graphs display the out-of-sample R^2 calculated from each point in time (the horizontal axis in the graphs) until 2018.

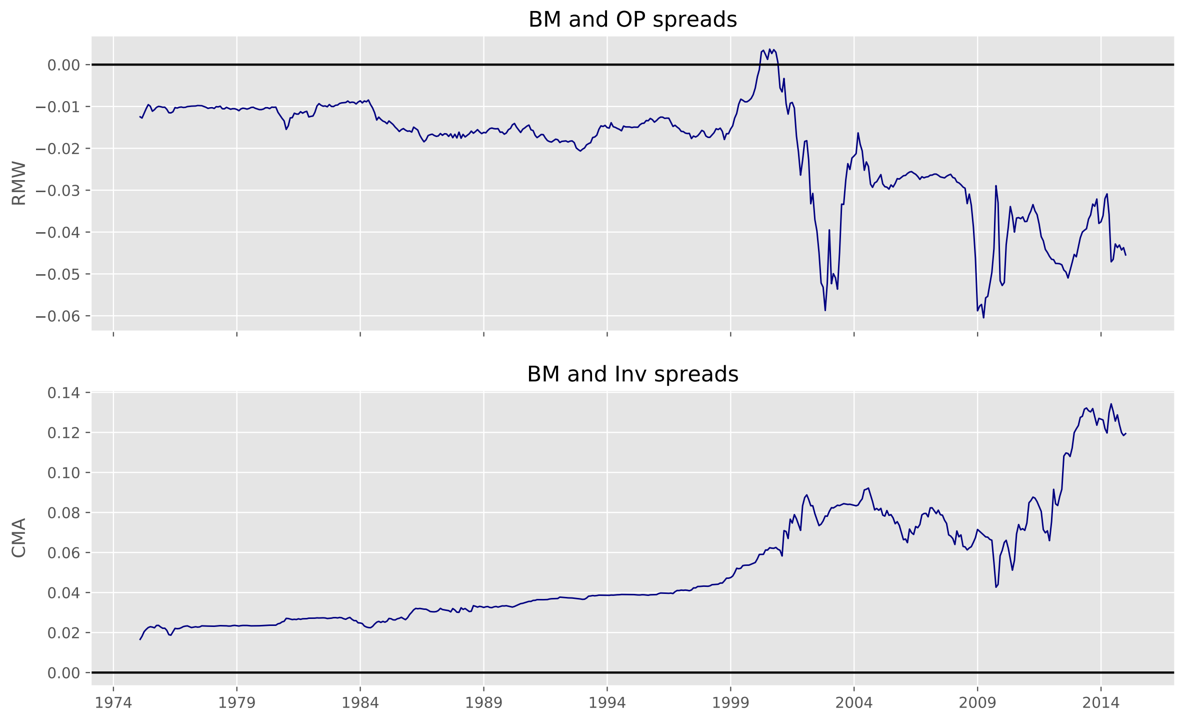

Figure 5: Out-of-sample R^2 (until August 2018) from regressions of the profitability or investment premiums on the BM spread in addition to either the profitability or the investment spreads. The top graph predicts the profitability premium from the BM and profitability spreads. The bottom graph predicts the investment premium from the BM and investment spreads. The recursive predictive regressions have the form

$$R_{i,t+1} = \alpha_i + \boldsymbol{\beta}_i^\top \boldsymbol{f}_t + \varepsilon_{i,t+1},$$

where $\boldsymbol{f}_t$ is a two-dimensional regressor. The graphs display the out-of-sample R^2 calculated from each point in time (the horizontal axis in the graphs) until 2018.

6 Factor timing

Daniel and Moskowitz (2016) argue that the optimal procedure to maximize the unconditional Sharpe ratio of an investment is a dynamic strategy that scales the portfolio weights at each point in time, so that the strategy's conditional volatility is proportional to its conditional Sharpe ratio. Without a model of conditional volatility (assuming homoscedasticity), this implies that the portfolios are levered up or down over time in proportion to their conditional expected returns.

I use this trading rule to create a series of factor timing strategies in this section. In particular, I set the weights of each factor portfolio i at time t to its OOS conditional expected return,

$$\omega_{i,t} = \mathrm{E}_t[R_{i,t+1}], \tag{23}$$

where $\mathrm{E}_t[R_{i,t+1}]$ is obtained recursively with information available until time t, as given by Eq. (19) and Eq. (20). Therefore, the realized return on each of the five managed portfolios i at time t is given by

$$R_{mpi,t} = \omega_{i,t-1} \times R_{i,t}. \tag{24}$$

The OOS Sharpe ratio for managed portfolio i, calculated from time t until T (August 2018), $SR_{i,tT}$, is given by

$$SR_{i,tT} = \frac{\overline{R}_{mpi,tT}}{s_{tT}}, \tag{25}$$

$$\overline{R}_{mpi,tT} = \frac{\sum_{h=t}^{T} R_{mpi,h}}{T-t}, \tag{26}$$

$$s_{tT} = \sqrt{\frac{\sum_{h=t}^{T}\left(R_{mpi,h} - \overline{R}_{mpi,tT}\right)^2}{T-t-1}}. \tag{27}$$

The graphs in this section are similar to the ones in the previous section. The month used to split the sample into training and evaluation subsamples is displayed on the horizontal axis and the Sharpe ratios calculated from that point until the end of the sample are displayed on the horizontal axis. The managed portfolios appear in blue and the unmanaged ones in orange.

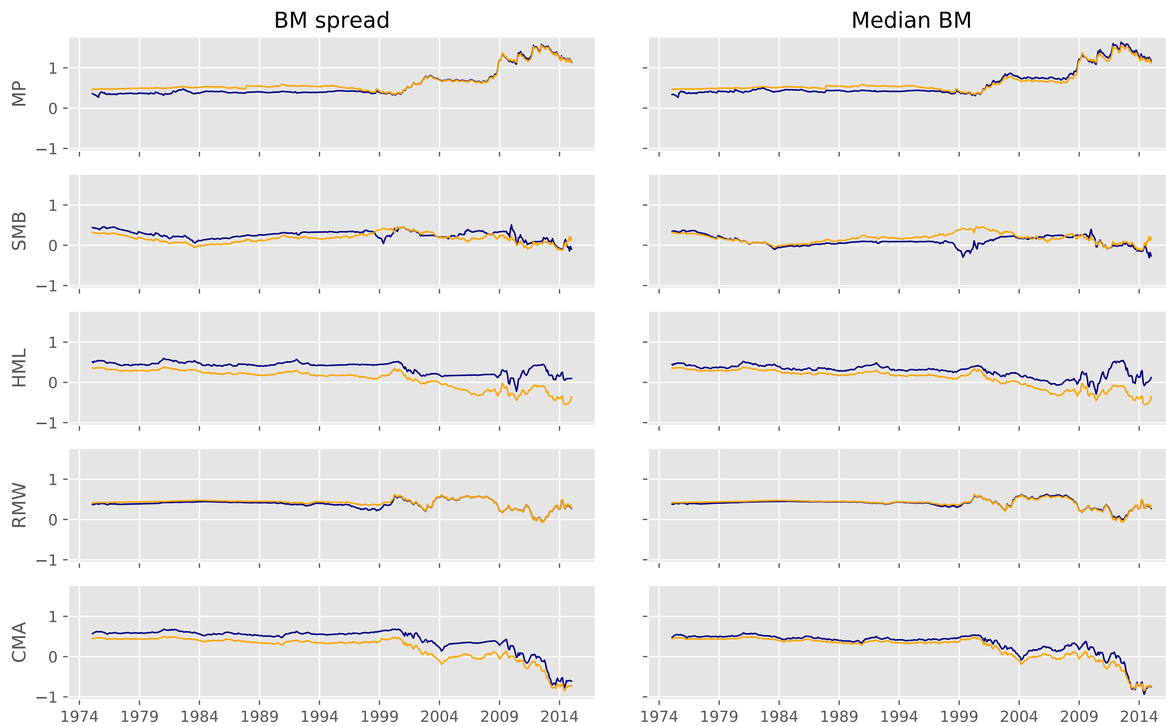

Figure 6: Out-of-sample Sharpe ratios (until August 2018) based on BM-related price of risk proxies – in blue. The graphs on the left-/right-hand side use the BM spread/median, f_t, to recursively forecast $R_{i,t+1}$, which is the market premium (the top graph, MP), the size premium (SMB), the value premium (HML), the profitability premium (RMW), or the investment premium (CMA) in regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The graphs display the out-of-sample Sharpe ratios calculated from each point in time (the horizontal axis in the graphs) until 2018 using portfolio weights proportional to the expected premiums each month to lever the portfolios up or down over time,

$$\omega_{i,t} = \mathrm{E}_t[R_{i,t+1}],$$

which is obtained recursively with information available until time t. I report the monthly Sharpe ratios multiplied by $\sqrt{12}$. Unconditional values appear in orange.

Fig. 6 shows that the factor timing strategies based on the BM-related price of risk proxies tend to deliver larger Sharpe ratios compared to the unconditional premiums. Again, the BM spread usually performs better than the BM median as a price of risk proxy. Compared to the unmanaged portfolio, the (BM spread) managed value portfolio has consistently larger Sharpe ratios regardless of the date used to split the sample. For example, this managed portfolio delivers positive (even if small) Sharpe ratios starting from almost every year after 2004. The unconditional value premium calculated from any year since 2004 is negative, in contrast. The (BM spread) managed investment portfolio also performs relatively better than its unconditional counterpart, but the difference is negligible for split dates towards the end of the sample. Finally, there is not much evidence of gains for size portfolios based on the trading rule in Daniel and Moskowitz (2016). Section 6.1 discusses another trading rule for the size premium. In line with the previous results, there are no improvements for the market and profitability premiums.

The gains from managing any of the factors based on the investment- or profitability-related factor risk proxies alone are very modest at best.[9] However, the managed investment portfolio based jointly on the BM and investment spreads has impressive Sharpe ratios compared to the unmanaged portfolio regardless of the split date. The difference becomes extreme towards the end of the sample in Fig. 7. Again, in line with the previous results, there is no evidence of improvements considering the profitability portfolio.

[9] Fig. 12 and Fig. 13 in Appendix A show these results.

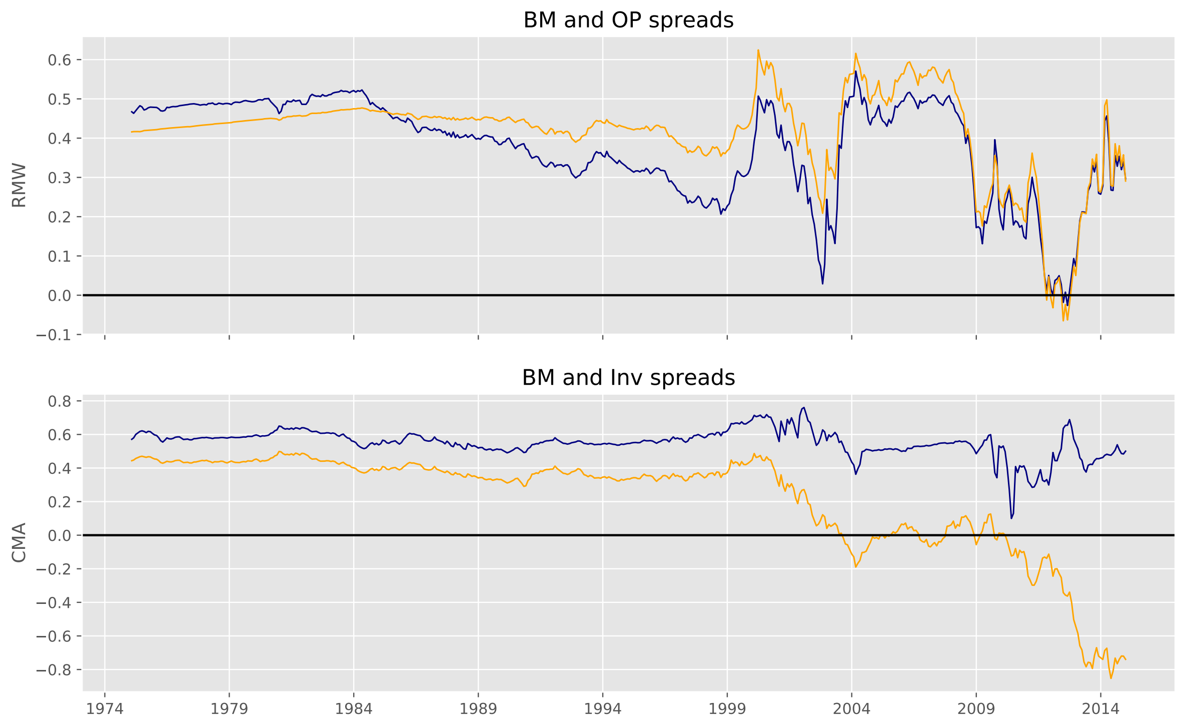

Figure 7: Out-of-sample Sharpe ratios (until August 2018) from regressions of the profitability or investment premiums on the BM spread in addition to either the profitability or the investment spreads – in blue. The top graph predicts the profitability premium from the BM and profitability spreads. The bottom graph predicts the investment premium from the BM and investment spreads. The recursive predictive regressions have the form

$$R_{i,t+1} = \alpha_i + \boldsymbol{\beta}_i^\top \boldsymbol{f}_t + \varepsilon_{i,t+1},$$

where $\boldsymbol{f}_t$ is a two-dimensional regressor. The graphs display the out-of-sample Sharpe ratios calculated from each point in time (the horizontal axis in the graphs) until 2018 using portfolio weights proportional to the expected premiums each month to lever the portfolios up or down over time,

$$\omega_{i,t} = \mathrm{E}_t[R_{i,t+1}],$$

which is obtained recursively with information available until time t. I report the monthly Sharpe ratios multiplied by $\sqrt{12}$. Unconditional values appear in orange.

6.1 Alternative trading rule

It is possible that the trading rule in Daniel and Moskowitz (2016) is not the most suitable one for the predictability that I document in the paper. A second alternative is, for example, to invest in a given factor portfolio only after observing a "good" signal, and not investing otherwise. For example, the strategy could be to invest in factor i only when the out-of-sample expected returns given by Eq. (19) and Eq. (20) are above a certain threshold, κ_i. I investigate this type of strategy in this section.

These strategies are equivalent to using an indicator function as the portfolio weights, $\omega_{i,t}$, to calculate the return on the managed portfolios with Eq. (24),

$$\omega_{i,t} = \begin{cases} 1, & \mathrm{E}_t[R_{i,t+1}] \geq \kappa_i \\ 0, & \mathrm{E}_t[R_{i,t+1}] < \kappa_i. \end{cases} \tag{28}$$

Unlike the previous section, here I evaluate all performances in the same 1990–2018 period; I vary the thresholds along the horizontal axis instead. I consider different threshold intervals for different premiums (two standard deviations away from their own average forecasts). These thresholds determine what a "good" signal is. For example, Fig. 8 shows the results for the SMB portfolio using annual return thresholds from −6% to 6%, displayed on the horizontal axis. The graphs on the left ("Invested") show the results involving the portfolios with expected returns above the threshold in question. The ones on the right show the results for the remaining ("Avoided") portfolios. The results are the Sharpe ratios, mean returns, skewness, and the proportion of months with expected returns above (on the left) or below the threshold (on the right). Intuitively, this proportion reflects how often the strategy has an active position in the market.

In general, the results in this section confirm that it is possible to use the predictability that I document to obtain better performances by avoiding investing in periods with expected premiums below certain thresholds. Both the Sharpe ratios and the mean returns tend to increase with the thresholds for all managed factors. For example, Fig. 8 shows

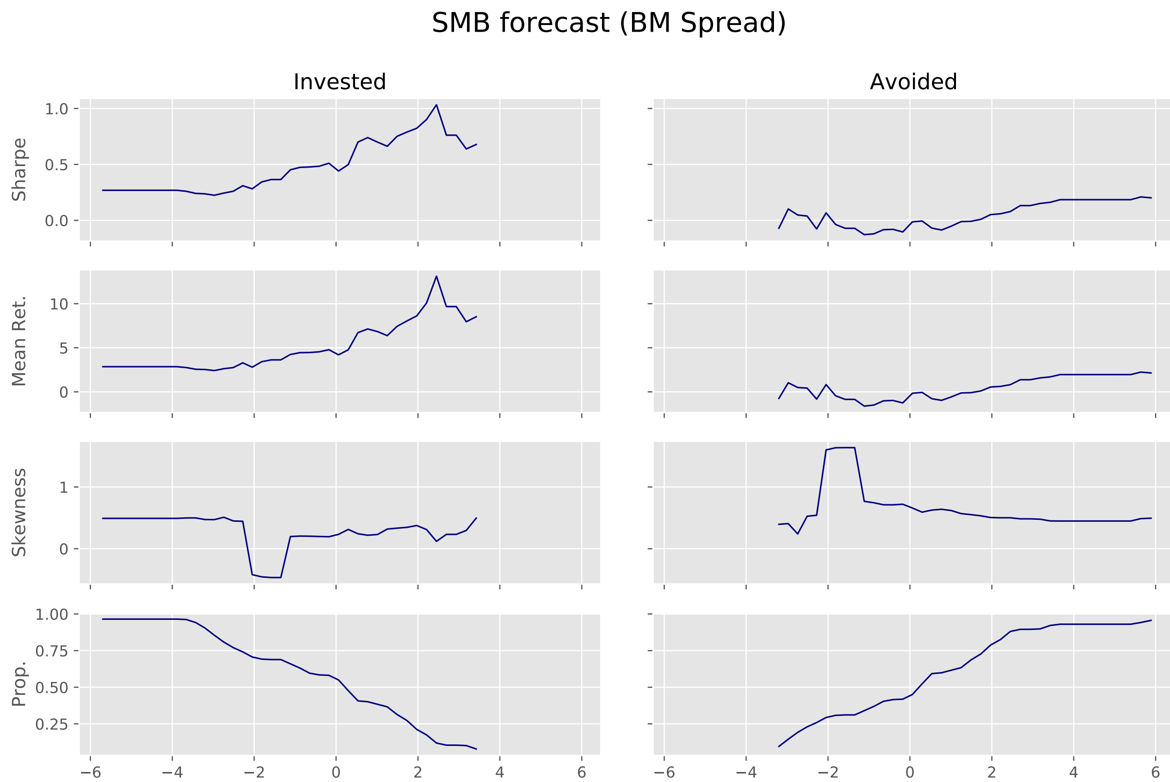

Figure 8: Summary statistics for the period 1990-2018 regarding the returns on the SMB portfolio with recursively estimated expectations above (Invested) or below (Avoided) the thresholds shown on the horizontal axis. The forecasts use the BM spread, $BM_{SP,t}$, to recursively forecast the size premium, $R_{smb,t+1}$, in regressions of the form

$$R_{smb,t+1} = \alpha + \beta BM_{SP,t} + \varepsilon_{t+1},$$

with information available until month t. The statistics are calculated between July 1990 and August 2018. The graphs display the Sharpe ratios (monthly values multiplied by $\sqrt{12}$), mean annualized returns (%), skewness, and the proportion of months corresponding to expected size premiums above or below the respective threshold on the horizontal axis.

that the portfolios avoided (on the right) tend to give very low and often negative returns for thresholds below 1% for the SMB portfolio. On the other hand, there is no clear pattern involving the skewness of these portfolios. Fig. 8 also shows that it is possible to obtain Sharpe ratios close to 0.5 by investing in SMB portfolios with expected returns at least a little (but not much) below zero, which happens around 60% of the time. Indeed, the graphs on the right show that the portfolios that are avoided in this case have slightly negative mean returns.

The results in Fig. 9 reflect the fact that the BM spread forecasts the value premium better. However, both the "invested" and the "avoided" Sharpe ratios and mean returns increase less monotonically with the thresholds (from −2% to 6%) because of the slightly positive return for a threshold just below zero as we see in the right column. Still, it is only after a threshold above 4% that the "avoided" mean returns become positive. Indeed, it is possible to obtain a Sharpe ratio above 1 by investing in the HML portfolio close to 20% of the time when the expected value premium is above 4%, for example.

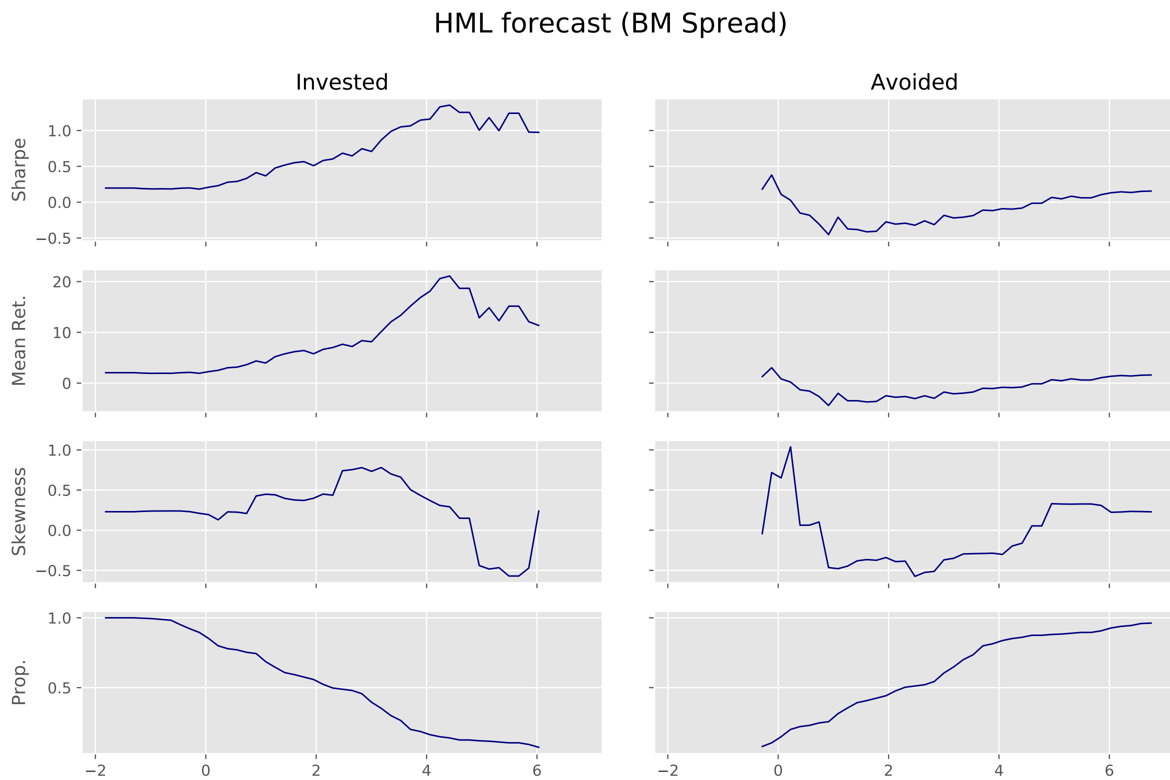

Figure 9: Summary statistics for the period 1990-2018 regarding the returns on the HML portfolio with recursively estimated expectations above (Invested) or below (Avoided) the thresholds shown on the horizontal axis. The forecasts use the BM spread, $BM_{SP,t}$, to recursively forecast the value premium, $R_{hml,t+1}$, in regressions of the form

$$R_{hml,t+1} = \alpha + \beta BM_{SP,t} + \varepsilon_{t+1},$$

with information available until month t. The statistics are calculated between July 1990 and August 2018. The graphs display the Sharpe ratios (monthly values multiplied by $\sqrt{12}$), mean annualized returns (%), skewness, and the proportion of months corresponding to expected value premiums above or below the respective threshold on the horizontal axis.

Finally, the results for the investment portfolio in Fig. 10 and Fig. 11 are similar to each other and to the previous results. There are two main differences, however: The skewness of the investment portfolio tends to become negative as the required return threshold increases. And the Sharpe ratios and mean returns increase more monotonically with the thresholds, which reflects the fact that the investment premium is highly predictable.

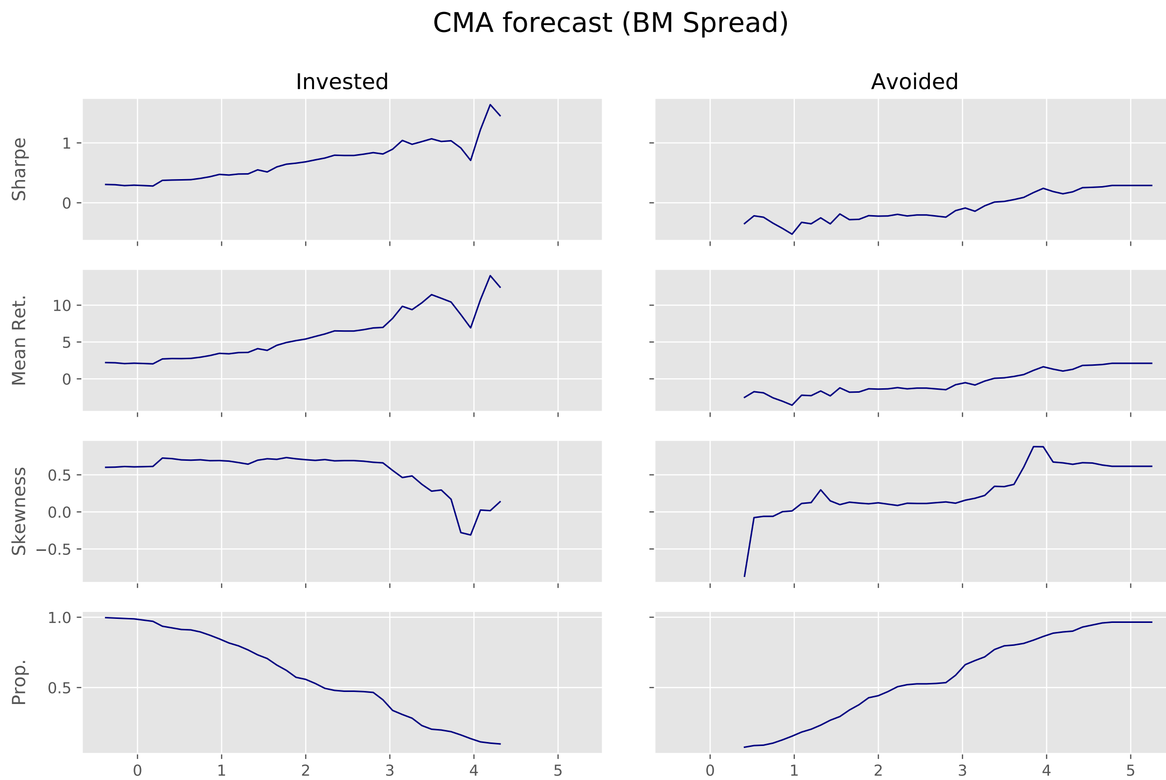

Figure 10: Summary statistics for the period 1990-2018 regarding the returns on the CMA portfolio with recursively estimated expectations above (Invested) or below (Avoided) the thresholds shown on the horizontal axis. The forecasts use the BM spread, $BM_{SP,t}$, to recursively forecast the investment premium, $R_{cma,t+1}$, in regressions of the form

$$R_{cma,t+1} = \alpha + \beta BM_{SP,t} + \varepsilon_{t+1},$$

with information available until month t. The statistics are calculated between July 1990 and August 2018. The graphs display the Sharpe ratios (monthly values multiplied by $\sqrt{12}$), mean annualized returns (%), skewness, and the proportion of months corresponding to expected investment premiums above or below the respective threshold on the horizontal axis.

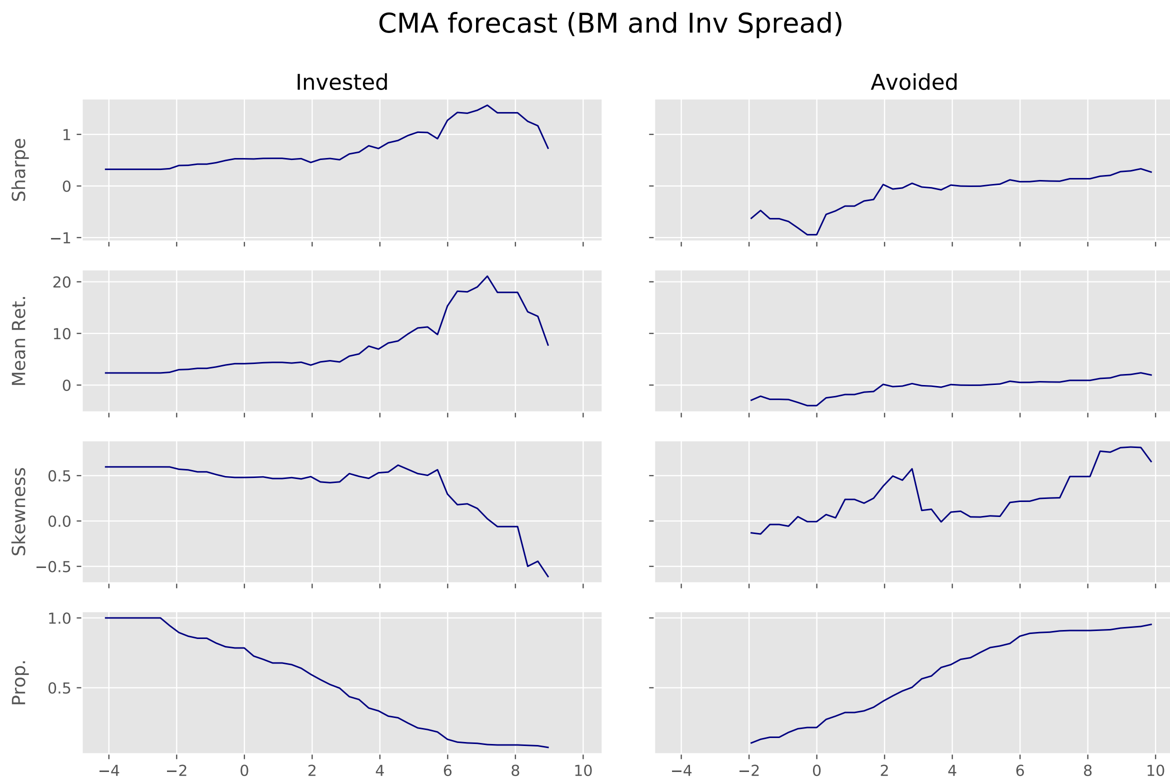

Figure 11: **Summary statistics for the period 1990-2018 regarding the returns on the CMA portfolio with recursively estimated expectations above (Invested) or below (Avoided) the thresholds shown on the horizontal axis.** The forecasts use the BM spread, $BM_{SP,t}$, and the investment spread, $Inv_{SP,t}$, to recursively forecast the investment premium, $R_{cma,t+1}$, in regressions of the form

$$R_{cma,t+1} = \alpha + \beta_{bm} BM_{SP,t} + \beta_{iv} Inv_{SP,t} + \varepsilon_{t+1},$$

with information available until month t. The statistics are calculated between July 1990 and August 2018. The graphs display the Sharpe ratios (monthly values multiplied by $\sqrt{12}$), mean annualized returns (%), skewness, and the proportion of months corresponding to expected investment premiums above or below the respective threshold on the horizontal axis.

7 Conclusion

In this paper we learn about the existence of a common variable driving all the expected factor premiums in Fama and French (2015), except profitability. We also learn that this variable is theoretically linked to the market price of risk, which implies that the premiums seem to arise as a compensation for systematic risk. This adds support to risk-based explanations of these factor premiums in general and provides broad support for standard macro-finance models that predict exactly this type of time variation in the price of risk.

In addition, we learn that the risk of the factors in Fama and French (2015) is not constant over time: The time variation in the spread of the characteristic used to create each factor relates to the time variation in the risk of the factor. Hence, the criticism against industry-related factors for not having constant risk over time (Fama and French, 1997) seems equally applicable to the factors in Fama and French (2015). This adds to similar evidence in de Oliveira Souza (2019c) about the factors in Fama and French (1996).

Finally, the paper provides out-of-sample evidence confirming the in-sample results. This is particularly important for investors who would like to implement the strategies that follow from the discoveries in the paper. In this sense, we learn that especially the investment factor is highly predictable both in sample and out of sample by a common market price of risk proxy (the BM spread) in addition to the factor risk proxy (the investment spread). The factor timing strategies that arise from this predictability generate substantial economic gains out of sample.

A Factor timing from factor risk proxies only

This appendix shows the out-of-sample performance of the factor timing strategies based on the investment- or profitability-related factor risk proxies alone.

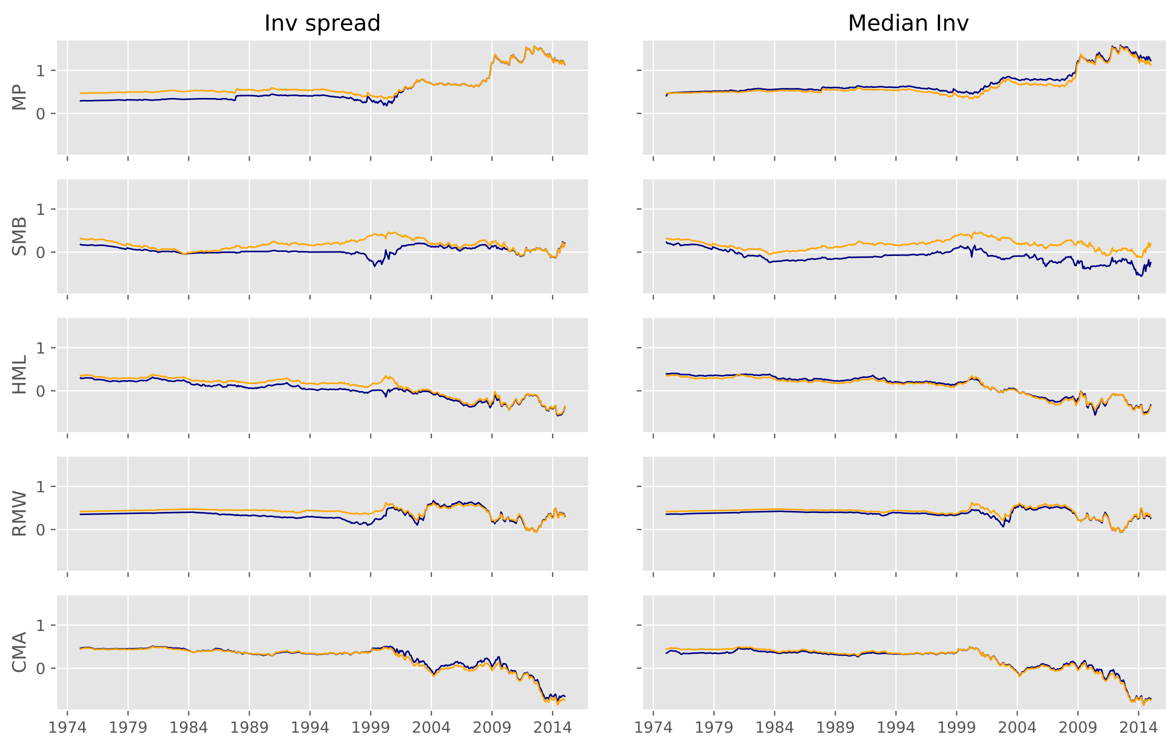

Figure 12: Out-of-sample Sharpe ratios (until August 2018) based on investment-related factor risk proxies – in blue. The graphs on the left-/right-hand side use the investment spread/median, f_t, to recursively forecast $R_{i,t+1}$, which is the market premium (the top graph, MP), the size premium (SMB), the value premium (HML), the profitability premium (RMW), or the investment premium (CMA) in regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The graphs display the out-of-sample Sharpe ratios calculated from each point in time (the horizontal axis in the graphs) until 2018 using portfolio weights proportional to the expected premiums each month to lever the portfolios up or down over time,

$$\omega_{i,t} = \mathrm{E}_t[R_{i,t+1}],$$

which is obtained recursively with information available until time t. I report the monthly Sharpe ratios multiplied by $\sqrt{12}$. Unconditional values appear in orange.

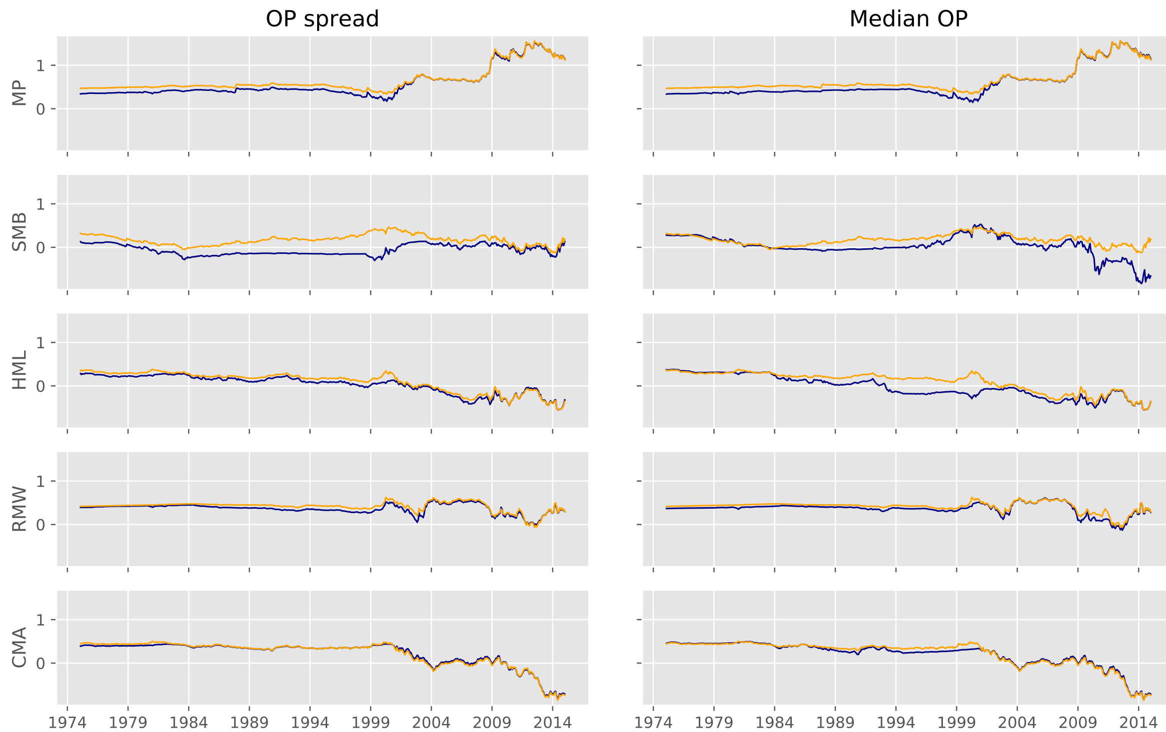

Figure 13: Out-of-sample Sharpe ratios (until August 2018) based on profitability-related factor risk proxies – in blue. The graphs on the left-/right-hand side use the profitability spread/median, f_t, to recursively forecast $R_{i,t+1}$, which is the market premium (the top graph, MP), the size premium (SMB), the value premium (HML), the profitability premium (RMW), or the investment premium (CMA) in regressions of the form

$$R_{i,t+1} = \alpha_i + \beta_i f_t + \varepsilon_{i,t+1}.$$

The graphs display the out-of-sample Sharpe ratios calculated from each point in time (the horizontal axis in the graphs) until 2018 using portfolio weights proportional to the expected premiums each month to lever the portfolios up or down over time,

$$\omega_{i,t} = \mathrm{E}_t[R_{i,t+1}],$$

which is obtained recursively with information available until time t. I report the monthly Sharpe ratios multiplied by $\sqrt{12}$. Unconditional values appear in orange.

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