Strategic tax and public service competition among local governments

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Strategic tax and public service competition among local governments

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Abstract. Tax and public service competition between local governments concerning localisation of new residents is analysed in a setting of economic spillovers which means that also a neighbouring region will benefit from localisation via demand of residents in a border region, (a so-called host region). We identify two basic Nash-equilibrium outcomes of the analysed tax-game. In one of these outcomes local tax rates will be different across the regions – a fact that appears important for (future) empirical studies of local tax competition. Due to the lack of adequate theoretical modelling, studies in this field have often demonstrated spatial dependence of local policy variables without identifying the source of interaction between decision-makers. Our theoretical findings prove to be robust to a range of important expansions of the basic simple framework.

Keywords: Local tax competition · Household locational choice · Spillover effects · Nash-equilibria

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1. Introduction

Over the past several years, theoretical and empirical studies have investigated various perspectives of tax and public service competition in a system of political jurisdictions. A range of important issues are considered in a variety of international policy fields, such as investment policy, trade policy, migration policy and environmental policy.\(^1\)

The focus in the present paper relates to household locational choices and tax competition between local governments. We provide an analysis of the strategic interaction between two local governments concerning a local policy variable that is assumed essential for the choice of jurisdiction of new residents planning to locate in the community defined as the regions covered by both jurisdictions. As for domestic household locational choices, a strategic setting is preferable even for small jurisdictions since households are likely to consider relatively few neighbouring regions as close substitutes. Households may thus typically possess relatively strong preferences about living in the local area nearby their workplace, their relatives, etc., whereas preferences are weaker concerning more specific locations in the local area. In situations where a household for some reason has decided to move and settle down in a geographic area, the specific choice of location between regions in the area may then to a large extent be dictated by tax and public service levels set by the local governments.

Our motivation for the following analysis is to clarify this kind of strategic interaction in the presence of economic spillover effects between regions meaning, for example, that benefits in a region depend on the demand of residents in surrounding regions. Although spillover models constitutes a major research line in the tax competition literature, only a few theoretic analyses are based on a setting of local governments.\(^2\) Considering the growing number of empirical studies on

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\(^1\) Analytically, an important distinction between models is whether strategic interactions are present or not. In the tax-competition literature, where the focus is on taxation of a mobile capital base, early models such as Wilson (1986) and Zodrow and Mieszkowski (1986) provide analyses in fully competitive settings: A tax on a locally employed capital base finances a local public good, jurisdictions are small, and are not in a position to affect after-tax-returns to capital. Mintz and Tulkens (1986) and Bucovetsky (1991) were among the first to introduce strategic behavior between units. Both forms of models basically conclude that public goods typically are underprovided since jurisdictions lower tax rates due to competition as they try to keep the local tax base intact. See Wilson (1999) for a survey of the early literature.

\(^2\) In these studies strategic interactions are, unlike the interactions in the present study, assumed to stem form voters’ views on the performance of local politicians which leads to so-called yardstick competition.
spatial correlation and tax competition effects on tax rates at local level there is a need for a better theoretical description and understanding of various sources of interactions at local level. This need is for instance reflected via the empirical specification of strategic interaction in a number of studies where spatial reaction functions appear to be similar for both tax competition and yardstick competition.³ Spillover effects are pervasive in some settings of tax competitions. As an example, environmental regulation constitutes an obvious case across national borders due to externalities of global pollution. In other settings spillover effects may be more pronounced among local regions than among national regions, and there is reason to believe this is the situation for the kind of effects considered in the present analysis, where spillover effects arise in the simplest form of the model via profits from local consumption accruing to economic agents in both regions. As we develop the basic model, spillovers will further include positive benefits of employment on the regions, as higher levels of employment in one jurisdiction are likely to lead to improved growth opportunities also for neighbouring regions within national borders.

In relation to residential location, a key feature of the model to be presented is the ‘locational surplus’, which a region achieves by being a host region for new residents rather than being a neighbouring region for these. As potential new residents in the two regions would want to settle down in the region with the lowest level of the policy variable, competition in this variable may lead to discontinuous jumps in benefits of a region because a change in the policy variable may change the status for a region in the form of a host or neighbouring region. This basically leads to two forms of Nash equilibria in the policy variables where in the first form the policy variables are equal and benefit levels are the same for the host and neighbouring region. In the other form of Nash-equilibrium the policy variables are different in the two regions and benefits are highest in the neighbouring region, which can be interpreted as a situation where the variable of the neighbouring region is set at a sufficiently high level to keep the other region in its role as a host. Given our formulation of strategic interaction between local governments, it is moreover noteworthy that we do not arrive at the conclusion that equilibrium policy variables will necessarily be set too low relative to the Pareto efficient level for the joint benefit of the two regions. This is, as mentioned, in contrast to most of the literature on tax and public service competition. The model presented in the

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³ See for example Edmark and Ågren (2008) for a discussion on this. The study is one of only a few empirical examinations attempting to uncover the underlying source of spatial correlation in taxes by making use of a reform of the central government system of grants in Sweden. See also Besley and Case (1995) and Bordignon et al. (2003). In a survey of empirical studies, Brueckner (2003) also points to the same ultimate empirical specifications in tests of different effects of strategic interaction.
following draws on Hoel (1997) where the related issue of a game in environmental taxes between the governments of two countries is analysed in relation to location decisions of producers regarding polluting production plants. We apply and gradually extend Hoel’s model to also account for real-world issues ignored in the basic analysis. For example, one can rarely identify a strict host region and a strict neighbouring region in practice, as jurisdictions are always to some extent hosts for all kinds of demographic groups, although some of these groups may be represented in a limited number. We hence consider how decision-making on multiple potential new households, rather than just one household, will change the Nash equilibria and outcomes of the basic tax game.

The fact that strategic policy variables may end up being different across regions in a Nash equilibrium is an important theoretical input for empirical studies of spatial correlations in tax and public service variables. Examinations of taxation decisions of competing jurisdictions are often constructed to search for evidence of tax mimicking, for instance by identifying a statistically significant positive correlation between a given local tax rate and tax rates in neighbouring jurisdictions. Authors are likely to consider cases where no such relations have been found as evidence of no strategic interaction in tax rates. In the light of the analysis in the present paper, no conclusion can be drawn on strategic interactions even in the absence of tax mimicking, insofar as interactions between local policy makers may result in a Nash equilibrium with different levels of strategic policy variables.

Before presenting the theoretical framework, in the next section we offer a first empirical insight into the issue of spatial interaction between locals, based on Danish data. Section 3 presents the model and outlines the two basic outcomes of the Nash equilibrium for the game. In section 4 the model is expanded in a variety of directions in order to examine the robustness of results in more realistic settings, and we finally prove that the basic findings on equilibrium outcomes of the model will carry over to more general frameworks. In particular, under relatively general formulations of the host and neighbour benefit functions there will be one, and only one, pure strategy Nash equilibrium of the tax game. Finally, section 5 concludes.
2. Related empirical estimations for the case of Denmark

We will briefly consider empirical data for Danish municipalities to provide some evidence of spatial dependence between potential local strategic variables. In Denmark, as well as other Scandinavian countries, the public sector is organised into three governmental levels, respectively the municipal level, the county level and the central state level. Given a relatively high level of decentralisation at the local level, with 275 municipalities having individual responsibility of a range of service variables interesting for our study, Denmark lends itself to empirical examinations of spatial correlations between local policy variables. The responsibilities of Danish municipalities both concern the provision and the financing of service variables, and, as to the present theoretical analysis, the strategic policy variable also gains interpretation in terms of service variables applied in attracting or deterring certain demographic groups. Some of these variables naturally appear to be more obvious than others as a strategic instrument. For example, a local region, with ambitions of raising the number of young families in its jurisdiction (due to an overall objective of increasing local labour supply), may choose child day care prices as a strategic variable rather than income tax rates. Below, we focus on both local income taxes and day care prices to estimate their mutual dependency across local jurisdictions.

Data

Data for the study were obtained from the Key Data Base maintained by the Danish Ministry of the Interior and consist of observations from 268 of the 275 Danish municipalities (7 small municipalities located on islands were excluded due to lack of data), obtained annually from 2000 through 2006. In 2007, a municipality reform changed the number of municipalities from 275 to 98, so that data from later years were inapplicable for the analysis. Table 1 provides an overview of the data for the study.
Table 1. Data for the study.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Municipal tax rate</td>
<td>Municipal tax percentage</td>
<td>21.02</td>
<td>1.30</td>
</tr>
<tr>
<td>Municipal tax base</td>
<td>Municipal tax base per resident (1,000 DKK)</td>
<td>124.56</td>
<td>24.36</td>
</tr>
<tr>
<td>Municipal child care expenditure</td>
<td>Total child care expenditures per child, 0-10 years of age (1,000 DKK)</td>
<td>40.06</td>
<td>74.90</td>
</tr>
<tr>
<td>User fee</td>
<td>Monthly child care user fee per child, 0-2 years of age (1,000 DKK)</td>
<td>2.03</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Note: 1) Observations for 268 municipalities, yearly 2000-2006; price adjusted (2000=100). 2) One DKK is approximately 0.13 €.

Three measures of municipal variables are included: The municipal income tax rate, the expenditure per child for the service variable child day care of 0-10 years of age, and the user fee for child day care of 0-2 years of age. Furthermore, to control for the economic ability of the municipality, and as a rough measure of the household income in the theoretical analysis, the per resident municipal tax base is included as a control variable.

Regression methods and results

To investigate spatial interactions across neighbourhood municipalities, we apply a spatial autoregressive specification from spatial econometrics. Initially, given $N = 268$ municipalities, an $N \times N$ matrix $W$ is set up such that $w_{ij}$ equals $1/k_i$, only if the municipalities $i$ and $j$ are neighbours, where $k_i$ is the number of neighbouring municipalities of municipality $i$. In case $i$ and $j$ are not neighbours, then $w_{ij}$ is set to 0. With $y_t$ being a service measure (tax rate, child care expenditure or user fee) of the 268 municipalities for a given year $t$, the product $W_{ij}y_t$ hence defines a variable, which for each municipality holds the average of the tax rate for the neighbourhood municipalities. The SAR specification (Anselin, 1988) reads as:

$$y_t = \lambda(W_{ij}) + X_t\beta + \nu_t,$$

(1)

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4 In line with other studies, we define two municipalities as neighbours only if they share borders. Implicitly, we thus assume that local decision-makers exclusively consider border municipalities as competitors.
where $\lambda$ is a parameter specifying the degree of spatial spillover, formally restricted to the interval between -1 and 1, but for most practical purposes is simply restricted to be non-negative, and $X_t$ contains exogenous variables (i.e. the municipal tax base and a constant term). A significantly positive spillover parameter will now be indicative of municipal service competition.

Furthermore, given that data are available for several years, it is possible (and indeed statistically efficient) to account for intra-municipal behavioural correlation across years, using a Seemingly Unrelated Regression (SUR) type specification to the residual of (1); see Anselin (1988). Between any two years, the residual covariance reads as

$$E(\nu_t \eta_t^s) = \sigma^2 \delta I, \ t, s = 2000, ..., 2006,$$

so that a joint SAR-SUR specification occurs. Given that the dependent variable occurs on the right hand side of (2), traditional OLS or GLS approaches will lead to biased estimates of the parameters. Rather, a Maximum Likelihood (ML) approach is applied (Anselin, 1988).

Table 2. Regression models for tax percentage.

<table>
<thead>
<tr>
<th>Model (Y)</th>
<th>Tax rate</th>
<th>Expenditure 0-10</th>
<th>User fee 0-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>15.10 (0.16)*</td>
<td>9.98 (0.74)*</td>
<td>0.56 (0.03)*</td>
</tr>
<tr>
<td>Spatial spillover</td>
<td>0.27 (0.005)*</td>
<td>0.53 (0.007)*</td>
<td>0.51 (0.004)*</td>
</tr>
<tr>
<td>Tax base</td>
<td>0.0007 (0.0005)</td>
<td>0.08 (0.005)*</td>
<td>0.003 (0.001)*</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-1813.23</td>
<td>-14970.56</td>
<td>-9775.01</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses are standard errors. Significance indicated by * (1%)

The estimation results are collected in Table 2. The spatial spillover is significant for all three service measures. The interaction seems to be most profound for child day care, as indicated by the relatively high coefficients of 0.53 for expenditure and 0.51 for the user fee. For the tax rate, the coefficient is of a relatively smaller magnitude of 0.27. The economic abilities of the municipality, as measured by the tax base, seem to impact daycare service, while they have no significant impact on the tax rate.\(^5\) We now turn to the theoretical analysis.

\(^5\) For another Scandinavian country, namely Sweden, Edmark and Ågren (2008), using an instrument variable approach, estimate the spatial coefficient of neighbouring tax rates to have a positive and significant effect on the own tax rate for municipalities. This study also finds a markedly higher coefficient of 0.74. See Allers and Elhorst (2005) for other recent studies providing evidence of spatial interaction processes in local tax rates.
3. The model

We consider two regions for localisation, each representing a local jurisdiction. New residents\(^6\) who locate within the regions face the respective local taxes and tariffs for public services and have the rights to enjoy public services supplied by the jurisdiction. A new resident in the area will locate in the region offering the highest flow of public services net of tax and tariff payments to this region from the resident. We assume that the total tax bill of a resident depends on the level of a policy variable \(\sigma\) set by local authorities. The following analysis does not necessitate further specification of the policy variable. The modelling practice in the tax competition literature is to focus on a capital tax as the local policy instrument – often interpreted as an important component of broader property taxes at local level. The present analysis is in line with these analyses as we essentially consider a situation with a single tax instrument levied on mobile factors. The set-up to be presented also allows interpretation of the choice variable in terms of a user fee, levied at the local level on those demanding public school or day care services, which may be applied in combination with public subsidies to finance such services. A higher level of \(\sigma\) may thus reflect a higher user fee or consumer price (equivalent to a lower public subsidy) for various publicly supported activities. The choice variable is referred to as a tax rate in the following.

In the light of national equity objectives, we also assume political feasibility restrictions on local policy design towards funds and services: to comply with national constitutions local policymakers need to formulate income dependent policies towards citizens so that, for a given level of the policy variable, public revenues and services will, respectively, rise and fall in income levels. Given these restrictions, we disregard situations where a local policy-maker chooses to provide higher net flows of services the higher the household income, in order to make attractive high income groups locate in the region (where ‘attractive’ could be seen in terms of high local buying power or high-value local labor supply).

Denote the public revenues from a new resident with income \(I\) by \(R(I, \sigma)\) and the cost to the region of the local service level towards this resident by \(S(I, \sigma)\), with \(R_I > 0, R_\sigma > 0, S_I < 0\) and \(S_\sigma > 0\). Revenues, \(R(I, \sigma)\) from the resident can be seen as the total tax bill to the resident at the local level

\(^6\) As the economic unit could also be seen as a household, we alternate between the terms ‘household’ and ‘resident’ in what follows.
including income taxes, property taxes and public user fees, and these revenues will naturally
depend on the income of the resident. The service costs, \( S(I, \sigma) \) are also assumed to be a function
of \( I \), as services offered to residents may in many cases depend on their level of income. We
moreover assume local budget concerns by making service costs depend on \( \sigma \) in order to mirror a
situation in which higher revenues to the jurisdiction provide a basis for higher service costs and
vice versa.\(^7\)

We shall first consider income levels of potential new residents to be exogenously given. Sub-
sequently, income is endogenised by making the supply of labour from the household depend on the
level of \( \sigma \), given that labor supply depends on after tax disposable income. Then consider
\[ N(I, \sigma) = R(I, \sigma) - S(I, \sigma), \]
which is the extra ‘net public fund’ of a region following a residential choice of this region. In the case of exogenous income, the disposable income as defined here will be
\[ I^d(I, \sigma) = I - N(I, \sigma), \]
which expresses the income left for consumption after net payments to the local region (thus disregarding state tax payments and services). Marginal disposable income with respect to \( I \) is assumed positive, that is \( I^d_I = 1 - N_I > 0 \) (or equivalently \( R_I - S_I < 1 \)), which seems reasonable, as in the opposite case any incentives of individuals to supply labour units will vanish. Further, disposable income will fall in \( \sigma \), meaning \( I^d_\sigma = -N_\sigma < 0 \) (or equivalently \( R_\sigma - S_\sigma > 0 \)). As we also ignore degressive tax policies (inclusive of public service amounts), \( R_\mu - S_\mu \geq 0 \). Assume further that the net public fund effects of a higher fund raising policy are declining in \( \sigma \), that is \( R_\sigma - S_\sigma \leq 0 \). Finally, the choice variable is normalised so that \( \sigma = 0 \) represents the most lenient tax policy of a jurisdiction. Assume that for any income there will always be a negative net public fund for this level, whereby \( N(I, 0) < 0, \forall I \). For both regions we will moreover assume a maximum level \( \bar{\sigma} \) of the policy variable. This level can be interpreted as an upper restriction on local public activity imposed on local jurisdictions by the national government having overall fiscal policy objectives for the country.

\(^7\) We do not incorporate an explicit budget constraint for the regions as the policy variable in focus is to be perceived as just one specific instrument for the locals among many others. We shall shortly introduce a cost of raising public funds which may reflect tax distortions from adjusting other tax instruments when the considered policy variable is changed.
As residents spend a major part of their disposable income locally, business in a region also benefits from localisation. Assume a commensurate good, representing goods from local retail, local culture, etc., being supplied in both regions. The price $P$ of this good is assumed constant and unaffected by extra demand from localisation. This assumption is only for reasons of simplicity and it means that policy makers in their choice of the local tax level ignore local price change effects on local benefits which seems reasonable relative to practice. Moreover, marginal production costs for the good are constant and also the same for all producers, simply given by $b$. There are two spillover effects of trade on the neighbouring region from the region hosting new residents: a share of profits from sales in the host region belongs to owners living in the neighbouring region, and new residents moreover spend a part of their income in the neighbouring region. We shall assume the aggregate neighbour spillover effect to be the same for both regions. In other words, in the position of a neighbour, regions will enjoy the same benefit from the private spending of new residents located in the host region, for a given $\sigma$ in this region. All owners of profits from the commensurate good are assumed to live within the two regions. This can be formalised as follows: denote by $\delta$ the part of owners who live in the region in which an amount $q$ of their product is traded, with $0 \leq \delta \leq 1$. The profit $\delta(p-b)q$ would then accrue to these owners while owners in the other region would achieve the profit $(1-\delta)(p-b)q$. For each unit $q$ of the good bought by new residents the shares $\gamma$ and $(1-\gamma)$ are, respectively, sold in the host and the neighbouring region. The total share of profits of the host region from new residents’ private consumption then becomes $\gamma \delta + (1-\gamma) (1-\delta) \leq 1$ where the last term represents the host region’s profits from trade in the neighbouring region. We henceforth denote this total share by $\theta$ with $0 \leq \theta \leq 1$ for $0 \leq \delta, \gamma \leq 1$. By analogous reasoning the profit shares of the neighbouring regions profit share becomes $\gamma(1-\delta) + (1-\gamma) \delta = 1-\theta$. Demand for the commensurate good is given by the relation

$$q = A - P + \alpha I^d, \quad (3)$$

where $0 < \alpha < 1$ and $A \geq P - \alpha I^d$. A higher disposable income will increase the demand of the locally produced good meaning that local policy makers are able to impact on local business activity through their decision on the level of $\sigma$.

Consider next the effects for the regions of the level of policy variable imposed on prospective new residents with income $I$ in one of the regions. Policymakers consider the total benefits for the region
in terms of the value of changes in public funds and business profits. The benefits for the region in which a household with income $I$ locates (the host region) is then

$$B^h(\sigma^h) = (1 + \lambda) N(I, \sigma^h) + \theta(P - b) q(\sigma^h, I^d),$$  \hspace{1cm} (4)$$

where we have assumed that there is a cost to the region of raising public funds, denoted $\lambda$, such as tax distortion in other sectors. Then a net public fund $N$ from a new household will be worth $(1 + \lambda)N$ to the region. Recall that $N$ may be negative, in which case the amount in (4) may be negative. Due to the neighbour spillover effects of private consumption there will be benefits also for the neighbouring region $B^n$. Given a region is a neighbour, the spillover effect yields

$$B^n(\sigma^h) = (1 - \theta)(P - b) q(\sigma^h, I^d).$$  \hspace{1cm} (5)$$

As all spillovers are generated from the demand of the new resident located in the other region, neighbour benefits in the tax game are thus a function of only the choice variable in the host region. (We suppress the ‘h’ topnote henceforth). It will be useful in what follows to consider the difference in benefits between the host and the neighbouring region, given by the relation

$$B^h(\sigma) - B^n(\sigma) = (1 + \lambda) N(I, \sigma) + (2\theta - 1)(P - b) q(\sigma, I^d).$$  \hspace{1cm} (6)$$

We restrict our attention to cases where for the most lenient tax policy level, $\sigma = 0$, the benefit of the host region is lower than the spillover benefits of the neighbour so that $B^h(0) - B^n(0) < 0$. From the expressions it follows by use of (3) that

$$\frac{dB^h}{d\sigma}(\sigma) = (1 + \lambda) N_\sigma - \alpha \theta (P - b) N_\sigma = [1 + \lambda - \alpha \theta (P - b)] N_\sigma$$  \hspace{1cm} (7)$$

and

$$\frac{dB^n}{d\sigma}(\sigma) = -\alpha (1 - \theta)(P - b) N_\sigma.$$  \hspace{1cm} (8)$$

The sign of (7) is ambiguous depending on parameter values, whereas (8) is negative for any $\sigma$. This opens up for various outcomes of strategic interaction as illustrated in Figure 1a-d where benefit
levels of the two regions are depicted as functions of the policy variable in the region chosen for localisation, that is \( \sigma^h \). Throughout the analysis, we will apply the curves for relations (4) and (5) to identify equilibrium tax variables and outcomes for the tax game between the local governments.

Suppose first, that \([1 + \lambda - \alpha \theta (P - b)] > 0 \) so that the host region gains from a higher \( \sigma \) meaning that the marginal gain in net revenue of public funds always exceeds the local marginal loss in producer surplus. In this situation there will be a unique Nash equilibrium between decision-makers in the regions that leads to the same levels of the policy variable with \((\sigma^h, \sigma^n) \leq \overline{\sigma}\). As illustrated in Figure 1a, for \((\sigma^h, \sigma^n)\) strictly less than \(\overline{\sigma}\), benefits are equal for the host region and the neighbouring region in this equilibrium, entailing that expression (6) above equals zero. To see that this is the unique equilibrium, first note that different values of the policy variables in the regions always induce region \(h\), (being the host and thereby having set the lowest policy variable), to raise \(\sigma\) to achieve a higher benefit. Second, no other equal levels of \(\sigma\) than \(\sigma^*\) where \(B^h(\sigma^*) = B^n(\sigma^*)\) would constitute a Nash-equilibrium: for alternative identical levels lower than \(\sigma^*\), the host region would have an incentive to raise its policy variable to take over the role of a neighbour region, and for identical levels higher than \(\sigma^*\), the neighbour region would in turn be better off by reducing its standard to become the host region. For \((\sigma^h, \sigma^n) = (\sigma^h, \sigma^n)\), both regions would in contrast reduce their benefits by changing \(\sigma\), taking the policy variable level, \(\sigma^*\), for the rival player as given. (For these levels, note in particular that if the host country raises \(\sigma\), it would become the neighbouring region and hence achieve a benefit change along the \(B^n(\sigma^h)\) path and not along the \(B^h(\sigma^h)\) path. Likewise, the present neighbouring region would become the host region if it lowers \(\sigma^n\), and therefore it faces a reduction in benefit from this.) We could think of this equilibrium case as a situation where regions compete in being the neighbour region rather than the host region for policy variables lower than \(\sigma^*\) whereas for levels beyond \(\sigma^*\), regions prefer being the host and then compete by lowering the policy variable. Note that in spite of competition on attracting residents, the benefits for both regions are positive in equilibrium.

Still for \([1 + \lambda - \alpha \theta (P - b)] > 0 \), a variant of the equilibrium depicted in Figure 1a occurs when the level of \(\sigma\) for which \(B^h(\sigma) = B^n(\sigma)\) exceeds \(\overline{\sigma}\), see figure 1b. This turns competition into the so-called case of ‘Not In My Backyard’ in which regions compete to avoid being the host since the role
of a neighbouring region is the most attractive for any choice of legal policy variable. This ends up in an equilibrium where both regions adopt the toughest legal local policy within the federation, represented by the policy variable $\sigma$. In equilibrium the host region therefore cannot evade its host role by tightening local policy even further, given $\sigma^h = \overline{\sigma}$. The neighbouring region’s best reply is obviously the toughest policy, as any reduction of $\sigma^n$ below $\overline{\sigma}$ would imply that it takes over as the host, which may lead to a discontinous (large) reduction in benefit. Hence, the strategy combination $(\sigma^h^*, \sigma^n^*) = (\overline{\sigma}, \overline{\sigma})$ is the only equilibrium. As to the payoff for this game, note that $B^h(\overline{\sigma}) < B^n(\overline{\sigma})$ and the benefit to the host region $B^h(\overline{\sigma})$ may be negative or positive, whereas $B^n(\overline{\sigma}) > 0$, given our assumptions.

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Figure 1a. Equal taxes and equal benefits in equilibrium

Figure 1b. The case of ‘Not in my backyard’

Now consider the case where $[1 + \lambda - \alpha\theta(P - h)] < 0$. Both benefit curves are decreasing in $\sigma^h$ as shown in figure 1c, and depending on the levels and shape of $B^h(\sigma)$ and $B^n(\sigma)$, these may or may not cut. Note here that for any $\sigma^h > 0$ and for $\sigma^n$ given, the host region always becomes better off by lowering $\sigma^h$, (as it remains a host), and therefore a Nash equilibrium may necessarily involve the host to set $\sigma^h = 0$. For $\sigma^h = 0$, any level of $\sigma^n$ lower than $\overline{\sigma}$, where $\overline{\sigma}$ is defined by $B^h(0) = B^n(\overline{\sigma})$, cannot be sustained as a Nash-equilibrium since if $\sigma^n < \overline{\sigma}$, the host would then deviate from $\sigma^h = 0$ and choose a policy variable higher than $\sigma^n$ and lower than $\overline{\sigma}$. It would then
become a neighbour and enjoy a benefit higher than $B^\sigma \left( \bar{\sigma} \right)$. All combinations of $\sigma^h = 0$ and $\sigma^n \geq \bar{\sigma}$ are then a Nash equilibrium in this case. In any equilibrium the regions gain $B^h \left( 0 \right)$ and $B^n \left( 0 \right)$ whereby both parties are seen to achieve maximum benefits, given their respective individual roles as host and neighbour. This form of equilibrium will be generated whether the curves cross or not. In the case where $B^h \left( 0 \right) < B^n \left( \bar{\sigma} \right)$ there exists no $\bar{\sigma} < \sigma$ for which $B^h \left( 0 \right) = B^n \left( \bar{\sigma} \right)$ and the unique Nash equilibrium will be the strategy profile $\left( \sigma^h, \sigma^n \right) = \left( 0, \bar{\sigma} \right)$. The neighbour sets $\sigma^n$ at its highest legal level which the host region is not allowed to exceed and, in consequence, $\sigma^h = 0$ is the best response for the host. This is the situation depicted in figure 1d.

*Figure 1c. Unequal taxes and unequal benefits in equilibrium*  
*Figure 1d. Equilibrium with ‘low’ host benefits*

By means of the above simple framework we have gained insight in basic competition between strategically dependent jurisdictions in recruiting new residents. Based on the income flow potential of localisation for the local community, regions will either compete in attracting or deterring localisation.

**Proposition 1.** For $\frac{dB^h}{d\sigma} > 0$ an ‘Equal Tax Equal Benefit’ (ETEB) equilibrium outcome will emerge in which the host and neighbouring regions impose the same tax level and achieve the same benefit level. For some income groups this may turn into a Not In My Backyard situation where
both regions impose the highest feasible tax level, whereas the host benefit is less than the neighbour benefit. For $dB^h/d\sigma < 0$ an ‘Unequal Tax Unequal Benefit’ (UTUB) equilibrium outcome will emerge in which tax levels as well as benefits are lowest in the host region. The tax in the neighbouring region is set at a sufficiently high level to deter the host region from striving for the neighbour’s role.\textsuperscript{8}

An interesting issue remains to be considered: will social benefits – interpreted as the sum of benefits in the two regions – be maximised in the identified equilibria? Within the framework of the outlined model the answer to this is relatively clear. It appears from figure 1a and 1b that this will normally not be the case when competition results in an ETEB equilibrium. Only by coincidence will these equilibria also maximise the joint benefits of the regions. There is, on the other hand no, economic arguments indicating that this will never be the case. In contrast, in a UTUB equilibrium social benefits will always be maximised as both regions, as mentioned, achieve maximum benefits in equilibrium. This result of maximum social benefits for the UTUB equilibrium, however, arises only because in our application both benefit functions are decreasing in $\sigma^h$. If benefits of the host region, for instance, were first increasing and then decreasing over the defined interval for $\sigma^h$, the UTUB equilibrium would obviously not be socially efficient. In other words, the result is not robust to realistic changes in assumptions in the form of spill over effects between the regions.

4. Equilibrium analysis in expanded frameworks

Asymmetric costs and benefits between regions

Regions are so far identical concerning costs and benefits of the local policy. Even though this may be a reasonable assumption for some settings, an obvious theoretical extension would be to examine tax equilibria under asymmetric net-benefits for the regions. In practice, there may be a number of reasons for asymmetry to arise; one region could benefit more than the other from additional labour resources, economies of scale in public production may appear in activities such as childcare and

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\textsuperscript{8} As in Proposition 1, we will henceforth consider the Not In My Backyard case as an ETEB equilibrium, as the incentives in equilibrium are equivalent to an ETEB equilibrium and are only restricted by external rules for the policy variable. This is, however, somewhat misleading since the two regions receive different benefits in equilibrium in the Not In My Backyard case.
schooling, and so forth. Examining the consequences of asymmetry is also obvious given that the impact from tax changes on the present residents in a region may differ substantially across regions. Due to obvious real policy non-discriminatory rules of citizens in the same jurisdiction several residents (rather than just new residents) in a region are affected by changes in policy variables even though we have considered the strategic variable in focus as a specific instrument for affecting locational decision-making. The asymmetric costs and benefits originating from this may be pronounced when regions differ as to various demographic factors of citizens such as age distribution and industry composition, as this is likely to entail high income variation between regions. To demonstrate the consequences of asymmetry, we introduce unequal host benefits while neighbour benefits remain to be equal for the regions. Assume that the benefits to a host region are generally higher in region 1 than in region 2, and denote these values by $B^h_1$ and $B^h_2$, respectively. Assume further that in situations with equal taxes in the regions, a new resident will choose the region with the highest benefit (region 1). It is obvious, that in case of rising benefits in the host region, see figure 2a, Nash equilibrium tax rates must be similar in the regions. (Suppose they were different. The host region could then increase benefits by raising its tax to the level given in the neighbouring region, as $B^h_1$ increases in $\sigma^h$. ) This leads to equilibria corresponding to the two basic situations already considered above. In addition, equal values of host and neighbour taxes within the interval $[\sigma^*, \sigma^{**}]$ now also constitute a Nash equilibrium. Consider for instance the values for $\sigma^h = \sigma^n = \bar{\sigma}$. These values will be equilibrium values since the neighbour (region 2) now has no incentive to lower its policy variable to take over the host role, as this would lead to a fall in benefit because $B^h_2(\sigma) < B^n(\sigma)$ between $\sigma^* and \sigma^{**}$. Social efficiency in equilibrium will still only be realised by coincidence. Note, though, that the assumption that households will locate in region 1 under equal taxes implies that the choice of region is efficient in equilibrium.

When conditions leading to a UTUB type of equilibrium are present, we can identify equilibria with each of the regions as hosts, respectively; see figure 2b. In both cases the tax rate of the host region will be set at $\sigma^{h*} = 0$ while the neighbour sets its tax rate at a level that deters the host from becoming a neighbour. With region 1 as a host, the neighbour will thus choose $\sigma^{n*} \geq \bar{\sigma}$, and with region 2 as host, we have $\sigma^{n*} \geq \bar{\sigma}$. Social welfare is maximised only in the equilibria in which region 1 becomes the host.
Competition for more than one potential new resident

Local tax policy in a region is of interest to many potentially new residents in practice. Dealing with only one prospective new resident may lead to unawareness of an important aspect of benefit functions relative to situations in practice where locals face ‘many’ new residents: in cases where regions set the same tax level, both regions will host some new residents, and we therefore need therefore to modify the understanding of a host and a neighbouring region. With equal tax levels the two regions would receive respective shares  and  of the new residents across the regions with , and with benefits modified accordingly. Assume that a given region receives the share  of an absolute number of new residents who want to locate in one of the two regions. The benefit to this region per ‘resident candidate’ now becomes

\[ B(\sigma, s) = sB^h(\sigma) + (1-s)B^n(\sigma) = B^n(\sigma) + s\left(B^h(\sigma) - B^n(\sigma)\right), \tag{9} \]
as the particular region obtains $s$ and $(1 - s)$ of its benefits per resident in the position of being, respectively, a host and a neighbour. Similarly, the benefits in the other region hosting the share $(1 - s)$ become

$$B(\sigma, 1 - s) = (1 - s)B^h(\sigma) + sB^n(\sigma) = B^h(\sigma) - s(B^h(\sigma) - B^n(\sigma)).$$

(10)

Comparing the last expressions on the right-hand side in (7) and (8), it is clear that, relative to the case with one resident, the difference in benefits between regions is reduced, as an amount of the difference under the case of one resident is now allocated to the region with the lowest benefit. Graphically, the distance between the curves in figure 1a–d is therefore reduced; see figure 3. It follows from this that exactly the same equilibrium tax levels will arise in a game with competition for several residents as in a game with competition for only one new resident since all incentives are intact for all levels of $\sigma^h$ and $\sigma^n$. Therefore, assume a situation with equal tax rates different from $(\sigma^h, \sigma^n)$, such as $\bar{\sigma}$ in figure 3 (in which case both regions will host new residents unless $s = 1$).

Both regions will always have incentives to change the tax in the same direction as in the one resident case (although the absolute benefit levels per resident may differ). This reasoning will be valid irrespective of the level of equal taxes and of whether benefits of the host region are increasing or falling in the policy variable. In figure 3 the case with several new residents is illustrated with cost structures like in figure 1a for the one-resident case. We leave out the figures for the other types of benefit structures considered above. We can conclude that the results from the analysis in section 2 all carry over to a setting with competition for several new residents. Like in the one-resident case, the equilibrium will only be socially efficient by coincidence.
It should be emphasised that this insight hinges upon linearity of total benefits rather than on concavity or convexity of this. Put differently, the benefits to a region of hosting say $M$ new residents needs to be $M$ times the one-resident benefit $B^h$; that is, $MB^h$ and linearity also apply for the neighbouring region. In practice, a local administration may obviously face economies (or diseconomies) of scale from hosting several new residents.\footnote{We can draw on the analysis of Hoel (1997) to obtain insight into the consequences of non-linearity of total benefit. Hoel examines the case of non-linearity of total benefits concerning country competition on several polluting firms. Non-linearity arises due to a convex environmental damage function. Essentially, Hoel identifies more equilibria than in the one-firm case, and in all equilibria taxes are the same for both countries. As for the present analysis, the result suggests that competition for residents among local regions makes identical tax structures in these regions more likely.}

**Proposition 2.** With asymmetric costs and benefits between regions new equilibria may arise relative to the symmetric case. For $dB^h/d\sigma > 0$, equilibrium tax rates are still equal for the host and the neighbour, and an interval of similar tax levels now constitutes equilibrium values. In these equilibria the two regions will have different benefits. For $dB^h/d\sigma < 0$ two UTUB equilibria exist. The basic results outlined in Proposition 1 carry over to a setting where regions compete on the benefits from not only one but more new residents.
Endogenising income levels of households

There are several reasons to examine whether our main results on the two basic types of equilibria would carry over to a more general setting of spillover effects. By allowing for various realistic spillover types between the regions, it seems evident that the monotone benefit functions so far considered cannot be maintained. The relevance of a non-monotone host benefit function can for example be demonstrated by endogenising income levels of residents. We have so far put aside an obvious positive effect of new residents for a region in the form of improved local growth perspectives arising from a higher supply of labour units. In the analysis, effects on employment could simply be treated as a constant positive contribution to benefits in the host region implying that the curves representing the host region in figure 1a-d all move upwards. A more realistic modelling, however, needs to incorporate an endogenous determination of individual labour supply and, hence, income from residents since a range of local tax and tariff levels may impact on incentives to supply labour units. One example is prices of public day care that may affect labour supply of residents with children. These residents would include in their calculation of after tax income not only direct income taxes but also the effects of local subsidies for day care, (recall that the policy variable may also represent local user fees). Changes in the local policy variable could therefore have relatively large and complex impacts on the above considered benefit levels of the regions, and this may, at first, imply other forms of equilibria than those considered above in the game between regions. The general nature of the specified net service function makes scope for analysing overall labour supply effects of a change in the policy variable.

Assume there is a benefit $Z(h)$ to the host region from a resident’s labour supply $h$ with $Z_h > 0$ and $Z_{hh} < 0$. Income now also depends on labour supply via the relation $I = w h(\sigma)$ where $w$ is the exogenously given wage rate which we normalise to $w = 1$ so that $I = h(\sigma)$. The benefit to the host region then becomes:

$$B^*(\sigma) = (1 + \lambda) N(I(h(\sigma)), \sigma) + \theta (P - b) q(I^d(h(\sigma), \sigma)) + Z(h(\sigma)) \tag{11}$$

and

$$B^*(\sigma) = (1 - \theta) (P - b) q(I^d(h(\sigma), \sigma)). \tag{12}$$
We now have that \( \frac{dN}{d\sigma} = N_i \frac{dh}{d\sigma} + N_{\sigma} \). Using that \( I^d(I,\sigma) = h(\sigma) - N(I,\sigma) \), differentiation of (9) and (10) yields:

\[
\begin{align*}
\frac{dB^h}{d\sigma}(\sigma) &= (1 + \lambda) \frac{dN}{d\sigma} + \alpha \theta (P-b) \left[ \frac{dh}{d\sigma} - \frac{dN}{d\sigma} \right] + z_h \frac{dh}{d\sigma} \\
&= [1 + \lambda - \alpha \theta (P-b)] \frac{dN}{d\sigma} + [\alpha \theta (P-b) + z_h] \frac{dh}{d\sigma}
\end{align*}
\]

(13)

and

\[
\frac{dB^n}{d\sigma}(\sigma) = \alpha(1-\theta)(P-b) \left[ \frac{dh}{d\sigma} - \frac{dN}{d\sigma} \right].
\]

(14)

Substituting \( dN/d\sigma \) and rearranging terms lead to:

\[
\begin{align*}
\frac{dB^h}{d\sigma}(\sigma) &= [1 + \lambda - \alpha \theta (P-b)] N_{\sigma} + [(1 + \lambda) N_i + \alpha \theta (P-b)(1-N_i) + z_h] \frac{dh}{d\sigma} \\
&= [1 + \lambda - \alpha \theta (P-b)] N_{\sigma} + [\alpha \theta (P-b)(1-N_i)] \frac{dh}{d\sigma}
\end{align*}
\]

(15)

and

\[
\frac{dB^n}{d\sigma}(\sigma) = -\alpha(1-\theta)(P-b) N_{\sigma} + [\alpha(1-\theta)(P-b)(1-N_i)] \frac{dh}{d\sigma}.
\]

(16)

For both \( B^h \) and \( B^n \) the derivative is now the sum of the derivative when no change in labour supply occurs (that is, the derivatives (7) and (8)) and a term being negative (positive) for \( dh/d\sigma < 0 \) (\( dh/d\sigma > 0 \)). The sign of \( dh/d\sigma \) may thus make the specific forms of equilibria considered above more or less likely. First, if \( dh/d\sigma < 0 \) for any value of \( \sigma \) in the interval \([0, \sigma]\), \( B^n(\sigma) \) is still a decreasing function and it is now more likely that \( B^h(\sigma) \) is falling, which makes the UTUB equilibria the result of the tax game. Conventional wisdom on tax distortions normally says that labour supply will fall when taxes rise, meaning that \( dh/d\sigma < 0 \) appears to be the most likely case in practice. We shall consider a more general structure arising when benefits of the
neighbouring region is falling in $\sigma$. From (16), it appears that this will be the case for 

$$\frac{dh}{d\sigma}(1-N_f) < 1^{10}$$

meaning that the following proposition applies even for increases in $\sigma$ that leads to (minor) increases in labour supply. This case hence includes a situation like the one captured in Figure 4a characterized by ‘several’ crosses between $B^h$ and $B^n$ as well as shifting intervals of $\sigma$ in which $B^h$ may fall and rise. Even for this case important insights can be established:

**Proposition 3.** Assume $B^s(\sigma)$ to be continuous and falling and assume $B^h(\sigma)$ to be non-linear and continuous over the interval $[0, \bar{\sigma}]$. Then there always exists a pure strategy Nash equilibrium in the tax game.

**Proof:** See Appendix.

**Proposition 4.** Under the same assumptions about $B^s(\sigma)$ and $B^h(\sigma)$ as in Proposition 3 and given that possible local maxima functional values are different for $B^h(\sigma)$, we have that: (i) Except for the particular case outlined in (ii), in case an equilibrium is of type ETEB in the tax game, this will be unique. In case an equilibrium is of type UTUB, an infinite number of weak Nash equilibria may exist of this type, each, however, with the same tax level in the host region. (ii) If both an ETEB, $(\sigma^{h*}, \sigma^{n*})$, and an UTUB equilibrium, $(\sigma^{h**}, \sigma^{n**})$, exist for the tax game, then $B^h(\sigma^*) = B^h(\sigma^{**}) = B^n(\sigma^{**})$. Further, $B^h(\sigma^*)$ is a local maximum for the host region benefit function and a maximum for this function must occur at a value of $\sigma$ higher than $\sigma^{**}$ (see figure 4b).

**Proof:** See Appendix.

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$^{10}$ In relation to (16) we can provide an economic interpretation of this condition. Note first that $dh/d\sigma$ can also be seen as the change in disposable income from an income change as $dh/d\sigma = dI^h / dI$. Then the condition for falling neighbour net benefits in $\sigma$ under endogenous labour supply will be that the direct effect of a demand reduction for the local good following from a higher $\sigma$ exceeds a possible countervailing effect on a demand rise stemming from the income increase that appears when individuals decide to raise their labour supply due to a higher $\sigma$. 
The propositions reveal that under quite general benefit structures, equilibrium outcomes of the tax game will be the same as the ones already identified in the analysis. In particular, benefit structures for the regions that may both lead to an ETEB equilibrium and an UTUB equilibrium (or other forms hitherto not identified) do not exist except for the most specific case under (ii), which is illustrated in figure 4b. The finding that either equilibria like the ones depicted in figure 1a-d or figure 3 will emerge is in other words intact. The specific functional forms behind the majority of the analysis therefore does not appear to invalidate the identification of equilibria, and the basic findings and the predictionary power of the analysis in explaining the behaviour of local policymakers in a competitive environment seem rather strong. This may prove to be useful knowledge also for future empirical investigation of issues of strategic interactions.

5. Conclusion and scope for further work

The primary aim of this paper is to add to the theoretical understanding of strategic competition of local governments when spillover effects across regions are present. Our motivation is not least to provide some important insights that may contribute to uncover the source of interaction behind the spatial correlations between taxes and other decision variables at local levels that several empirical studies have identified.
One important message from the analysis is that tax competition between regions for new residents need not lead to an equalisation of policy variables across these regions. With spillover effects an equilibrium may emerge in which one region chooses a strategy of deterring localisation for some types of residents and therefore imposes a level of the policy variable that deviates sufficiently from the level set in surrounding regions to keep away these residents. While such strategies are well known in environmental policy to deter or attract dirty industries, it is important to reflect on the influence of the same strategies on local government behaviour in understanding localisation decisions of, for instance, certain demographic groups. In consequence, to find evidence of strategic interaction one cannot solely consider the slope of reaction functions among jurisdictions.

As for spillover effects from new residents, another important equilibrium outcome identified above is the ‘not in my backyard’ case in which both regions try to evade the host role for (expected) ‘costly’ new residents. To focus on the basic equilibria and the strategies behind them, the issues have been cast in a simple framework. The findings above proved, on the other hand, to be robust to a range of more realistic assumptions including asymmetry of cost, endogenous labour supply and competition for more than one household. We have moreover shown that under more general assumptions there always exists a unique Nash equilibrium with an outcome to be categorised as either an ETEB or a UTUB type.

Still, it would be useful to extend the setup in various directions. For example, we have only considered competition among two regions, and also the basic one-stage game applied could purposefully be developed to encompass more than simultaneous moves. This is left for further analysis. An issue for empirical investigation is the practical occurrence of the outlined equilibrium outcomes with unequal policy variables. This is ignored in our own empirical regressions above, where the immediate aim is confined to demonstrating the spatial correlations between policy variables of neighbouring local jurisdictions also for the case of Denmark. In all, more solid empirical examinations of the various kinds of policy equilibria derived in the theoretical analysis are generally useful for determining whether they are more than theoretical constructions.
APPENDIX

Proof of Proposition 3

First, assume that for every value of $\sigma$ in the interval $[0, \bar{\sigma}]$, $B''(\sigma) > B^h(\sigma)$. Then an equilibrium will exist if the host chooses the value of $\sigma$, where $B^h(\sigma)$ reaches its maximum value and where the neighbour sets a value of $\sigma$ high enough to prevent the host from setting a higher value in order to become a neighbour. This is in other words an UTUB equilibrium.

Second, assume that one or more values of $\sigma$ exist where $B''(\sigma) \leq B^h(\sigma)$. Then also one or more values of $\sigma$ exist for which $B''(\sigma) = B^h(\sigma)$. Consider the lowest of these values in the interval $[0, \bar{\sigma}]$, and denote this value by $\sigma_L$. Assume first that $B''(\sigma_L) \geq 0$ and that no value of $\sigma < \sigma_L$ exists for which $B^h(\sigma) > B^h(\sigma_L)$. Then a Nash equilibrium of type ETEB obviously exists for both the host and the neighbour choosing $\sigma_L$, as in this situation the host cannot lower its policy variable (given $\sigma'' = \sigma_L$) and achieve a higher benefit or alternatively can raise its policy variable without losing the host role and thereby achieve lower benefit as $B''(\sigma)$ is falling. Similarly, the neighbour cannot raise or lower its policy variable without having to accept a lower benefit.

Still assuming $B''(\sigma_L) \geq 0$, if values of $\sigma < \sigma_L$ exist for which $B^h(\sigma) > B^h(\sigma_L)$, an equilibrium of type ETEB cannot be generated where both regions choose $\sigma_L$ (because the host can raise its benefit by choosing a value lower than $\sigma_L$ and still keep the host role). But then an equilibrium of type UTUB exists in which the host chooses the value of $\sigma < \sigma_L$ providing the highest host benefit while the neighbour chooses a value that is sufficiently low to make it irrational for the host to choose an even higher value to take over the neighbour’s role, again meaning that the neighbour sets $\sigma''$ higher than $\bar{\sigma}$, where $\bar{\sigma}$ is defined by $B^h(\sigma) = B^h(\bar{\sigma})$.

Now assume that $B''(\sigma_L) < 0$. Then there always exist values of $\sigma < \sigma_L$ for which $B^h(\sigma) > B^h(\sigma_L)$ again entailing that an equilibrium of type UTUB exists. Again the host chooses the value of $\sigma$ yielding the highest value of $B^h(\sigma)$. 
We have been through all cases, and in each case an equilibrium is identified. This proves the proposition.

Proof of Proposition 4

To prove Proposition 4 we shall first show that for given benefit functions $B^h(\sigma)$ and $B^u(\sigma)$ there can be only one value of $\sigma^h$ in the interval $[0, \sigma]$ which can be an equilibrium tax rate for the host region. Therefore, assume that two equilibria exist with two different levels of the tax in the host region. Denote the equilibrium strategy profiles by respectively $(\sigma^h, \sigma^u)$ and $(\sigma^{h*}, \sigma^{u*})$ and assume that $\sigma^h < \sigma^{h*}$. It is clear that for these values $B^h(\sigma^h) \leq B^h(\sigma^{h*})$. Suppose they are not. Then $\sigma^{h*}$ cannot be a Nash equilibrium value as the host region, for a given level of $\sigma^{u*}$ can lower its tax to $\sigma^h$ (without losing its host role) and achieve a higher benefit. Moreover, in any equilibrium, benefits to the host region must be equal to or lower than benefits to the neighbouring region. In case not, the neighbour would always be able to raise its benefit by setting its tax rate marginally lower than the present host, and then take over the host role and achieve the host benefit. For the two assumed equilibria we thus have $B^u(\sigma^{h*}) \leq B^h(\sigma^h)$ and $B^u(\sigma^{h*}) \leq B^h(\sigma^{h*})$.

Assume for a while that $B^h(\sigma^{h*}) < B^h(\sigma^{h**})$. This contradicts the assumption that $\sigma^{h*}$ is a Nash-equilibrium value, because for this value the host could raise its tax level to $\sigma^{h*}$ and raise its benefit to either $B^h(\sigma^{h*})$ (if it keeps the host role), or to a value of $B^u(\sigma^h)$ (if it becomes a neighbour). For this value we have that $B^u(\sigma^h) > B^h(\sigma^{h*})$ because $B^u(\sigma^h)$ belongs to the open interval $B^u(\sigma^{h*})$, $B^u(\sigma^{h**})$ (since, given the former host is now a neighbour, the new host necessarily holds a value of $\sigma$ in the interval $[\sigma^h, \sigma^{h*}]$, and in $B^u(\sigma^h), B^u(\sigma^{h*})$ all values are higher than $B^h(\sigma^{h*})$. Therefore, whether the original host keeps it role as host or not, it has an incentive to deviate from $\sigma^{h*}$ so there cannot be two equilibrium values of $\sigma^h$ when $B^h(\sigma^{h*}) < B^h(\sigma^{h**})$. 

For $B^h(\sigma^h) = B^h(\sigma^{h**})$, consider the Nash equilibrium with tax rate $\sigma^h$ for the host. For this value $B^h$ must clearly have a local maximum. Otherwise, there will be values of $B^h$ higher than $B^h(\sigma^{h**})$ for values of $\sigma$ lower than $\sigma^{h**}$ whereby $\sigma^{h**}$ cannot be an equilibrium value. As we assume local maximum values to be different, $B^h$ will not have a local maximum for $\sigma^h = \sigma^{h**}$, but will be increasing for this value meaning that $\sigma^{h**}$ is only an equilibrium value if also the neighbouring region sets $\sigma^{n**} = \sigma^{h**}$. The strategy profile $(\sigma^{h**}, \sigma^{n**})$ is necessarily an equilibrium of the ETEB form implying that $B^h(\sigma^{h**}) = B^n(\sigma^{h**})$. As $B^u(\sigma)$ is falling, it also follows that in the equilibrium $(\sigma^{h**}, \sigma^{n**})$ we must have that $\sigma^{n**}$ is set at a level leaving no incentives for the host in this equilibrium to raise its policy variable. This is only fulfilled for $\sigma^{n**} = \sigma^{h**}$. We can conclude that with $B^h(\sigma^h) = B^h(\sigma^{h**})$ two equilibria may exist, the one being of the UTUB type, the other being of the ETEB type. This very specific case is illustrated in figure 4b.

Finally, in relation to Proposition 3 (i), if only one value of the host policy variable constitutes an equilibrium value, say, $\sigma^{h*}$ in case $B^h(\sigma^{h*}) = B^n(\sigma^{h*})$ the equilibrium must be an ETEB, since if $B^h(\sigma^{h*}) < 0$ we cannot have an equilibrium (because the host then will lower its tax rate to raise benefits). This ETEB is clearly unique since neither an UTUB or several ETEB equilibria can exist for the same equilibrium value. Further, in case $B^h(\sigma^h) < B^n(\sigma^h)$, an UTUB equilibrium exists, and an interval of values for the neighbour may constitute an equilibrium value, given $\sigma^h = \sigma^{h*}$, since $B^n(\sigma^h)$ is falling, so this is the situation illustrated in figure 1c. This proves the proposition.

References


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