

Monopolistic Trades, Accuracy of Beliefs and the Persistence of Long-Run Profit

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Abstract

In a model that encompasses a general equilibrium framework, we consider a monopolist (a producer) with subjective beliefs that endogenously hedges against fluctuations in input prices in a complete market. We introduce a notion of entropy of beliefs, and we characterize long-run optimal rational investments with this entropy. For irrational beliefs, we show that long-run profits are a decreasing function of this entropy. However, long-run profits always remain positive as long as the entropy remains finite despite the Market Selection Hypothesis that would predict long-run 0-profit. Allowing for Cournot competition in this economy, we show that if at least one entrant makes accurate predictions, the monopolist must also make accurate predictions or else its long-run profit will converge to zero for almost every path. In this latter case, the whole market power switches to the firm making the most accurate predictions.

Keywords: Market selection hypothesis, survival,
monopoly, entrants.

JEL codes: G3, D82, D84

1 Introduction

The question of long-run survival of economic agents is central to Economic Theory, and in particular the problem of determinants of this survival. The much-debated *Market Selection Hypothesis* advocates the idea that economic behavior leading to long-run survival must be consistent with rational maximization of expected returns. For instance, Friedman in [8], p. 22, argues that

“Whenever this determinant [of business behavior] happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not, the business will tend to lose resources and can be kept in existence only by the addition of resources from outside. The process of “natural selection” thus helps to validate the hypothesis or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgement that it summarizes appropriately the conditions of survival.”

This view is formalized in Sandroni [12] in the case of complete financial markets, where the only traders to eventually survive are the ones making the most accurate predictions. Again in complete financial markets, Leoni [10] later showed that an agent with market power must make more accurate predictions than the market in order to survive, showing that the Market Selection Hypothesis does not hinge on perfect competition as argued for

instance in Alchian [1]. The assumption of extreme market power in this last reference suggests that the Market Selection Hypothesis holds in fairly general settings, but the current study aims at providing some limits to this result by showing that the Hypothesis is true only when competitors are present on the market, and when they implement behavior consistent with rational and informed maximization of profits.

In a model encompassing a GE framework, we consider a monopolistic producer with subjective beliefs that hedges against shocks in input price and/or consumer demand. For this monopolist, we show that rational assessment of economic uncertainty is not a determinant of survival. This monopolistic producer will be negatively affected by irrational beliefs, but the long-run profits obtained through retained earnings will always remain positive unless highly erratic beliefs. The Market Selection Hypothesis would predict in this case long-run disappearance of the monopolist, in sharp contrast to our result. When we allow for Cournot competition in this setting, through the possibility of entries on the output market, the Market Selection Hypothesis remains valid. We show that if at least one entrant makes accurate predictions and the monopolist does not, the monopolist will eventually disappear almost surely. That is, when facing an entrant making accurate predictions, the monopolist must also make accurate or else its long-run profit will converge to zero for almost every path. In this later case, the market power will entirely switch to the firm making the most accurate predictions.

This example of failure of the Market Selection Hypothesis sets yet another restriction on its validity, since we essentially show that the rationale

for the selection of agents making the most accurate predictions is valid only when agents are in competition. In absence of competition, the determinants of survival are different, in particular the need to implement rational and informed maximization of profits is no longer necessary. The main conclusion from our results at heuristic level is that the market structure, in our case the presence or not of competitors, is also a key determinant of survival.

Other kinds of failure of the Market Selection Hypothesis have been previously found. Blume and Easley [5] first pointed out that markets must be complete for the Hypothesis to hold, in sharp contrast with Sandroni [12] and Leoni [10] where the results critically depend on the completeness of markets. Moreover, profit maximizing behavior as a determinant of survival depends on the market micro-structure; for instance, Beker [2] gives an example of a market where entrepreneurs using inefficient technologies end up dominating those using efficient technologies.

The view that most accurate forecasts are a determinant to survival, regardless of the market structure, has already been challenged. Nelson and Winter [11], p. 58, point out that the coevolution of firm behavior and its economic environment can hardly be dissociated, since “... the relative profitability ranking of decision rules may not be invariant with respect to market conditions.” This criticism suggests that rational and informed maximization of profit as a determinant of long-run survival may critically depend on the market structure where the economic agents interact. In this paper, we thus develop further the intuition that the market structure is at the heart of the Hypothesis by formalizing a general economic environment where

1. irrational assessments of economic environment do not trigger long-run disappearance when future investment decisions are undertaken with retained earnings (unless highly erratic beliefs),
2. there is a correlation between the accuracy of those assessments and long-run profits, and
3. rational and informed maximization of returns matters only when competitors or entrants implement this business behavior.

In more details, we consider a monopolist with subjective beliefs and facing uncertainty about consumer demand and/or cost of processing, in a framework that encompasses a general equilibrium model. Input prices also depend on those shocks, and the monopolist has access to a complete financial market to purchase future contracts on input delivery to hedge against those shocks. We introduce a notion of entropy of beliefs, which can be regarded as a measure of accuracy of beliefs. We find a condition on this entropy that characterizes long-run rational profit-maximizing investments. We also show that the long-run profit of the monopolist is a decreasing function of this entropy, in the sense that the worst the beliefs in our sense, the lower the long-run profit. However, profits eventually become null only for infinite entropy.

Once this framework is fully analyzed, we allow for the possibility of entries in the first period. A finite number firms may decide to enter in a Cournot competition with the firm already in place, without the possibility of exit at some point in the future. The problems faced by those firms is

identical to that of the monopolist, and to simplify matters we assume that the firms differ only on their assessment of uncertainty. We use the notion of making accurate profits along an infinite history introduced in Sandroni [12] to analyze the possible transfer of market power after a successful entry. We characterize this notion of accuracy of predictions using our concept of entropy, and we show that if at least one entrant makes accurate predictions then the monopolist must also make accurate predictions or otherwise its profit will converge to 0 almost surely.

The intuition of those results can be summarized as follows. First, the monopolist alone (that is, without entrants) is not in the situation of a zero-sum game with other agents, in contrast to Leoni [10]. In other words, the monopolist does not face any competitive pressure per se, even if imperfect. Given so, one should expect the monopolist to maintain market superiority and thus positive long-run profit regardless of the soundness its investment decisions, provided that those decisions are not exuberantly irrational. However, one should also expect realized profits to depend on the accuracy of beliefs, in the sense described here. The only case leading to eventual disappearance, as predicted by the Market Selection Hypothesis, is that of extreme exuberant irrationality corresponding to infinite entropy. With entries, the market shares depend critically on the investment levels on a given path. Failure to make accurate predictions will result in relative lower investments on the paths that are the most likely to occur, leading to eventual 0-profits when one competitor dominates the most likely infinite histories.

Our study also implicitly suggests that, for Friedman’s view to be a basis for the Market Selection Hypothesis, a business must face competitors to eventually disappear as a result of repeatedly erroneous choices. However, facing competition is not a sufficient condition for the Market Selection Hypothesis to hold, as indicated in the previous references. The question of knowing which economic conditions lead to the evolutionary selection of agents based on beliefs accuracy remains open.

The paper is organized as follows. In Section 2 we described the general model and we introduce our notions of entropy beliefs, in Section 3 we present our formal results about the monopolist without competitors, in Section 4 we analyze the possible transfer of market power with entrants, and finally Section 5 contains some concluding remarks. The technical proofs are given in the Appendix.

2 The model

In this section, we formalize the model and we define the relevant notion of accuracy of beliefs and entropy. Time is discrete and continues forever. In every period $t \in N_+$, a state is drawn by nature from a set $S = \{1, \dots, L\}$, where L is strictly greater than 1. Before defining how nature draws the states, we first need to introduce some notations.

Denote by S^t ($t \in N \cup \{\infty\}$) the t -Cartesian product of S . For every history $s_t \in S^t$ ($t \in N$), a cylinder with base on s_t is defined to be the set $C(s_t) = \{s \in S^\infty \mid s = (s_t, \dots)\}$ of all infinite histories whose t initial

elements coincide with s_t . Define the set Γ_t ($t \in N$) to be the σ -algebra which consists of all finite unions of cylinders with base on S^t .¹ The sequence $(\Gamma_t)_{t \in N}$ generates a filtration, and define Γ to be the σ -algebra generated by $\cup_{t \in N} \Gamma_t$. Given an arbitrary probability measure Q on (S^∞, Γ) , we define $dQ_0 \equiv 1$ and dQ_t to be the Γ_t -measurable function defined for every $s_t \in S^t$ ($t \in N_+$) as

$$dQ_t(s) = Q(C(s_t)) \text{ where } s = (s_t, \dots).$$

Given data up to and at period $t - 1$ ($t \in N$), the probability according to Q of a state of nature at period t , denoted by Q_t , is

$$Q_t(s) = \frac{dQ_t(s)}{dQ_{t-1}(s)} \text{ for every } s \in S^\infty,$$

with the convention that if $dQ_{t-1}(s) = 0$ then $Q_t(s)$ is defined arbitrarily.

The posterior probability of Q given a finite history $s_t \in S^t$ ($t \in N$), denoted by Q_{s_t} , is

$$Q_{s_t}(A) = \frac{Q(A_{s_t})}{Q(C(s_t))} \text{ for every } A \in \Gamma,$$

where A_{s_t} is the set of all paths $s \in S^\infty$ such that $s = (s_t, s')$ and $s' \in A$.²

In every period and for every finite history, nature draws a state of nature according to an arbitrary probability distribution P on (S^∞, Γ) .

¹The set Γ_0 is defined to be the trivial σ -algebra, and $\Gamma_{-1} = \Gamma_0$.

²If $Q(C(s_t)) = 0$, then Q_{s_t} is defined to be an arbitrary probability measure on Γ .

2.1 The agents

We now formally describe the interaction of the agents. In the next section, we explain how our model can be re-interpreted in terms of a General Equilibrium, even if our analysis goes beyond this framework.

There are two goods available in every period, an output good $x \in \mathbb{R}$ and an input good $y \in \mathbb{R}$. There is a producer that lives forever and produces the output good in every period. The producer is in a situation of monopoly for its production. In every period, an arbitrary number of consumers is born and will live for this period only; consumers own the input good y and seek the output good x . The assumption that consumers live for one period only simplifies the analysis by avoiding the problem of commitment to future prices as in Gul *et al.* [7]; it can also be justified as a one-time buy on the consumer side. One can also easily extend the framework to an overlapping generation model, this issue is omitted to simplify the analysis and similar results obtain in the later case.

The producer owns a quantity y_0 of input good in period 0. In every period t , a new market opens with an arbitrary number of new consumers who live for this period only. For a given level of input good, the producer produces the output good in this same period, and the output good is delivered to current consumers. We thus avoid without loss of generality the issue of delay in production.

We assume that the *gross profit* to the monopolist in every period t , which breaks down to the proceeds from delivering the good x to the consumers

less of the cost of processing the input good, depends on the history of shocks s_t ;³ it is represented by the function $Q_{s_t}(y)$ where y the quantity of input good being used. Shocks can be, for instance, on the cost of production or the consumers' demand, and they can be correlated with shocks in other histories in the same period. For every finite history s , we assume that the function Q_s is positive, strictly increasing, concave, differentiable and satisfies $Q_s(0) = 0$ (this last assumption simplifies the analysis, and it captures the idea that there is no sunk cost in production). Those assumptions are consistent with possible risk aversion on the production side, because concavity may also capture risk aversion with von Neuman-Morgenstern utility function.

Example 1 *We can consider as a particular case of our model profit functions in every history s of the form*

$$Q_s(q) = \tilde{Q}_s(q, q_{-s}) = f_s(q)d_s(q, q_{-s}) - c_s(q) \text{ for any given } q_{-s} \in \mathfrak{R}^{S-1} \quad (1)$$

where the vector q_{-s} represents input supply level in every other contingency, the function $d_s(q, q_{-s})$ is the consumers' demand for output in s that depends on other variables q_{-s} , the function f_s is the production function and c_s is the cost of production both in state s .

Input is purchased by the producer in the current period for the next period, taking as given the price of the input good. We assume that the producer has access to a complete market where she can purchase *futures*

³The gross profit does not include the cost of purchasing the input goods, which is described later.

contracts on input delivery next period. The price of a future contract purchased in history s_t and paying off one unit of input if event i occurs next period, and 0 otherwise, is denoted by $q_{s_t}^i$. We denote the quantity of this contract held by the monopolist by $\theta_{s_t}^i$.⁴ The monopolist is a price-taker on the futures market, and we assume that its volume of trade does not affect prices to simplify the analysis. We assume that $q_{s_t}^i > 0$ for every s_t and i .

The financial market is not modelled to simplify the exposition, and without loss of generality since the monopolist is assumed to be a price-taker and our results hold for arbitrary positive nets of security prices. Those prices can stem from, albeit without being restricted to, market clearing conditions on demand functions consistent with rational investors endowed with standard von Neuman-Morgenstern utility and facing standard budget constraints.

The monopolist purchases those contracts through retained earnings, while seeking to extract dividends from the proceeds. Let e_{s_t} denote the *dividends* extracted by the monopolist in history s_t . The dividends to the monopolist in history $s_t = (s_{t-1}, j)$ satisfy, or equivalently can be defined as

$$e_{s_t} + \sum_i \theta_{s_t}^i \cdot q_{s_t}^i \leq Q_{s_t}(\theta_{s_{t-1}}^j) \quad (2)$$

$$\theta_{s_t}^i \geq 0 \text{ for every } i.$$

The monopolist has subjective beliefs about the uncertainty in the economy, which is denoted by the probability measure M defined on (S^∞, Γ) . As standard in finance, we assume that the monopolist seeks to maximize the

⁴Without loss of generality, We can restrict our analysis to this type of contracts, also known as Arrow securities, since markets are complete.

(subjective) expected net present value of the firm; i.e., the monopolist seeks to maximize the expression

$$E^M \left(\sum_t \beta^t \cdot e_t \right), \quad (3)$$

where $\beta \in (0, 1)$ is the intertemporal discount factor, and where $E^M(\cdot)$ is the expectation operator associated with the probability measure M . We could have assumed that the monopolist is risk-averse on dividends payments without changing the qualitative nature of our results.

The objective of the monopolist is to maximize (3) subject to (2); that is, the monopolist seeks to maximize the subjective net present value of its future stream of dividends taking as given the exogenous prices of future contracts to deliver input. We implicitly assume that the monopolist considers its belief to be correct, or at least its learning process to be the most appropriate, without any explicit consistency check with observed realizations of events. This issue is not overly restrictive, since the monopolist can be endowed before trades with the final posterior generated by its learning experience over time, leading to beliefs consistent with equilibrium learning and trading.

Our model encompasses a standard general equilibrium model, with an infinitely-lived monopolist and consumers living for one-period only with standard preferences. Supply (of the input good) and demand (for the production good) functions from the consumers' side, and stemming from standard maximization problem, can be regarded as already embedded in the gross profit function of the monopolist. We can easily extend those demand functions to be consistent with other settings such as the emergence of

monopsonist behavior from consumers (strategic behavior meant to reduce purchasing prices) or tax effects, as typically done in economic theory. The arbitrary prices that we will allow throughout the paper can be chosen so as to be market-clearing prices both on financial markets and goods markets for those supply and demand functions, as long as equilibrium market prices remain positive.

Those supply and demand functions can be made consistent with maximizing behavior of consumers endowed with von Neuman-Morgenstern utility functions, whose main characteristic is that decisions in a given history depend on the decisions in other histories occurring during the same period. Profit functions as in Eq. (1) allow to choose those functions leading to contingent decisions consistent with one another within a given period for every possible decision, as well as past and future decisions if we want to consider an overlapping generation model and thus an intertemporal framework for decision-making for consumers. We do not develop this point to simplify the exposition, although it is important to remember that those standard frameworks in economic theory are a particular case of ours.

2.2 Accuracy of beliefs

Our analysis relies on a notion of accuracy of beliefs (or predictions) described next. We introduce two concepts of entropy, well fitted for long-run analysis. First, we need to ensure that both nature and the monopolist's beliefs assign strictly positive probability to every event.

Definition 2 *The entropy of the belief of the monopolist at period t ($t \in N$) along a path $s \in S$ is defined by*

$$\Pi_t(s) = \frac{dP_{s_t}}{dM_{s_t}}$$

if $dM_{s_t} > 0$ and an arbitrary finite real otherwise.

We next introduce two notions capturing the long-run evolution of the above entropy.

Definition 3 *The upper entropy of beliefs of the monopolist along a path $s \in S$ is the function $\overline{\Pi}$ defined by*

$$\overline{\Pi}(s) = \overline{\lim}_t \Pi_t(s)$$

The lower entropy of beliefs along this path s is the function $\underline{\Pi}$ defined by

$$\underline{\Pi}(s) = \underline{\lim}_t \Pi_t(s)$$

The basic motivation for introducing two distinct notions of entropy, involving both the \liminf and \limsup of the ratio above, is that learning processes used to form individual beliefs may not converge or may also display erratic behavior around accurate beliefs. Given so, the ratio of beliefs may not have a limit for every learning process forcing us to make this distinction. It is also important to notice that, in the above definition, the evolution of long-run beliefs only matter. Any particular belief formed early in the past does not influence the entropy. This represents an important departure from the concept of entropy introduced in Lehrer and Smorodinsky [9], which considers

a weighted average of all previous entropy at any point in time (the entropy at any point in time differs from ours in this last concept).

We will also use a notion of accuracy of predictions when analyzing the effects of entrants of monopolistic power; this notion was already introduced in Sandroni [12].

Definition 4 *An agent with beliefs M makes accurate predictions on a path $s \in S^\infty$ if $\|M_{s_t} - P_{s_t}\| \rightarrow 0$.*

Definition 4 says that a firm makes accurate prediction on a given path if the posterior subjective probabilities along this path become arbitrarily close to those of nature for the sup-norm. The following lemma makes the link between this notion and our concept of entropy.

Lemma 5 *P -almost surely, an agent makes accurate predictions on a path s if and only if*

$$\infty > \bar{\Pi}(s) = \underline{\Pi}(s) > 0$$

The above lemma states that, for P -almost every path, a firm makes accurate predictions along a path if and only the upper and lower entropy coincide and are finite.

3 Long-run investment

This section is devoted to proving the equivalence between long-run optimal investment and next-period accuracy of beliefs. We also extend the equivalence to the notion of entropy introduced earlier. Let (\tilde{e}, \tilde{y}) be the solution

to the program consisting of maximizing (3) subject to (2) at correct belief P , we define an *optimal investment plan* to be \tilde{y} . We say that the monopolist, with dividends and investment streams (e, y) solution to the program consisting of maximizing (3) subject to (2) at belief M , *eventually makes rational investments* along a path s if $\lim_t |y_{st} - \tilde{y}_{st}| \rightarrow 0$. It is important to notice that rational investments need not converge (they can even display chaotic behavior); what matters in the analysis, and what represents the source of our technical difficulties, is that individual investments mimic the asymptotic behavior of rational investments to be eventually rational.

For the following result only, we need the following assumption. Before stating it, we define any *subinterval* of \mathfrak{R}_+ to be an interval of the form $[0, a]$, for some $a > 0$.

Assumption 6 *For every **finite** history s , the function Q'_s is equi-continuous on every subinterval of \mathfrak{R}_+ .*

Equi-continuity is a common assumption, and it is not particularly restrictive when required on sub-intervals of \mathfrak{R}_+ . For instance, any C^1 function satisfies this requirement, and thus Assumption 6 is satisfied by standard demand functions.

Proposition 7 *Under Assumption 6, the following statements are equivalent. For every path s*

1. *the monopolist eventually makes rational investments along s ,*
2. $\bar{\Pi}(s) = \underline{\Pi}(s) = 1$.

The above result shows that our notion of accuracy of predictions characterizes eventually rational investments pathwise, and it thus establishes the relevance of the concept for our analysis. We next analyze how long-run performances are affected by inaccurate beliefs.

Proposition 8 *Consider two beliefs M^1 and M^2 for the monopolist such that $\bar{\Pi}^1(s) < \bar{\Pi}^2(s) < \infty$, and consider a path $s \in S$. Denote by $(y_{s_t}^i)_{s_t \in S^t, t \in \mathbb{N}}$ the equilibrium associated with the belief M^i ($i = 1, 2$). The following relation holds:*

$$\liminf_t y_{s_t}^2 \leq \liminf_t y_{s_t}^1.$$

Proof. See Appendix. ■

The above result shows that the lim inf of subjective investments is a decreasing function of the entropy of the monopolist' beliefs pathwise, provided that the entropy of beliefs is finite. We must use the notion of lim inf, with all of its restrictions, since subjective investments have to particular reasons to converge. What we get instead is that the worst-case under-investment scenario (provided that the lower entropy is greater than 1) leads to this ranking in term investment level.

The above result encompasses the case of both over-investments and under-investments, since from Proposition 7 we know that the optimal investment corresponds to an overall of 1. In particular, Proposition 8 shows that the monopolist increasingly under-invests when the entropy converges to 0, and it increasingly under-invests when the entropy becomes greater than 1. It is relatively straightforward to show that, when the entropy increases to

infinity with finite values, the lim inf of subjective investments converges to 0 but always remain positive. In particular, this shows that eventual profits remain positive as long as the entropy is finite along a path.

The next proposition analyzes equilibrium investments when the entropy becomes arbitrarily large. We restrict our attention to the paths where it is *rational to eventually invest*, defined as infinite paths $(s_t)_t$ such that $\inf_t \tilde{y}_{s_t} > 0$ (recall that $(\tilde{y}_{s_t})_t$ is the optimal investment sequence for an agent with rational expectations). We also assume for the next result only that, for every finite history s , the profit function satisfies $Q'_s(y) \rightarrow +\infty$ as $y \rightarrow 0$, with interpretation similar to that of the standard Inada condition.

Proposition 9 *Consider a path s where it is rational to eventually invest, and a sequence of beliefs $(M^i)_{i \in N}$ such that the associated sequence $(\bar{\Pi}^i(s))_{i \in N}$ converges to ∞ . Denote by $(y_{s_t}^i)_{s_t \in S^t, t \in N}$ the equilibrium associated with the belief M^i ($i \in N$). We have that*

$$\lim_i \lim_t y_{s_t}^i = 0.$$

Proof. See Appendix. ■

The above result states that when the entropy becomes arbitrarily high on a given path, then in the long-run the monopolist will reduce its investment until it eventually invests arbitrarily small quantities infinitely often. It is not true in general that investments will converge to 0, because beliefs may temporarily become less exuberant and thus subjectively optimal investment will remain strictly positive for the time beliefs are not too erratic. Therefore,

the inf limit of the one-period profits becomes arbitrary small in the long-run for arbitrarily large lower entropy.

We can also prove that if the upper entropy converges to infinity, which implies that the lower entropy also converges to 0, then the equilibrium investment sequence converges to 0. In other words, strict convergence to 0 occurs for well behaved and erratic beliefs. We prefer to give the above version of the result, which encompasses the possibility of non-convergent learning processes, to keep our framework as general as possible.

4 Entry and loss of market power

We now analyze how bad beliefs may affect the monopolist when there are potential entrants. In particular, we show that when at least one entrant makes P -almost surely accurate predictions the monopolist must also make accurate predictions or otherwise vanquish almost-surely. The notion of vanquishing means that the long-run profits of the monopolist will converge to 0 P -almost surely.

We consider now a finite number of potential entrants. Entries occur in period 0 without the possibility of exit to simplify matters. Competitors will play a Cournot game over the infinite horizon; that is, they will compete with the monopolist in quantity. Every firm i has a belief M^i about the uncertainty of nature, and it faces the same budget constraint as the monopolist. The entrants are price-takers on the financial and input markets. We maintain the assumption that the net of futures prices $(q_s)_s$ is bounded away from 0. We

need to restrict the class of gross profit functions for those entrants and the monopolists to obtain to our result, although the new class satisfies standard textbook assumptions. For a net of input $y = (y^M, y^1, \dots, y^I)$ chosen by all of the firms in any history s , we will focus on gross profit functions for any firm i of the form

$$Q_s^i(y_s^i, y_s^{-i}) = f_s(y_s^i) p_s(y_s^i, y_s^{-i}), \quad (4)$$

where f_s is the production function and p_s is the price function given the choice of input by all of the firms. This price function incorporates various issues about price formation given the output level, in particular this price formation may go beyond the direct and usual dependency on aggregate output. We assume for every s that f_s is differentiable, concave and increasing, with $f_s'(0) = \infty$. Any standard production function of the form $f_s = \ln$ or $f_s(y) = y^\alpha \cdot l^{1-\alpha}$ for some $\alpha \in (0, 1)$ and any given labor input l will satisfy those assumptions. For every s , the price function is assumed to be decreasing in every variable and differentiable. Every firm may have idiosyncratic profit functions without loss of generality, we will assume that they are all identical to simplify the notations.

Every entrant seeks to maximize its subjective expected profit as before. We focus on any Nash equilibrium of this game. We can now state our result about the survival of a monopolist that does not make accurate predictions.

Proposition 10 *Assume that at least one entrant makes accurate predictions P -almost surely. In every equilibrium, If the monopolist does not make accurate predictions then its profit will converge to 0 P -almost surely.*

Proof. See Appendix. ■

Proposition 10 states that it is enough to have one entrant making accurate predictions to force the monopolist to have accurate predictions. In this case, failure to accurately forecast will drive away the monopolist from the market. Eventually, the domination of the market will switch to this entrant with superior predictions.

A corollary of this result is that the minimal cost of entry for which there is **no** possible entry is exactly the intertemporal profit of a monopolist. If the cost of entry is below this value, then an entrant making accurate prediction will eventually dominate the market against an inaccurate monopolist and it will thus seize its profits.

5 Conclusion

We have studied a general situation where a monopolist makes arbitrarily inaccurate (but not completely erratic) predictions and still realizes some profits in the long-run, whereas the Market Selection Hypothesis would predict eventual disappearance in this case. We introduce a measure of accuracy of predictions, and we show that the long-run profit is correlated with this measure of accuracy. The correlation between long-run performances and accuracy of beliefs has intuitive content: one should expect an agent to suffer

from bad beliefs, although the level of losses should be related to the level of inaccuracy of the beliefs.

However, it is critical to analyze the market structure to decide whether the Market Selection Hypothesis is true. When allowing for Cournot competition in this economy, we show that if at least one entrant makes accurate predictions, the monopolist must also make accurate or else its long-run profit will converge to zero for almost every path. The point is to show that the relevance of rational and informed maximization of profits as a determinant of survival critically depends on the presence of competitors implementing this strategy.

Our work can also be regarded as a counterexample to the evolutionary theory developed by Alchian and Friedman among many others. Our point is that rational profit maximization as a determinant of long-run survival cannot be dissociated from the market micro-structure. The idea that we advocate here is that most (if not all) of the determinants of long-run survival critically depend on the organization of markets, and therefore the long-run behavior of the economy in terms of the domination of some agents depend critically on the very nature of trades. We believe that it is important to identify economic situations for which some determinants such as accuracy of beliefs are critical to survival.

A Appendix

We now prove all the results stated earlier.

A.1 Proof of Lemma 5

This technical lemma making the link between making accurate predictions and our notion of entropy is a direct consequence of Proposition B.3 in Sandroni [12]. It is shown there that, P -almost surely, making accurate predictions along a path s is equivalent to

$$\infty > \lim_t \frac{dM_t(s)}{dP_t(s)} > 0.$$

This is equivalent to saying that the upper and lower entropy along this path s coincide and are finite. The proof is complete.

A.2 Long-run behavior

We first derive a fundamental equation describing the evolution of equilibrium variables along a given infinite path, as a function of individual beliefs.

By rearranging terms, the program consisting of maximizing (3) subject to (2) can be rewritten as

$$\text{Max} \sum_{s_t=(s_{t-1},j)} \beta^t \cdot M_{s_t} \cdot [Q_{s_t}(\theta_{s_{t-1}}^j) - \sum_i \theta_{s_t}^i q_t] \quad (5)$$

Taking the first-order condition of the above program directly gives that

$$\beta \cdot dM_{s_t} \cdot Q'_{s_t}(\theta_{s_{t-1}}^j) = q_{s_t}^j, \quad (6)$$

for every j and every event s_t . Denote now by $\tilde{\theta} = (\tilde{\theta}_{s_t})_{s_t,t}$ the optimal hedging plan when the monopolist has correct beliefs P . Since the monopolist takes asset prices as given and does not affect the volume of trade, the net $\tilde{\theta}$ also satisfies (6) and we thus have that

$$\frac{dM_{s_t}}{dP_{s_t}} \cdot \frac{Q'_{s_t}(\theta_{s_{t-1}}^j)}{Q'_{s_t}(\tilde{\theta}_{s_{t-1}}^j)} = 1 \text{ for every } j \text{ and every } s_t. \quad (7)$$

We next give an intermediary result useful for proving Proposition 7. The result is relatively well-known and interesting because it characterizes our class of gross profit functions in terms of functional compactness; its proof is given on page 192 of Brobowski [6].

Lemma 11 *A set in $C(\mathfrak{R}_+)$ is composed of functions equi-continuous on every sub-interval of \mathfrak{R}_+ if and only if it is relatively compact.*

A.2.1 Proof of Proposition 7

We first prove $1 \Rightarrow 2$. We proceed by way of contradiction, by assuming that entropies are different along a path. We then extract subsequences converging uniformly using Lemma 11 and we derive a contradiction to Eq. (7) by a simple continuity argument.

Fix any infinite history s , and assume that $\bar{\Pi}(s) > \underline{\Pi}(s) > 1$; a similar argument can be used for the other possible cases in the previous inequality. In particular, this last assumption implies that there exist a subsequence $(p_t)_t$ extracted from $(s_t)_t$ and a constant α such that

$$\frac{dM_{p_t}}{dP_{p_t}} \leq \alpha < 1 \text{ for every } p_t. \quad (8)$$

Since by Assumption 6 and Lemma 11 the sequence $(Q'_{p_t})_t$ is relatively compact, it follows that there exist a function Q and subsequence $(r_t)_t$ extracted from $(p_t)_t$ such that $(Q'_{r_t})_t$ converges to Q uniformly.

Since the monopolist eventually makes accurate predictions, it then follows from a straightforward continuity argument that $\frac{Q'_{r_t}(\theta_{r_t-1}^j)}{Q'_{r_t}(\tilde{\theta}_{r_t-1}^j)}$ converges to 1 for every j . Therefore, by Eq. (8) there exists $\bar{t} > 0$ such that

$$\frac{dM_{r_t}}{dP_{r_t}} \cdot \frac{Q'_{r_t}(\theta_{r_t-1}^j)}{Q'_{r_t}(\tilde{\theta}_{r_t-1}^j)} < 1 \text{ for every } j \text{ and every } t \geq \bar{t}. \quad (9)$$

This contradicts Eq. (7), and the implication is proven.

We now prove $2 \Rightarrow 1$ by way of contradiction. Fix any infinite path s , and assume that there exist \bar{t} and $\alpha > 0$ such that $|\theta_{s_t} - \tilde{\theta}_{s_t}| > \alpha$ for every $t \geq \bar{t}$ (up to an extracted sequence, omitted here to simplify notations). By Lemma 11, there exist a subsequence $(p_t)_t$ extracted from $(s_t)_t$ and a continuous function Q such that $(Q'_{p_t})_t$ converges to Q uniformly.

By a simple continuity argument, it then follows that there exist $t' > 0$ and a constant $\tau > 1$ such that $\frac{Q'_{p_t}(\theta_{p_t-1}^j)}{Q'_{p_t}(\tilde{\theta}_{p_t-1}^j)} \geq \tau$ for every j and every $t \geq t'$ (a similar argument can be used for the case where the previous ratio is less than 1). From our previous remarks, there must exist $t'' > 0$ such that

$$\frac{dM_{p_t}}{dP_{p_t}} \cdot \frac{Q'_{p_t}(\theta_{p_t-1}^j)}{Q'_{p_t}(\tilde{\theta}_{p_t-1}^j)} > 1 \text{ for every } j \text{ and every } t \geq t''. \quad (10)$$

This violates Eq. (7), and the proof is complete.

A.2.2 Proof of Proposition 8

From now on, we will (sometimes) omit the superscript j to simplify notations; this superscript will refer to individual beliefs. The reader can easily recast the argument in its initial formulation.

Consider two beliefs M^1 and M^2 and a path $s \in S$ such that $\bar{\Pi}^1(s) < \bar{\Pi}^2(s) < \infty$. In particular, this implies that there exist a subsequence $(s_t)_t$ such that $\frac{\tilde{Q}_{s_t}}{M_t^2} > \frac{\tilde{Q}_{s_t}}{M_t^1}$ for t large enough. We next prove that $\liminf_t y_{s_t}^1 \geq \liminf_t y_{s_t}^2$.

We proceed by way of contradiction. Assume that $\liminf_t y_{s_t}^1 < \liminf_t y_{s_t}^2$. It follows that there exists a sequence $(p_t)_{p \in N}$, extracted from $(s_t)_t$, such that $y_{p_t}^1 < y_{p_t}^2$ for all t . To show this, consider any subsequence $(y_{v_t}^1)_t$ converging to $\liminf_t y_{s_t}^1$. There must also exist a subsequence of $(y_{v_t}^2)_t$, denoted by $(y_{r_t}^2)_t$ that converges to real a such that $a > \liminf_t y_{s_t}^1$ by definition of the limit inf and the inequality linking the two respective limit inf. Therefore, there exists $\bar{t} > 0$ such that $y_{r_t}^1 < y_{r_t}^2$ for every $t \geq \bar{t}$; this is the subsequence starting at \bar{t} that we require.

Consider now Eq. (6) for any belief. This equation directly yields that, with the omission of the subscript j that is replaced by equilibrium variables corresponding to the appropriate belief,

$$\frac{dM_{p_t}^2}{dM_{p_t}^1} \cdot \frac{Q'_{p_t}(y_{p_{t-1}}^2)}{Q'_{p_t}(y_{p_{t-1}}^1)} = 1 \text{ for every } p_t, \quad (11)$$

where the subsequence p_t is chosen as above. By our property on beliefs, it follows from Eq. (11) that $Q'_{p_t}(y_{p_{t-1}}^2) > Q'_{p_t}(y_{p_{t-1}}^1)$. Moreover, the concavity of Q_{p_t} implies that Q'_{p_t} is decreasing, and thus we have that $y_{p_{t-1}}^1 > y_{p_{t-1}}^2$. This contradicts the property from which the sequences $(y_{p_{t-1}}^2)_t$ and $(y_{p_{t-1}}^1)_t$

are constructed, and the proof is now complete.

A.2.3 Proof of Proposition 9

In order to prove the result, we extract a subsequence from the sequence from the sequence $(y_{s_t}^i)_{t \in N, i \in N}$ and prove that this sequence converges to 0.

Since the sequence $\Pi^i(s)$ converges to $+\infty$, there exists a strictly increasing sequence $\left(\frac{\tilde{Q}_{s_t}}{M_{s_t}^i}\right)_{t, i \in N}$ that converges to $+\infty$. Consider the equilibrium sequence $(y_{s_t}^i)_{t, i \in N}$ associated with the previous sequence of beliefs. For Eq. (7) to hold, it must be true that $\frac{Q'_{s_t}(y_{s_{t-1}})}{Q'_{s_t}(\tilde{y}_{s_{t-1}})}$ converges to ∞ . Since along s it is rational to eventually invest, the sequence $(\tilde{y}_{s_{t-1}})_t$ is bounded away from 0 and so is $[Q'_{s_t}(\tilde{y}_{s_{t-1}})]_t$. Therefore, the sequence $[Q'_{s_t}(y_{s_{t-1}}^i)]_t$ must converge to ∞ . By our assumption on $[Q'_{s_t}]_t$, this occurs if and only if $(y_{s_{t-1}}^i)_t$ converges to 0. The proof is now complete.

A.2.4 Proof of Proposition 10

Following an argument similar to that in the proof of Proposition 7, one can show that the solution in every history s of the Cournot game $(y_s^1, \dots, y_s^I, y_s^M)$ with I entrants and the monopolist must satisfy for every $j = 1, \dots, I, M$

$$\beta \cdot dM_s^j \cdot \left(f'_s(y_s^j) p_s(y_s^j, y_s^{-j}) + f_s \frac{\partial p_s}{\partial y_s^j}(y_s^j, y_s^{-j}) \right) = q_s. \quad (12)$$

Assume that the entrant i makes accurate predictions. It follows from Eq. (12) that, for every t

$$\frac{dM_{s_t}^i}{dM_{s_t}} \cdot \frac{f'_{s_t}(y_{s_t}^i) p_{s_t}(y_{s_t}^i, y_{s_t}^{-i}) + f_{s_t}(y_{s_t}^i) \frac{\partial p_{s_t}}{\partial y_{s_t}^i}(y_{s_t}^i, y_{s_t}^{-i})}{f'_{s_t}(y_{s_t}^M) p_{s_t}(y_{s_t}^M, y_{s_t}^{-M}) + f_{s_t}(y_{s_t}^M) \frac{\partial p_{s_t}}{\partial y_{s_t}^M}(y_{s_t}^M, y_{s_t}^{-M})} = 1. \quad (13)$$

Assume that the monopolist does not make accurate prediction. Then by Lemma 5 the fraction $\frac{dM_{s_t}^i}{dM_{s_t}}$ must converge to $+\infty$ for P -almost every path. We now claim that the numerator $f'_{s_t}(y_{s_t}^i)p_{s_t}(y_{s_t}^i, y_{s_t}^{-i}) + f_{s_t}(y_{s_t}^i)\frac{\partial p_{s_t}}{\partial y_{s_t}^i}(y_{s_t}^i, y_{s_t}^{-i})$ is bounded away from 0. Indeed, assume by way of contradiction that it converges to 0. Since the term $\beta \cdot dM_{s_t}^i$ is bounded above, the left-hand side of Eq. (12) converges to 0. The sequence $(q_{s_t})_t$ is bounded away from 0, so there exists a time t_0 after which Eq. (12) cannot hold. This is a contradiction, and thus the numerator must be bounded away from 0.

Therefore, it must be true for Eq. (13) to hold that

$$f'_{s_t}(y_{s_t}^M)p_{s_t}(y_{s_t}^M, y_{s_t}^{-M}) + f_{s_t}(y_{s_t}^M)\frac{\partial p_{s_t}}{\partial y_{s_t}^M}(y_{s_t}^M, y_{s_t}^{-M}) \rightarrow_t +\infty. \quad (14)$$

Since $\frac{\partial p_{s_t}}{\partial y_{s_t}^i}(y_{s_t}^i, y_{s_t}^{-i}) < 0$ by assumption, it must be true that $f'_{s_t}(y_{s_t}^M)p_{s_t}(y_{s_t}^M, y_{s_t}^{-M})$ converges to $+\infty$. Since the price function is always finite, and since $f'_{s_t}(0) = +\infty$, it must be true for Eq. (13) to hold that $(y_{s_t}^M) \rightarrow 0$.

In other words, the profits of the monopolist converges to 0 P -almost surely, and the proof is now complete.

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