

# A Basic Model of the Residential Market, Where Ownership Matters

by

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## *Abstract*

A model where ownership is essential for the utility of consumption of residential units is presented. The consumer has an ideal and unique variant of the unit, which can only be realized by incurring a sunk cost. Equilibrium on the residential market and allocation of consumers between owners and tenants are derived under perfect elastic and fixed supply, respectively. The model shows that owners occupy bigger housing units than tenants, and that ownership improves welfare. Congestion leads to higher rent, smaller housing units, and a larger share of tenants among the consumers. A variant of the model with ownership of secondary homes restricted to a group of consumers shows that such restriction secures a more than proportional share of the market for the privileged group.

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*Key words:* Residential market, ownership, ideal variant, sunk cost, ownerspecific consumer surplus.

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## 1. Introduction

Consumers get utility from a broad range of durable goods. Measured in shares of household budgets, the most important durable goods for most consumers is (imputed) outlays for primary homes, but secondary homes (summerhouses), cars, household equipment such as furniture, and leisure goods such as boats, campers, etc. may also take a significant part of household budgets.

It is the flow of services from the durable good, which affects utility through the process of consumption. In principle a dual market therefore exists for each durable good: a leasing market or 'flow' market where the services are traded and a 'stock' market where ownership of the durable goods as such is traded. The two markets are interrelated. Under simplifying assumptions the stock price reflects the present value of the expected future net leasing yields of the good and the main analytical interest of analysing both markets is thus related to the expected future flows of leasing yields. Ownership is relegated only to be a question about portfolio management.

The consumer of a durable good very often has a Lancaster (1979) type of an ideal variant of the good. This variant might, contrary to Lancaster's analysis, be unique and not available at the market, and the only possibility to consume the ideal variant is to make a supplementary investment so that the good is adapted to the consumer's specific taste. As the preferences are individually very differentiated, the consumer cannot expect to recover the expenditures for adapting the good and the supplementary investments is therefore perceived as sunk costs.

The aim of this paper is to analyze the role of ownership of durable goods in a model based on a Lancaster type of ideal variant. This makes ownership crucially important. Only by the legal protection of ownership has the consumer the incentive to incur the supplementary costs. This approach seems especially plausible for homes and the analysis presented in the following will therefore be presented in the context of a model for homes. However, the relevance of the analysis also applies for similar types of durable goods such as cars or leisure boats.

The utility effect of adapting the good is assumed to differ between individuals. Some individuals increase their utility more than others when they adapt their home to their

'personal' style by the design of the kitchen, bathrooms, choice of furniture, ex- and interior decoration, and the way the garden looks like.

The presented model allows for an analysis of the individual's optimisation. The market will 'screen' individuals so that only those with strong ownerspecific preferences for adapting their houses will make up the actual owner group, leaving the rest of the individuals to the leasing market. It appears from the analysis that owners with strong ownership preferences obtain an ownerspecific consumer surplus in addition to the to the consumer surplus they get as implicit tenants, and ownership therefore matters for welfare. The stock price of the house is determined by supply costs in the case of a perfect elastic supply, but will exceed the present value of the rent at the leasing market if scarcity of houses exists, e. g. If it is assumed that the number of houses is exogenously given. This last case has relevance for congested cities and for summerhouses where land restrictions have limited the number of houses.

The literature contains other models of housing markets with ownership and rental markets included, see i.e. the survey paper by Smith, Rosen and Fallis (1988). A model with some similarities with the model presented here is Swan (1983). However, Swan's model assumes, that individuals have no preference for renting or owning and puts emphasis on marginal tax rates, inflation expectations, capital gains, etc. and studies the implications of changes in these variables for the relative size of the owner-occupied vis-à-vis rental housing and other endogenous variables. The model of the present paper is basic in the sense, that the propositions of the model does not follow from variations of marginal tax rates, inflation expectations, capital gains, etc., but from the specification of the preferences of the consumers.

The paper is organised as follows. Section 2 presents the basic model of residential demand of owners and tenants respectively. Section 3 analyses the market equilibrium in with a variable number of residential units assuming that the supply is perfectly elastic with respect to the price of units. Market equilibrium is derived after identifying the group of owners who derive utility by using their own house after adapting it to the ideal variant. Section 4 looks at the equilibrium with a fixed supply of residential units. Market equilibrium is derived and the effects on market

equilibrium of alternative endowments of houses are analysed. Section 5 turns to an analysis of the specific Danish case where ownership of secondary homes is restricted to individuals who are permanently living in Denmark. It is shown that this influences not only the price determination but also the *de facto* use of secondary homes, as Danish citizens will use a larger share of disposable time in such homes. Section 6 concludes.

## 2. A Basic Model for Residential Demand

The economy consists of  $N$  consumers with preferences for consumption of a composite good called a *residential unit*, which the consumer occupies either as tenant or as owner. The residential unit is measured in square meters per year, i.e. a variable, which counts the volume of the good residential living. It is assumed, that each consumer demands and gets only one *housing unit* consisting of the demanded number of square meters per year on the market, and that discontinuities, e.g. because of minimum leasing times, house and apartment sizes etc. does not disturb the functions. To ease the reading, Home or House will be used in the following synonymously with Housing unit.

The  $i$ 'th consumer's utility is given by the linear-quadratic utility function

$$U_i = y_i x - \frac{x^2}{2} + z \quad ; \quad 0 < \alpha \leq y_i \leq \delta \quad (1)$$

Where  $x$  is consumption of residential units, and  $z$  is consumption of all other goods in units with price equal to one, and  $\alpha$  and  $\delta$  fixed parameters.  $y_i$  is person specific in the case where the consumer adapt the home to his specific taste.

When the consumer lease the home (housing unit)  $y_i$  takes the value  $\alpha$ . Preferences for leasing a home is thus identical for all consumers and the marginal utility of leasing a unit for consumer  $i$  ( $i = 1, 2, \dots, N$ ) is a linearly decreasing function of the number of residential units per year  $x$ , i.e.:

$$\frac{\partial U_i}{\partial x} = \alpha - x_i. \quad (2)$$

Besides renting, each consumer has the possibility to buy his home (housing unit). Moreover, each consumer is assumed to have an ideal variant of how the home should be equipped, and the only possibility to realise this is through ownership. Only when he owns the home is it worth to adapt it to his specific tastes. An investment is required to realize these “accessories” and, as the preferences are individually differentiated, the consumer perceives this investment as sunk cost, i.e. the extra costs he incurs will never be recovered when he sells the house. To keep the analysis simple this investment is exogenously given at the level  $K$  per housing unit and the corresponding yearly costs is  $\kappa$  (which equals the real interest rate times  $K$  in case of infinite lifetime).

The gain of utility consumers obtain in case of ownership varies among the consumers so that the marginal utility of an owner  $i$  is given by:

$$\frac{\partial U_i}{\partial x} = y_i - x_i. \quad (3)$$

$y_i$  will be used in the following as a parameter to characterise the (maximum marginal owner augmented) utility of a consumer.

The consumer is characterised by the utility parameter  $y_i$  in case of ownership. All consumers are assumed to be equally (rectangular) distributed in the interval  $[\alpha, \delta]$  with respect to  $y_i$  and hence, the density of consumers in this interval is given by  $N/(\delta - \alpha)$ . However, as appears from the following not all consumers will become actual owners. A marginal owner is defined as a consumer who is indifferent between the option to own and adapt his home to his preferences or to abstain from ownership. In this section of presentation of the basic model of residential markets the following three assumptions are made:

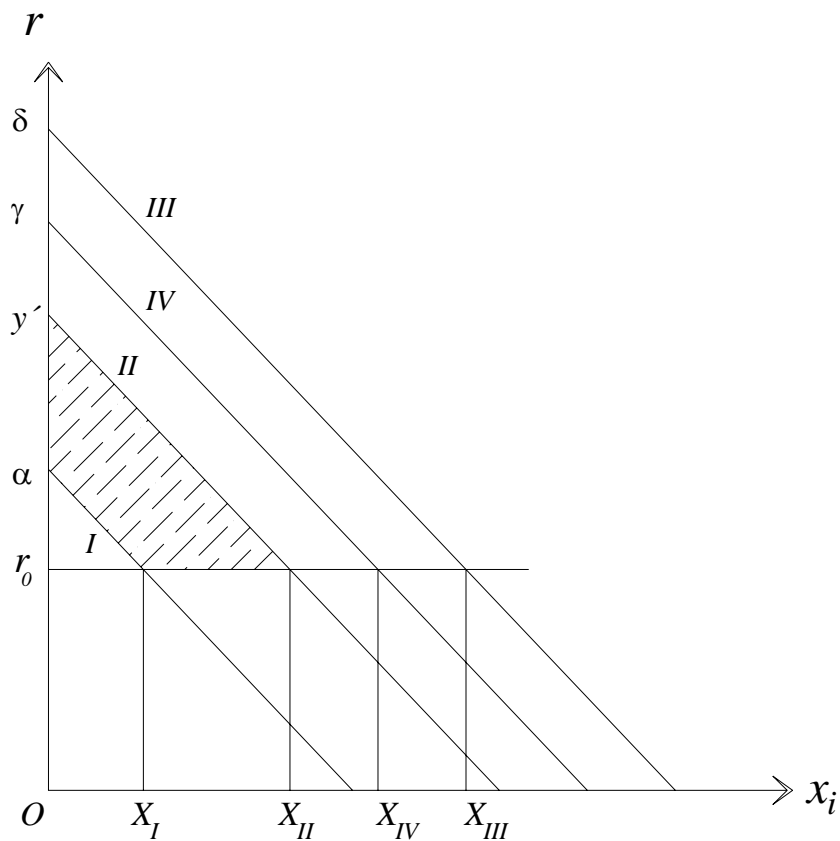
- Rent does not reach the maximum rent a tenant will pay for the first unit of residential consumption, i.e.  $r < \alpha$ .

- The equivalent flow price of a bought house equals the rent in case of leasing, i.e. the ownership of the house commands no specific scarcity price.
- At least some consumers prefer to take ownership and adapt the house for own use. As appears from the following a sufficient condition for this is that  $\kappa < (\delta - \alpha)^2 / 2$ .

These three assumptions secure the existence of a dual market of tenants who lease their house and owners who live in their house after having adapted it to their tastes. Finally, identical stationary expectations are assumed for all consumers such that prices and the real interest rate is expected to prevail at the present level in the future.

Figure 1 illustrates the individual marginal utility curve for tenants (*I*), the marginal owner (*II*) and the individual with the strongest ownership preference (*III*).

Figure 1: Marginal utility and individual demand curves



### *Demand*

The individual consumer on the leasing market optimises his consumption by leasing a number  $x$  of units on a perfect competitive market where the marginal utility of the last unit equalises the rent  $r$ . Thus, using (2), the individual demand function for tenants is given by:

$$x_i = (\alpha - r) \quad (4)$$

In Figure 1 the individual demand of a tenant follows the curve  $I$ . At the rent  $r_0$  demand is  $X_I$ .

The demand of an owner  $i$  is:

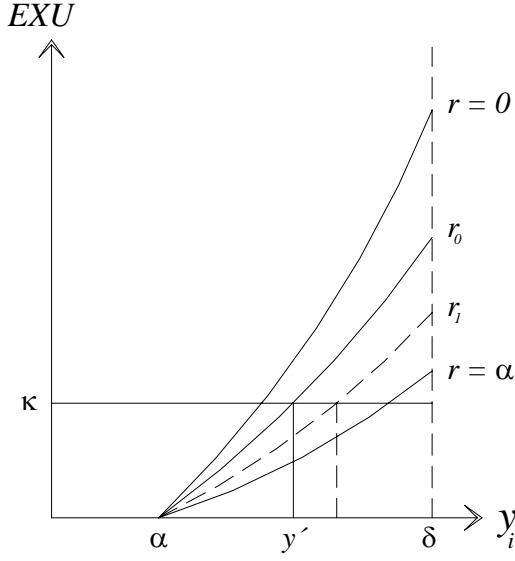
$$x_i = (y_i - r) \quad (5)$$

The first step on the way to derive residential demand is to identify the marginal owner characterized by the preference  $y'$ . The marginal owner's demand function is illustrated in Figure 1 by the curve II, which is situated between the demand curves of tenants I corresponding to the preference  $\alpha$  and the curve III of the owner with the strongest preference  $\delta$ . Assuming that  $r_0$  is the market equilibrium rent, the shaded area illustrates the marginal owner's extra utility per year compared to the utility of a tenant. In general, for a consumer with owner specific preference  $y_i$  the area is the difference between two triangles:

$$EXU = [(y_i - r)^2 - (\alpha - r)^2] / 2. \quad (6)$$

Where  $EXU$  stands for the owners extra utility. Figure 2 shows the relation between  $EXU$  and  $y_i$ .

Figure 2: The owner specific extra utility and  $y_i$



For the marginal owner, the extra utility of ownership  $EXU$  equals the flow equivalent of the sunk costs of adaptation of the house. Assuming that the consumer's alternative is to rent the house, the marginal owner is identified by the equation  $EXU = \kappa$ , which gives

$$y' = [2\kappa + (\alpha - r)^2]^{1/2} + r. \quad (7)$$

Where  $y_r' > 0$ ,  $y_{\kappa}' > 0$ , and  $y_{\alpha}' > 0$ . The dashed curves of Figure 2 are for  $r_i > r_0$ . The marginal owner is indifferent between owning and renting a house because the investment he has to make is equal to the present value of his ownerspecific utility and this has the implication that the stock value of a house only equals the present value of the stream of rent  $r_0$  when there is no scarcity of houses. As the maximum preference of the owner is limited to  $\delta$  and the maximum rent is  $\alpha$  provided there are tenants in the market, it follows from (7) for  $y' = \delta$  and  $r = \alpha$  that  $\kappa < (\delta - \alpha)^2/2$ .  $v$  is the fraction of consumers that will be actual owners:

$$v = \frac{\delta - y'}{\delta - \alpha}. \quad (8)$$



Where  $v_\delta > 0$ ,  $v_{y'} < 0$ , and  $v_\alpha > 0$ . The number of owners in the market is thus  $vN$ .

Because  $y'$  is an increasing function of  $r$ ,  $v$  is a decreasing function of  $r$ .

The aggregate demand for  $x$  from owners with preferences in an interval  $[y, y + dy]$ , remembering that the density of owners is  $N/(\delta - \alpha)$ , is:

$$(y - r) \frac{N}{(\delta - \alpha)} dy$$

And hence, aggregate demand for  $x$  from all actual owners  $X_o^d$  is determined by

$$X_o^d = \frac{N}{(\delta - \alpha)} \int_{y'}^{\delta} (y - r) dy \quad (9)$$

By solving (9) the aggregate demand function for owners appears to be:

$$X_o^d = (\gamma - r)vN. \quad (10)$$

Where

$$\gamma = (\delta + y')/2. \quad (11)$$

is the preference of the average owner. It follows that  $\gamma_\delta > 0$  and  $\gamma_{y'} > 0$ .

Demand from tenants  $X_t^d$  will be

$$X_t^d = (\alpha - r)(1 - v)N. \quad (12)$$

Before adding  $X_o^d$  and  $X_t^d$  to total demand two interesting propositions follow from the model. Firstly, other things equal, owners occupy bigger housing units compared to tenants, see Figure 1. Empirical analysis will have to confirm this. However, Skifter Andersen (2002) notes that housing units are comparatively bigger on the relatively liberalised American market vis-à-vis the more regulated Danish. There are, no doubt, many factors behind this difference; one may be a higher degree of ownership in the USA. Secondly, prohibition of ownership reduces welfare. Even

though the Soviet communist Empire has collapsed this is still a useful observation, and should be kept in mind where public authorities impose regulations on a local residential markets.

By addition, total demand  $X^d$  becomes

$$X^d = [(\gamma - r)v + (\alpha - r)(1 - v)]N \Rightarrow$$

$$X^d = (\mu - r)N. \quad (13)$$

Setting  $\mu = \gamma v + \alpha(1 - v).$  (14)

Where  $\mu_\gamma > 0$ ,  $\mu_v > 0$ , and  $\mu_\alpha > 0$ .  $\mu - \alpha$  is the average demand per consumer that follows from the higher demand of owners. If all consumers had identical preferences, e. g.  $y_i = \alpha$ , no one would want to be owners and  $\mu = \alpha$ .

Differentiating  $X^d$  with respect to  $r$ , remembering  $y_r' > 0$ , gives

$$\frac{\partial X^d}{\partial r} = -\left(\frac{y' - \alpha}{\delta - \alpha} \frac{\partial y'}{\partial r} + 1\right)N < 0. \quad (15)$$

I. e. total demand is a decreasing function of the rent  $r$ . The reduction of demand that follows from a rise of  $r$  can be cut into two parts. Firstly, both tenants' and owners' demand will fall. This is captured by the part  $-rN$  of (13). Secondly, some owners will go out of ownership and become tenants because a rise of  $r$  will raise  $y'$  and reduce the share  $v$  of owners among consumers. This movement contributes to a further fall of the demand.

The position of the demand curve with respect to  $r$  depends on the size of the parameters  $\delta$ ,  $\alpha$ , and  $\kappa$ . Differentiating  $X^d$  with respect to  $\delta$  gives

$$\frac{\partial X^d}{\partial \delta} = N \frac{\partial \mu}{\partial \delta} = N \left( \frac{v}{2} + \frac{(\gamma - \alpha)(y' - \alpha)}{(\delta - \alpha)^2} \right) > 0. \quad (16)$$

I.e. an increase of the parameter for the owner specific utility increases total demand.

A reduction of  $\alpha$  also increases owners' preferences relative to tenants. From (13) it is clear that  $\partial X^d / \partial \alpha = \mu_\alpha N > 0$ . But this covers that a change of  $\alpha$  has implications for  $y'$ ,  $\gamma$ , and  $v$ . It may be more revealing to write

$$\frac{\partial X^d}{\partial \alpha} = N[(1 - v) + (\gamma - \alpha)v_\alpha + v\gamma_\alpha] > 0. \quad (17)$$

I.e. an increase of the level of marginal utility for tenants increases total demand.

Moreover,

$$\frac{\partial X^d}{\partial \kappa} = N \frac{\partial \mu}{\partial \kappa} = -N \left( \frac{y' - \alpha}{\delta - \alpha} \frac{\partial y'}{\partial \kappa} \right) < 0. \quad (18)$$

Which shows that an increase of the flow equivalent of the required investment by owners will make it less attractive to invest and hence reduces total demand.

### 3. Market Equilibrium with Perfect Elastic Supply

This section assumes no scarcity of land for residential purposes, and the supply of residential units  $X^s$  is assumed to be perfect elastic with respect to the stock price of residential units. The equivalent flow price or rent  $r$  of the stock price is thus exogenously given i.e.

$$r = r_0. \quad (19)$$

A market equilibrium with perfect elastic supply of residential units at a given rent  $r_0$  that corresponds to the price of producing new units may be considered the long-term equilibrium of the market. The long run equilibrium condition is thus

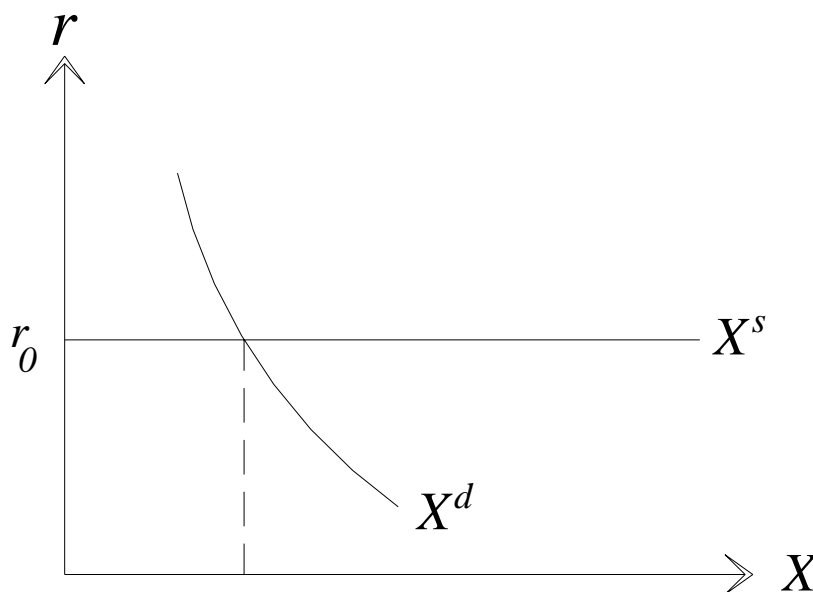
$$X = (\mu - r_0)N. \quad (20)$$

Figure 3 illustrates the market equilibrium with perfect flexible supply.

*Comparative static*

A change of the number of consumers  $N$  generates a proportionate change of the number of houses, as the supply of houses  $H$  is perfect elastic with respect to the rent  $r$  at the rent  $r_0$ . Hence, the preference of the marginal owner,  $y'$ , is unchanged as well as the share,  $v$ , of residential owners of the total population. A change in the rent of houses appears if the real interest rate in the economy changes or if the stock price of houses changes relative to other goods, i.e. because of different rate of growth of productivity between the construction sector and the rest of the economy. It follows from (15) that a rise of  $r_0$  will lower demand and raise the number of tenants as it increases  $y'$  and  $v$ , see Figure 2 and (8).

*Figure 3: Market equilibrium with perfect elastic supply*



An increase of owner specific utility, illustrated by an increase of the parameter  $\delta$ , lifts the preference of the average owner  $\gamma$ , by half of the increase in  $\delta$ , and hence the average owner's demand, and raises the share of owners (the fraction  $v$ ). Total demand will increase, and the owners' share of the market goes up.

Finally, the effect of an increase of the required investments for an owner, i.e.  $\kappa$ , will raise  $y'$ , reduce the number of owners and so owners demand. Total demand will fall and so will owners' share. It will, however, raise the average owner's demand  $\gamma$ .

#### 4. Market equilibrium with fixed supply of homes

Limited supply of residential units  $x$  may be the case in big cities. Congestion and environmental problems may force authorities to stop the building of new units. The case of limited supply is therefore most interesting for big cities and generally wherever scarcity of land and building restrictions limit the supply.

Total supply of residential units is now assumed to be completely inelastic with respect to the price, i.e.

$$X^s = H. \quad (21)$$

Where  $H$  denotes the given endowment of residential units. To have tenants in the market it is also assumed that the rent  $r < \alpha$  in equilibrium. The equilibrium is now given by the condition

$$H = (\mu - r)N \Rightarrow \frac{H}{N} = (\mu - r). \quad (22)$$

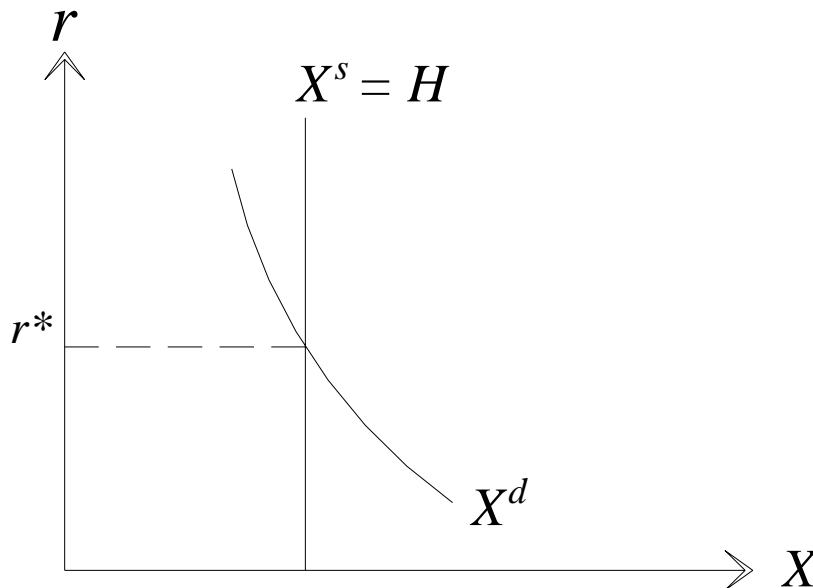
(22) is very similar to (20) but now  $H$  and  $N$  are exogenous and  $r$  endogenous.

Because of this, the implications of changes of the exogenous can be traced using the above-deducted relations. (22) shows that a reduction of the supply  $H$  has the same implications as an inflow of new inhabitants, i.e. an increase of  $N$ , where supply is limited.

The short run effects of increased congestion, and so a rise of  $N$ , will be an increase of the rent. However, short and long run effects may well be different. The reason for this is that in the short run present owners, having already invested  $K$  in their homes will stay in their houses, also in cases where the owner specific extra benefit is too

small to cover  $\kappa$ . As a consequence, the number of owners will remain unchanged and inflowing new citizens will be forced into the limited supply of residential units for tenants. This will raise the rent and the stock price for houses.

Figure 4: Market equilibrium with fixed supply



In the long run, some owners move out of their houses for various reasons. They sell the houses with a capital gain because of the increased rent (which in this case opens a possibility for being compensated for their initial investment  $K$ ). On the one hand, the high price of houses, and the high rent  $r$ , has the implication that owners should have higher owner specific preferences to be willing to buy a house and invest  $K$ ; this will reduce the number of owners. On the other hand, there are potential owners with high owner specific preferences among the incoming citizens and this, by itself, tends to raise the number of owners. The net effect on owners share among consumers can be found from (7) and (8), which show that this share will fall. The explanation is the same as the explanation following equation (15), a rise of  $r$  reduces the demand from owners and tenants equally (the equality is due to the specification of the utility function (1)), but at the same time it reduces demand by shifting marginal owners into tenants. In conclusion, in the long run, congestion leads to higher rent, smaller housing units, and an increase in the share of tenants among consumers. This is an interesting and testable hypothesis, which will have to be confirmed by empirical

evidence. Old owners selling their houses cash a windfall gain. In the same way, on the capital market, old owners of residential units for renting may cash a windfall gain. Skifter Andersen (2002) compares the velocity of changes in the residential structure in the relatively liberalised American market with the more regulated Danish and observes that close to 40 per cent of the Danish housing units are from 1940 or older compared to only 20 per cent in the USA. This indicates a speedier adjustment to the long run equilibrium in the USA than in Denmark.

The formal analysis above assumes static expectations. If more congestion is expected this will create expectations of future capital gains to owners and increase owners demand further, which may out weight the prescribed increase in the share of tenants among consumers.

## 5. The case of restrictions on ownership of Danish secondary homes

The model makes up a suitable framework for analysis of the Danish case of restrictions on ownership of summerhouses. Denmark has from nature been favoured with coastal areas of a recreative quality. The long coast line along the West coast of Jutland with beaches of high standard and the many islands with ample possibilities for all kind of leisure activities at the seaside has for a long time made those areas attractive for establishment of summerhouses and spending the holidays in a secondary house has become a part of many Danes' way of life.

The neighbouring countries, especially Germany, are less endowed with areas of similar quality. A Danish membership of the European Union where Denmark should respect the principle of free mobility of capital might therefore lead to a substantial buying up of summerhouses by foreigners. Denmark therefore demanded and got accepted an exemption to the principle of free mobility of capital related to foreigners possibility to own summerhouses. This exemption was laid down in special protocol the Treaty on the European Union and, in accordance with the protocol; only persons living permanently in Denmark are allowed to own a secondary home. The aim was primarily to protect the Danish citizens possibility to spend their holiday in the Danish summerhouses.

It is only ownership, which is restricted. The leasing market is free, as foreigners have right to lease a summerhouse on equal foot with Danish citizens. If ownership did not play any role for utility, restrictions exclusively on ownership would not have the wanted impact, namely to give Danish citizens priority in the use of Danish summerhouses. However, restrictions on the leasing market are very rigid and problematic to administer and such radical proposal has not been considered. Furthermore, the model presented in this section show that the ownership restrictions tends to fulfil the aim of the politicians: to favourite the use of houses for Danish citizens. The reason is that the owners on average use their houses more than tenants because of the efficiency effect of ownership. Restrictions of the ownership therefore turn the demand towards demand of the Danish citizens.

In a model by Skak (2000) restrictions has been analysed in a quite different set up as it focuses on effects of differences of incomes and prices, but with no specific owner utility.

#### *A slightly changed model*

To proceed, for summerhouses the analysis is now related to the individual consumer's intensity of the use of the house during a year. Thus the dimension of the good changes from square meters used over one year to number of week's use of one summerhouse. For primary homes the intensity is typically the same during the year and the consumers optimisation is related to the quantity of the composite good only. The reason for this is that the transaction costs of allowing others to use the primary home temporarily during the year is too high. For secondary homes the situation is different. The consumer frequently only uses the house in a limited period of the year and is therefore more willing to rent it out in the rest of the year. The equipment in the secondary home is often less valuable also in eyes of the owner and the transaction costs of allowing others to use the house is therefore less and the owner therefore more willing to let the house for a part of the year.

It is now assumed, that the total number of housing units is fixed and equal to  $H$ . All housing units (houses) are identical and each house is available for  $\lambda$  weeks per year, i.e. the total supply of weeks for all houses is  $\lambda H$  per year. As the owner is the only



person who can reap the owner specific utility no one wants to own more than one house for living. The number of consumers is assumed to exceed that of the number of houses ( $N > H$ ). Only the fraction  $\varphi$  of the  $N$  consumers is assumed to have preferences for owning their homes and the other fraction,  $(1-\varphi)$ , is ignorant whether to lease or to own a house. Specifically, it is assumed that  $\varphi N > H$ . The utility function (1) is still employed and the density of owners in the interval  $[\alpha, \delta]$  is thus  $\varphi N/(\delta - \alpha)$ . It is further assumed that owner preferences are so high that all  $H$  houses will be bought by owners having no problems to get marginal owner extra benefit above  $\kappa$ . In this case, the market price for houses will sort out the group of owners among the  $N$  consumers. Formally, only owners with ownerspecific preferences at or above  $y'$  will be in the owner group where  $y'$  is given by the equation:

$$(\delta - y') \frac{\varphi N}{(\delta - \alpha)} = H \Rightarrow \quad (23)$$

$$vN = H. \quad (24)$$

Where  $v$ , as before, is the fraction of consumers that are owners:

$$v = \varphi \frac{\delta - y'}{\delta - \alpha}. \quad (25)$$

In this variant of the model  $y'$  can be derived from (23) to be

$$y' = \delta - \frac{H(\delta - \alpha)}{\varphi N} = \delta - \frac{\delta - \alpha}{\varphi} v. \quad (26)$$

Neither  $y'$  nor  $v$  is a function of  $r$ , but given by the parameters and the ratio  $H/N$ .

### *Demand*

Using (24) and (25), equation (10) of the total demand of owners can now be shortened to

$$X_o^d = (\gamma - r)H. \quad (27)$$

It is assumed that the leasing market is not empty and hence, rent at the leasing market does not exceed  $\alpha$ . The number of consumers on the leasing market is  $(N - H)$  and the aggregate demand function on the leasing market is thus:

$$X_t^d = (\alpha - r)(N - H). \quad (28)$$

Where  $X_t^d$  denotes the number of weeks demanded by all tenants. For a given rent the aggregate demand of tenants is proportionate to their number  $(N - H)$ .

Using (26) and (27), the aggregate demand of tenants and owners  $X^d = X_o^d + X_t^d$  appears to be:

$$\begin{aligned} X^d &= (\gamma - r)H + (\alpha - r)(N - H) \Rightarrow \\ X^d &= [\gamma + \alpha(1 - v) - r]N \end{aligned} \quad (29)$$

(29) is equal to (13) remembering that  $\mu = \gamma + \alpha(1 - v)$ . However, (29) is more convenient for comparative static.

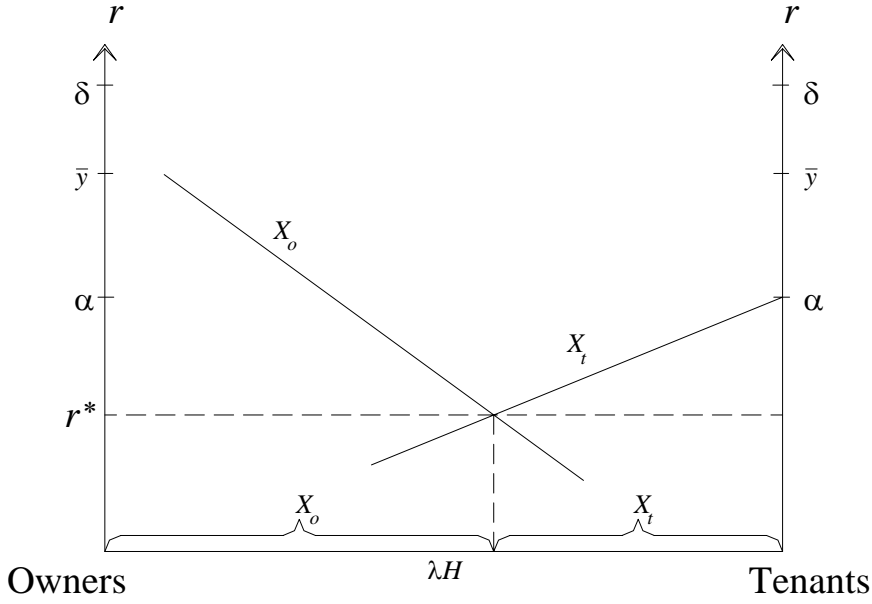
### *Supply and equilibrium*

The total supply is  $\lambda H$  and market equilibrium is thus determined by the condition:

$$\begin{aligned} [\gamma + \alpha(1 - v) - r^*]N &= \lambda H \Rightarrow \\ r^* &= \gamma + \alpha(1 - v) - \lambda v. \end{aligned} \quad (30)$$

Where  $r^*$  is the equilibrium rent. Figure 5 illustrates the market equilibrium and the allocation of the total supply  $\lambda H$  between owners demand  $X_o^d$  and tenants demand  $X_t^d$ .

Figure 5: Market equilibrium



To have tenants in the market, it is assumed that  $r^* < \alpha$ , and so from (30)  $\gamma - \alpha < \lambda$ .

This inequality can be rewritten as

$$2\varphi \frac{\delta - \alpha - \lambda}{\delta - \alpha} < v. \quad (31)$$

Which shows that congestion, i. e. a low  $v$ , may push tenants out of the market.

#### *Comparative static*

The parameters and exogenous variables, that influence the market equilibrium appears from (30). Attention here will be on the effects of changes in  $H$ ,  $N$  and  $\varphi$  only.

A proportionate change in  $H$  and  $N$ , leaving  $v$  unchanged, will only scale up the whole market with the equilibrium rent unchanged. The structure of the owner group will be the same, see (26), as will the prices for houses for the owners. What matters are changes in the relative endowments of houses  $v = H/N$ . It may intuitively be expected that if endowments of houses increases relative to the number of consumers, rent decrease. This is also the case. Differentiating (30) partially with respect to  $v$  gives:

$$\frac{\partial r^*}{\partial v} = y' - \alpha - \lambda < 0. \quad (32)$$

(32) is negative because it follows from (30), and the condition  $r^* < \alpha$ , that  $\gamma < \alpha + \lambda$ . So, with  $y' < \gamma$ , one gets  $y' < \alpha + \lambda$ . Consequently, an increase of the number of individuals  $N$  on the market with unchanged  $\varphi$ , and so a fall of  $v$ , will lead to an increase of the rent  $r^*$ .

The effect on the aggregate demand of tenants relative to total demand  $s = X_t^d / \lambda H$ , of an increase of the total number of consumers relative to the endowment of houses, i.e. a fall of  $H/N = v$ , may be an increase at first, but will gradually turn into a fall. To see this, divide (28) with  $\lambda H$  to get

$$s = (1 - v) \left( 1 - \frac{\gamma - \alpha}{\lambda} \right). \quad (33)$$

Using  $\partial(\gamma - \alpha) / \partial v = -(\delta - \alpha) / 2\varphi$ , the derivative with respect to  $v$  is

$$\frac{\partial s}{\partial v} = \frac{\delta - \alpha}{\lambda} \left( 1 + \frac{1 - 2v}{2\varphi} \right) - 1. \quad (34)$$

For (34) to be positive, it is necessary that

$$1 + \frac{1 - 2v}{2\varphi} > \frac{\lambda}{\delta - \alpha} \Rightarrow$$

$$\frac{1}{2} + \varphi \frac{\delta - \alpha - \lambda}{\delta - \alpha} > v. \quad (35)$$

(35) and (31) put the following limits<sup>1</sup> on  $v$

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<sup>1</sup> From previous assumptions  $H/N = v < \varphi$ , and so  $2\varphi(\delta - \alpha - \lambda\beta) / (\delta - \alpha) < \varphi \Rightarrow (\delta - \alpha - \lambda\beta) / (\delta - \alpha) < 1/2$ , and  $\varphi(\delta - \alpha - \lambda\beta) / (\delta - \alpha) < 1/2$  because  $\varphi$  is a fraction. From this follows  $2\varphi(\delta - \alpha - \lambda\beta) / (\delta - \alpha) < 1/2 + \varphi(\delta - \alpha - \lambda\beta) / (\delta - \alpha)$ , which leaves room for  $v$  inside the restrictions of (36).

$$2\varphi \frac{\delta - \alpha - \lambda}{\delta - \alpha} < v < \frac{1}{2} + \varphi \frac{\delta - \alpha - \lambda}{\delta - \alpha}. \quad (36)$$

(36) shows that, as congestion increases, i. e.  $v$  falls, from some point tenants share will begin to fall and finally they will be pushed completely out of the market. The illustration of Figure 6 assumes that (36) holds.

Figure 6: Effects of an increase of the relative endowment of houses

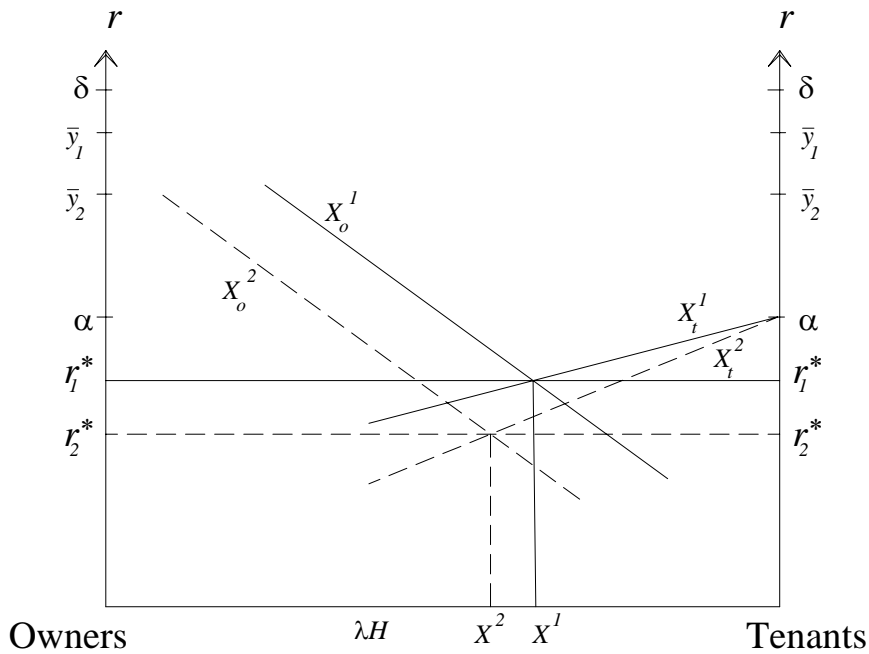


Figure 6 shows the effects of an increase of  $H/N$  from  $(H/N)_1$  to  $(H/N)_2$ . For expositional reasons the increase in  $H/N$  is caused by a decrease in  $N$  so  $H$  is unchanged. The increase in  $H/N$  shifts both demand curves downwards from  $X_o^1$  to  $X_o^2$  and  $X_t^1$  to  $X_t^2$ , respectively. The rent falls from  $r_1^*$  to  $r_2^*$ , the tenants' share grows as the equilibrium moves from  $X^1$  to  $X^2$ , and the marginal owners preferences decreases from  $y_1'$  to  $y_2'$ .

An increase in the share of consumers with ownerspecific preferences, i.e. an increase of  $\varphi$ , increases  $y'$ ,  $\gamma$  and hence  $r^*$ . This follows from (26), (11) and (29).

### *Restrictions on ownership*

This slightly changed version of the basic model makes up a suitable framework for analysing restrictions on ownership of secondary homes. Let assume that the total numbers of consumers  $N$  consists of national consumers  $N_D$  and foreign consumers  $N_F$ , i. e.:

$$N_D + N_F = N. \quad (37)$$

All consumers are assumed to have identical preferences given by (1), i. e. all consumers have ownerspecific preferences. The relative endowments of summerhouses in the two countries are different and to keep the analysis simple the total number of summerhouses  $H$  is located exclusively in the domestic country.

The fraction of consumers who are owners is thus

$$v = \frac{H}{N}. \quad (38)$$

If domestic and foreign consumers are allowed to lease or buy summerhouses on equal foot, the market equilibrium is described by (22) – (29) for  $\varphi = 1$ . Rent in the no-restriction case, using the supscript  $NR$ , is thus:

$$r^{*NR} = \gamma^{NR}v + \alpha(1 - v) - \lambda v. \quad (39)$$

Where, using (11) and (26), the average owners preference is given by:

$$\gamma^{NR} = \delta - v(\delta - \alpha)/2. \quad (40)$$

Because of symmetry of the preference structure of the two countries the relation between domestic and foreign occupation of weeks in the summerhouses for owners and tenants respectively and together,  $w$ , equals the relation between the size of the populations:

$$w = \frac{X_{oD}^d + X_{tD}^d}{X_{oF}^d + X_{tF}^d} = \frac{X_{oD}^d}{X_{oF}^d} = \frac{X_{tD}^d}{X_{tF}^d} = \frac{N_D}{N_F}. \quad (41)$$

If the domestic country wants to preserve a larger part of consumption of weeks in the summerhouses for its own citizens restrictions on the leasing market is not a feasible solution, but restricting ownership to national citizens only will provide the aimed result. Formally, market equilibrium in this case is specified by (22) – (29) for  $0 < \varphi < 1$  where

$$\varphi = \frac{N_D}{N_D + N_F}. \quad (42)$$

Using the superscript  $R$  in this case, rent is given by:

$$r^{*R} = \gamma^R v + \alpha(1 - v) - \lambda v. \quad (43)$$

Where the average owner's preference is given by:

$$\gamma^R = \delta - v(\delta - \alpha) / 2\varphi. \quad (44)$$

Comparing (39) and (40) with (43) and (44) shows that

$$\gamma^{NR} > \gamma^R. \quad (45)$$

And hence

$$r^{NR} > r^R. \quad (46)$$

In this case only a limited group of consumers are allowed to own the summerhouses. This lowers the preference of the average owner compared to the case of a full-liberalized market. Moreover, the rent is also lower with restrictions.

However, even though foreigners as tenants demand more, the share of total consumption of weeks in summerhouses is twisted towards the national citizens because owners demand more than tenants. Formally this follows from:

$$w = \frac{X_{oD}^d + X_{tD}^d}{X_{tF}^d} = \frac{H(\gamma^R - r^{*R}) + (N_D - H)(\alpha - r^{*R})}{N_F(\alpha - r^{*R})} \Rightarrow$$

$$w = \frac{N_D + H(\gamma^R - \alpha)/(\alpha - r^{*R})}{N_F} > \frac{N_D}{N_F}. \quad (47)$$

## 6 Concluding remarks

In the seminal paper of Lancaster (1979) on monopolistic competition the menu of product variants is exogenously given at the consumer level and hence the market will only offer the consumer his ideal variant by chance. In this paper it is assumed that the ideal variant represents a large number of attributes, which is unique for the consumer. The market will therefore never offer the ideal variant but leave it to the consumer to make adaptation of the good which turn it to his specific tastes. As the investment to adapt the good is sunk cost ownership is assumed to be the necessary institutional framework for the consumer for considering making the investment. The ideal variant is not only unique for the individual consumer but the extra utility of consumption of the ideal variant is also assumed to be different across consumers. Consumers with strong preferences for the ideal variant might therefore realise an ownerspecific consumer surplus when they buy and adapt the good. The model is applicable for both the market for primary houses, and the market for secondary houses; but the model may also have relevance for the demand for other durable goods.

From the basic model follows the propositions that owners occupy bigger housing units than tenants. Restriction on ownership reduces welfare, and congestion leads to higher rent, smaller housing units, and a relative growth of the number of tenants. A variant of the basic model describing restrictions on ownership of secondary homes to a group of consumers shows that such restriction secures a proportional big share of



the market for this group. In this case increased congestion may throw tenants out of the market.

The model does not state that the merry owner is happier than the tenant. For such conclusions interpersonal utility comparisons must be possible and this has not been assumed in the model. What the model shows is only that potential owners of a durable good in general will be better off if ownership is institutionalised.

Ownership may also influence efficiency of production activities. In a specific production activity total factor productivity may be improved by adaptation if the fixed equipment of the firm. The real capital typically will be heterogeneous in a unique form confined to the individual firm. The firm therefore perceives the investment of adapting its capital as sunk cost and hence ownership might be a necessary institutional precondition. Future research is needed to develop this further.

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