Preventing Runs with Fees and Gates

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Abstract

I study whether gates and redemption fees as recently introduced into US and EU money market fund (MMF) regulations achieve their goal of making MMFs less susceptible to runs. I focus on purely self-fulfilling runs in a setting without fundamental risk. There is a tension between eliminating run equilibria á-la Diamond and Dybvig (1983) on the one hand and eliminating deposit-access panics á-la Engineer (1989) on the other hand. The results suggest that the tools available to MMFs under the new regulations are effective in eliminating the first-mover advantage at MMFs whose assets are relatively liquid and/or whose investors are not characterized by a high degree of liquidity preference. Giving MMFs more flexibility how to use fees and gates may allow to eliminate the first-mover advantage at funds that likely cannot prevent runs with the tools currently available.

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1. Introduction

Money market funds (MMF) issue liabilities which are redeemable on demand and promise a high degree of principal stability, providing investors with an alternative to traditional bank deposits. The financial crisis of 2007/08 highlighted that this issuance of 'money-like' (or 'bank deposit-like') liabilities outside the regulatory safety net provided to traditional banks makes money market funds susceptible to runs.[1] After the financial crisis, reforms were adopted in both the United States and the European Union with the goal of making money market funds less run-prone. A key element of these reforms is that money market funds can/ought to take various measures if they run low on liquid assets. Specifically, money market funds can suspend convertibility (also referred to as 'gates') and charge fees on withdrawals.[2]

In this paper, I study whether fees and gates are effective tools to eliminate the first-mover advantage in redemptions, thus fulfilling the goal of making money market funds less susceptible to runs.[3] I put particular emphasis on the possibility of *deposit-access panics* à-la Engineer (1989) which are run equilibria where depositors (MMF investors) withdraw preemptively even though they know that they will never incur credit losses on their deposits (MMF shares). Depositors run because they correctly anticipate that, if many others withdraw, they might not be able to access their deposits in the future at the specific point in time when they need to, or they may only be able to withdraw if they pay a redemption fee. Both gates and redemption fees can give rise to deposit-access panics.

I study runs on a single intermediary (MMF) which acts as a social planner (or mechanism designer) distributing asset returns to a continuum of depositors. Depositors experience idiosyncratic, privately observed liquidity shocks. They arrive at the MMF sequentially and need to be served on the spot. The setting is a simplified version of the Engineer (1989) model, which is essentially a Di-

[1]In the days after the Lehman bankruptcy in September 2008, money market mutual funds in the United States experienced a modern-day bank run. See Schmidt et al. (2016) for a detailed account of the episode.

[2]SEC (2014) provide a detailed description of the US regulation, adopted in 2014. Gesley (2018) provides a useful summary of the EU regulation, adopted in 2017. With regard to fees and gates, the two regulations are similar in spirit but there are also important differences. For instance, in the US, it is never mandatory for money market mutual funds to impose fees or gates. In the EU, money market funds are obliged to impose fees or gates under certain conditions.

[3]The regulator clearly seems to believe that this is the case. For instance, recital 48 of the EU regulation (Regulation (EU) No 1131/2017) states that "[..] MMFs should have in place provisions for liquidity fees and redemption gates to ensure investor protection and prevent a 'first mover advantage'".

amond and Dybvig (1983) model with one additional period. The model-intermediary resembles a money market fund with stable net asset value (NAV) shares although the analysis should also have relevance for flexible NAV funds.[4] The focus of this paper is on unique implementation of the first best allocation. I study whether the fund can augment its demandable debt contracts with fees and gates- clauses in such a way as to implement first best without admitting for other (run-) equilibria. Runs are modelled as purely self-fulfilling events ('panics') in the absence of any fundamental risk. The MMF's investments are riskfree and aggregate liquidity needs of depositors are known.[5]

Since the fund is modelled as a social planner, the interests of the fund do not deviate from the interests of the depositors or of society at large. The question whether MMFs should be *obliged* to impose fees or gates is therefore outside the scope of this paper. In any case, before studying whether the regulator should mandate the use of gates or redemption fees, it is important to know if and how a benevolent money market fund can prevent runs with these tools.

It is useful to frame the analysis in terms of two types of run equilibria that can occur in an Engineer setting without fundamental risk. The first type of run equilibrium is the classical Diamond-Dybvig type panic which can occur if the MMF pays out redeeming depositors by liquidating assets while imposing the liquidation losses on depositors who remain in the fund. Restricting withdrawals if the fund runs low on liquid assets (e.g. by imposing fees or gates) can help to prevent Diamond-Dybvig type runs. For the model-fund studied in this paper, imposing such restrictions if liquid assets are exhausted will be necessary to eliminate Diamond-Dybvig type run equilibria. The other type of run equilibrium that can occur in the Engineer setting are deposit-access panics, which are unrelated to liquidation losses or indeed any credit losses incurred by depositors. The difficulty is that the restrictions on withdrawals that help to eliminate Diamond-Dybvig type panics can give rise to deposit-access panics. The question is then whether a MMF can use fees and gates in such a way as to eliminate both types of run equilibria simultaneously.

[4] An important part of both the US and EU regulation is that the use of stable net asset value (NAV) shares has been restricted (in the US more so than in the EU). This paper focuses on the 'fees and gates' part of the regulations and does not address the difference between flexible and stable NAV funds. In the EU, the 'fees and gates' provisions apply only to stable NAV and low volatility NAV funds (a hybrid between stable and flexible NAV). In the US, they apply to flexible NAV (prime-) funds while stable NAV (government-) funds can opt into them.

[5] Abstracting from fundamental risk is a simplification that helps to focus on the aspects most relevant for this paper. Empirically it is hard to disentangle the effect of fundamental risk from coordination failure in financial crises (see for instance the survey in Goldstein (2013)).

The results in this paper suggest that fees and gates of the sort introduced into the US and EU regulations are effective in eliminating run equilibria if either the fund's assets are relatively liquid or if depositors' liquidity preference is not very strong. The results also suggest that, if used in the way intended in the US and EU regulations, fees and gates will not be effective in preventing runs on all funds, notably on funds whose assets are relatively illiquid.

Regulators in both the US and the EU prescribe a narrow role for redemption fees as a means to ensure that depositors who redeem from a fund internalize the liquidation losses which the fund incurs by paying them out.[6] The results in this paper suggest that redemption fees used in this manner can only prevent runs if the fund's assets are relatively liquid. If a fund keeps paying out depositors even though liquid assets are depleted, a necessary condition to prevent Diamond-Dybvig type runs is that the fund charge redemption fees which are high enough to ensure that redeeming depositors internalize liquidation losses. If the fund's assets are relatively illiquid, the fee required to prevent Diamond-Dybvig type runs is so high that even depositors who experience a liquidity shock in the given period are not willing to pay it. Such a prohibitively high fee will then give rise to deposit-access panics.

On the positive side I show that, independent of asset liquidity, funds can always eliminate run equilibria in the present setting by using fees and gates in the right manner. I show how redemption fees can be combined with gates to implement a redistribution among depositors that incentivizes depositors (off the equilibrium path) to remain in the fund if others run. Used in this manner, fees and gates can prevent run equilibria without ever resorting to asset liquidations. In sum, the results suggest that fees and gates are powerful tools to prevent runs but it may be necessary to give MMFs more flexibility in how they can use them.[7]

Related Literature This paper is most closely related to a recent theoretical literature examining the effect of redemption fees and/or gates on investors' propensity to run on a bank or a fund. Cipriani et al. (2014) and Lenkey and Song (2016) study settings in which the fundamental

[6]Both the US and the EU regulation allow MMFs to charge redemption fees if (and only if) their liquid assets drop below a certain threshold. In the US, the redemption fee is limited to 2%. In the EU there is no explicit ceiling on the redemption fee but the fee should "[..] adequately reflect the cost to the MMF of achieving liquidity and ensure that investors who remain in the fund are not unfairly disadvantaged when other investors redeem their units or shares during the period." (See article 34(1) of Regulation (EU) No 1131/2017).

[7]For instance, the wording in recital 48 of Regulation (EU) No 1131/2017 seems to rule out explicitly the use redemption fees as a means to implement a redistribution among depositors.

return to the bank's investment is uncertain, with some depositors being better informed about the true return than others. In Cipriani et al. (2014) granting a bank the possibility to impose fees or gates can increase informed investors' propensity to withdraw preemptively once they learn that uncertainty about asset returns has increased. In Lenkey and Song (2016) redemption fees affect uninformed investors' signal extraction problem in which they try to infer the success of the bank's investment by observing withdrawals of other (informed) investors. Whether redemptions fees increase or decrease the propensity to run is ambiguous. Zeng (2017) studies runs on a fund that invests in illiquid but fundamentally riskfree assets and that issues shares with a flexible NAV. Early redemptions require the fund to rebuild its cash-buffer in the future, leading to predictable liquidation losses in the future caused by redemptions today. Despite the flexible NAV, withdrawal decisions can therefore be strategic complements. Fees and gates reduce (but not eliminate) run risk by making early redemptions more costly (fees) or more difficult (gates). Finally, Ennis and Keister (2009a, 2010) show that imposing gates may not be optimal ex-post and the effectiveness of gates in preventing runs is severely limited if the bank or the regulatory authority cannot credibly commit to take measures that hurt depositors ex-post.

Compared to the papers mentioned above the present paper highlights that fees and gates can be useful to eliminate Diamond-Dybvig type panic equilibria but, by compromising depositors' ability to withdraw their deposits at the specific point in time when they need to, they can lead to deposit-access panics. This paper abstracts from many other aspects that are relevant when studying the effectiveness of fees and gates to prevent runs and, in this sense, should be seen as complementary to the papers listed above.

More broadly, this paper is part of a literature that studies panic equilibria in settings with risk-free investment returns where the bank is required to follow sequential service when paying out depositors. This literature has largely focused on the case with two types of depositors ('patient' and 'impatient') and with uncertainty regarding aggregate liquidity needs. Good summaries can be found in Ennis and Keister (2009b) and Andolfatto et al. (2017). In models with sequential service and unknown aggregate liquidity needs, the efficient payout schedule may feature decreasing payouts.[8] One way to implement the efficient payout schedule would be to charge progressively increasing redemption fees. The role of the decreasing payout schedule in this class of models is fundamentally different than in the present model however; in the present model, redemption fees are a means to prevent multiple equilibria and, in the efficient allocation, are only charged off the equilibrium path.[9]

2. The Model

The economy lasts for three periods $t = 1, 2, 3$ and is endowed with three (infinitely divisible) assets: a '1-asset' that pays out an amount $\frac{1}{3}$ of the consumption good at date 1, a '2-asset' that pays out $\frac{1}{3}$ at date 2, and a '3-asset' that pays out $\frac{1}{3}$ at date 3. There is a unit measure of ex-ante identical depositors. Ex post, each depositor turns out to be of type 1,2 or 3 with probability $\frac{1}{3}$ each. A depositor of type t wants to consume most at date t. Payoffs equal $c_1 + \delta c_2 + \delta^2 c_3$ for type 1 depositors, $c_2 + \delta c_3$ for type 2 depositors and c_3 for type 3 depositors, where c_t denotes consumption in period t and $\delta \in (0, 1)$ represents liquidity preference. The lower δ, the higher liquidity preference. At date 1, depositors privately learn whether they are type 1 or not. At date 2, depositors who are not type 1 privately learn whether they are of type 2 or 3. Denote by Θ_t the collection of information sets with regard to the own type at date t. We have $\Theta_1 = \{1, \text{not}1\}$ and $\Theta_2 = \Theta_3 = \{1, 2, 3\}$. In the aggregate, by a law of large numbers, $\frac{1}{3}$ of depositors will be of type 1, $\frac{1}{3}$ of type 2 and $\frac{1}{3}$ of type 3.[10]

There is an intermediary ('fund') that acts as a social planner (or mechanism designer) distributing asset returns to depositors. Depositors can only communicate with the fund; they cannot communicate or trade with each other and they do not observe any actions taken by other depositors. When distributing asset returns to depositors, the fund is restricted to follow sequential service, as will be described in detail below. The fund can liquidate assets prematurely, which yields a fraction $\lambda \in (0, 1)$ of the final return of the asset. For simplicity, I assume that (i) the liquidation return is

[8] See for instance Green and Lin (2003) and Peck and Shell (2003). Offering lower payouts for depositors who show up late in the queue has sometimes been referred to as *partial suspension*, a term coined by Wallace (1990).

[9] I thank an anonymous referee for pointing out this difference.

[10] There are well known technical problems with the law of large numbers in continuum economies. See Al-Najjar (2008) for a discussion of measurability issues in continuum-player games.

the same whether an asset is liquidated one or two periods before maturity and (ii) whenever the fund liquidates assets at date 1, it liquidates the 2- and 3-asset in equal amounts.[11] The fund can store the consumption good across periods at no cost. Depositors can store the consumption good across periods as well but they incur a one-time fixed cost $\kappa > 0$ of accessing the storage facility.[12]

First-best First-best is defined as the allocation that maximizes the expected payoff of depositors subject to the economy's resource constraint, under the restriction that all depositors of the same type must get the same payoff. It is not hard to see that the first-best allocation is to give each depositor of type t one unit at date t and nothing in other periods. This implies that in first-best, the entire date 1 return (or 'cash-flow') is paid to type 1 depositors, the entire date 2 cash-flow is paid to type 2 depositors, and the entire date 3 cash-flow to type 3 depositors.

Sequential Service Sequential service is modelled in a similar fashion as in Ennis and Keister (2010), with some adaptations. In each period, the depositors who choose to withdraw from the fund arrive at the fund sequentially and the fund needs to pay them out on the spot. Depositors who choose not to withdraw from the fund stay at home in the given period and do not contact the fund. Showing up at the fund is therefore equivalent to withdrawing from the fund. Since payments need to be made on the spot, the payment to a depositor that shows up at the fund can only be made contingent on the number of depositors that showed up at the fund so far (within the same period and in previous periods) but not on withdrawal orders of depositors who arrive later in the queue. The *payout policy* specifies for date 1 a function $f_1 : [0, 1] \mapsto \mathbb{R}_+ \cup \{S\}$, where $f_1(z)$ is the payment to the z^{th} depositor to show up at the fund in period 1. Setting $f(\cdot) = S$ means that convertibility is suspended. A depositor who shows up at the fund at date 1 with arrival point z receives $f_1(z)$ and cannot show up again in future periods. The depositor then either consumes $f_1(z)$ or stores the good for future consumption, incurring the storage fixed cost κ. The only exception is if the fund has suspended convertibility: if a depositor arrives at the fund with arrival point $\hat{z}$ and the payout policy is such that $f_1(\hat{z}) = S$, then the depositor receives nothing and can show up again in future periods.

[11] Alternatively one could assume that the liquidation returns of the 2-asset and the 3-asset at date 1 are different, with an average liquidation return of λ, and the fund can choose which asset to liquidate. This would slightly complicate the analysis without affecting any of the main results.

[12] The fixed cost κ captures the cost of withdrawing 'too early' from the fund. This may stand for transaction- or search costs of changing to a different fund or bank, foregone return if deposits are withdrawn early, etc.

Denote $\overline{z}_1$ as the total number (measure) of depositors that withdraw at date 1. The number of depositors left in the fund at date 2 equals $1 - \overline{z}_1$. Analogous to date 1, the payout policy specifies for date 2 a function $f_2(z; \overline{z}_1)$ which maps $[0, 1 - \overline{z}_1]$ into $\mathbb{R}_+ \cup \{S\}$. As before, $f_2(z; \overline{z}_1)$ is the payment made to the z^{th} depositor to show up in period 2. The date 2 payment schedule $f_2(z)$ can be made contingent on total date 1 withdrawals $\overline{z}_1$. Denoting $\overline{z}_2$ as total withdrawals at date 2, the payout policy finally specifies a function $f_3(z; \overline{z}_1, \overline{z}_2)$, where $f_3(z)$ is the payment to the z^{th} depositor to show up in period 3. The date 3 payment schedule $f_3(z)$ can be made contingent on total withdrawals at dates 1 and 2 ($\overline{z}_1$ and $\overline{z}_2$). The entire payout policy is then given by $f = (f_1, f_2, f_3)$. The payout policy can in principle be such that the fund's resource constraint is violated if too many depositors show up. If a depositor shows up at the fund and the fund does not have the resources necessary to make the payment specified in the payout policy, then the fund defaults and all depositors which are still in the fund receive nothing.[13] Depositors know the structure of the game and are thus aware of the fund's resource constraint.

Withdrawal Game and Equilibrium Given the payout policy $f = (f_1, f_2, f_3)$, depositors choose their withdrawal strategy in a non-cooperative game. At the beginning of each period, the depositors who are still in the fund (those who did not withdraw in previous periods) wake up in a random order. The wake-up-order at date t is captured by an index i_t that is randomly allocated to all depositors who are still in the fund at date t. The date 1 indices are given by $i_1 \in [0, 1]$, date 2 indices by $i_2 \in [0, 1 - \overline{z}_1]$ and date 3 indices by $i_3 \in [0, 1 - \overline{z}_1 - \overline{z}_2]$. Each depositor who is still in the fund at date t is allocated each position in the corresponding interval with identical probability, independent of the depositor's type and independent of the indices in previous periods. Depositors do not observe their own index i_t or any indices allocated to other depositors.[14] Upon waking up, a depositor sees how much she can withdraw from the fund, and she has two actions available: 'withdraw' (which is equivalent to 'go to fund') or 'not withdraw' (which is equivalent to 'stay at home'). Hence in any given period t, the depositors with the lowest indices i_t are first to decide whether to withdraw, after which those with higher indices follow, etc. This implies that, in every period, the arrival point z of a depositor will be weakly lower than her index i_t. If some

[13]Conversely, if the fund has consumption good left after everybody withdrew, the leftover amount is lost.

[14]It is possible that the fund could set its payout policy such that the indices are revealed to depositors. This will not play a role in the following analysis however.

depositors with lower indices decide not to withdraw, the arrival point will be strictly lower than the index.

The position in the wake-up-order is similar to the 'position in the queue' common in bank run models. Indeed, the setting here is isomorphic to a setting where (i) depositors first decide whether or not to go the fund; (ii) the depositors who go to the fund arrive at the fund in a random order and (iii) once a depositor arrives at the fund and sees how much she can withdraw, she decides whether or not to withdraw.

When making the withdrawal decision, a depositor's information set consists of four elements: (i) the time period; (ii) the information about the own type; (iii) the amount that can be withdrawn from the fund and (iv) the observed history of events up to this point. Depositors do not observe any actions taken by other depositors. The observed history is the history of amounts which a depositor could have withdrawn in previous periods. For instance, a depositor who is still in the fund at date 2 knows how much she could have withdrawn at date 1. The observed history may in principle give depositors information about how many other depositors withdrew from the fund in previous periods, as well as information about their own indices i_t in previous periods.[15]

A depositor's behavior strategy maps her information sets into probability distributions over the two possible actions 'withdraw' and 'not withdraw'. To streamline notation, I denote by $s_t(x, y)$ a depositor's withdrawal strategy given that she finds herself in period t, the information about her own type is x, and the fund pays an amount y if she withdraws. The function s_t then prescribes behavior at date t for any observed history up to point t, given (x, y). As we shall see, this is without loss of generality (see also footnote 15). Denote $R(f_t)$ as the range of f_t, that is, $R(f_t)$ is the set of payments which the fund may potentially offer to a depositor at date t. The date t strategy s_t is then given by

$$s_t : \overbrace{\Theta_t}^{\substack{\text{information}\\ \text{about the}\\ \text{own type}}} \times \overbrace{R(f_t)}^{\substack{\text{amount that}\\ \text{can be withdrawn}\\ \text{from fund}}} \mapsto \Delta\{\text{‘withdraw’}, \text{‘not withdraw’}\} \tag{1}$$

[15]Note however that there is no risk on the aggregate level so that play on the aggregate level proceeds in a deterministic fashion for any strategy profile. The only information depositors effectively learn from the observed history relates to the indices i_t they were allocated in previous periods. This is unrelated to future indices and thus does not convey any payoff-relevant information.

where Δ is the simplex of the pure strategy set. A depositor's (behavior) strategy is given by $s = (s_1, s_2, s_3)$. Note that payoffs depend only on the aggregate behavior of other depositors, not on actions by individual other depositors. Since there is no risk on the aggregate level, play proceeds in a deterministic fashion on the aggregate level for any strategy profile s. Each player thus essentially plays against a deterministic continuum.

Throughout the paper I will limit attention to symmetric equilibria in which all depositors choose the same strategy s. An equilibrium is defined to be a strategy s^* such that (i) s^* maximizes each depositor's expected payoff given that all others play s^* and (ii) s^* is sequentially rational in the sense that depositors do not play strategies that are strictly dominated conditional on their information set having been reached. The equilibrium refinement in (ii) concerns behavior in information sets off the equilibrium path and corresponds to the restriction on off-equilibrium behavior imposed by the *weak perfect Bayesian equilibrium* concept. I will sometimes be a bit loose in the terminology and say that a strategy is 'strictly dominated' if it violates requirement (ii).

The remainder of the paper will be concerned with finding payout policies f that uniquely implement first-best; more precisely, payout policies f which are such that all equilibria s^* of the withdrawal game under payout policy f implement first-best. Implementing first best requires that depositors withdraw from the fund only at the date that corresponds to their type. A *run equilibrium* denotes an equilibrium of the withdrawal game in which a strictly positive measure of depositors withdraw from the fund at a date that does not correspond to their type.

Implementing first-best requires that the fund pay one unit to the type 1 depositors who show up at date 1. Consider now a payout policy where the fund pays out one unit to everybody who shows up at the fund at date 1, that is, the fund sets $f_1(z) = 1$ for all $z \leqslant 1$. If all depositors play 'withdraw' at date 1 irrespective of their type, the fund defaults during date 1 and depositors who do not withdraw at date 1 receive nothing. Given that all other depositors play 'withdraw' at date 1 irrespective of their type, doing the same is the best response and the withdrawal game exhibits a run equilibrium. Put differently, if the fund never imposes restrictions on withdrawals, it will be susceptible to runs. In the remainder of the paper, I examine if and how fees and gates -clauses can be used to implement first-best as the unique equilibrium of the withdrawal game.

3. Gates and Fees

The new regulatory provisions in the US and the EU allow/mandate MMFs to impose fees or gates if (and only if) liquid assets fall below some threshold.[16] In this section, I first study payout policies that capture gates (section 3.1) and fees (section 3.2) as they are available to MMFs in the new regulatory framework. I show under which circumstances such payout policies can eliminate run equilibria. I then derive a relatively simple payout policy involving both fees and gates that can always eliminate run equilibria in the current setting(section refsubsectioncombinedpolicy).

3.1. Gates (Suspension of Convertibility)

Consider the following payout policy (dubbed *suspension policy*) in which the fund suspends convertibility until the next period once liquid assets are depleted:[17]

$$f_t\left(z;\left(\overline{z_t}\right)_{j=1}^{t-1}\right)=\begin{cases}1 & \text{if } z\leqslant \frac{1}{3}t-\sum\limits_{j=1}^{t-1}\overline{z}_j \\ S & \text{otherwise}\end{cases} \qquad \text{for} \quad t=1,2,3 \tag{2}$$

The suspension policy (2) satisfies the fund's resource constraint for all profiles of withdrawal strategies. Furthermore, the policy implements the first-best allocation if all depositors withdraw at the date that corresponds to their type. It is not hard to see that there are equilibria of the withdrawal game that implement first-best. In particular, type 2 and 3 depositors have no incentive to withdraw early if no other type 2- and 3 depositors do so.

As shown by Engineer (1989), the withdrawal game under suspension policy (2) can exhibit run equilibria. To see why, consider a scenario where all depositors try to withdraw from the fund at date 1 independent of their type (that is, all depositors play $s_1(\cdot,1)=$'withdraw'). In this scenario, only the depositors with indices $i_1\leqslant\frac{1}{3}$ can actually withdraw from the fund at date 1. Hence a measure $\frac{2}{3}$ of depositors will be left in the fund at date 2. Since the position in the line at date 1 (the index i_1) is independent of depositors' types, $\frac{1}{3}$ of the depositors left in the fund at date 2 will

[16] In both the US and the EU, liquid assets are (roughly speaking) defined as assets maturing within one week, with special provisions for government assets. The EU regulation additionally requires that net daily redemptions from the fund be above a certain threshold for fees/gates to be activated.

[17] In the US, funds can impose gates for up to 10 working days, in the EU for up to 15 working days.

be of type 1, $\frac{1}{3}$ of type 2 and $\frac{1}{3}$ of type 3. Both type 1 and 2 depositors value consumption at date 2 higher than consumption at date 1 and will therefore again play 'withdraw' at date 2. (Playing $s_2(1,1) = s_2(2,1) =$'withdraw' is strictly dominant.) This means that the measure of depositors that try to withdraw at date 2 equals $\frac{2}{3}\left(\frac{1}{3}+\frac{1}{3}\right)=\frac{4}{9}$. Since the fund suspends convertibility after a measure $\frac{1}{3}$ of depositors have shown up at date 2, only a fraction $\frac{1}{3}\left(\frac{4}{9}\right)^{-1}=\frac{3}{4}$ of the type 1 and 2 depositors left in the fund at date 2 can withdraw at date 2. Figure (1) illustrates a run on the fund at date 1 under the suspension policy. A run at date 1 leads to a backlog of type 1 depositors left in the fund at date 2 which implies that too many depositors will try to withdraw at date 2 relative to the fund's date 2 cash-inflow. The fund will therefore again have to suspend convertibility at date 2. As a result, some of the type 1 and 2 depositors left in the fund at date 2 will not be able to withdraw at date 2.

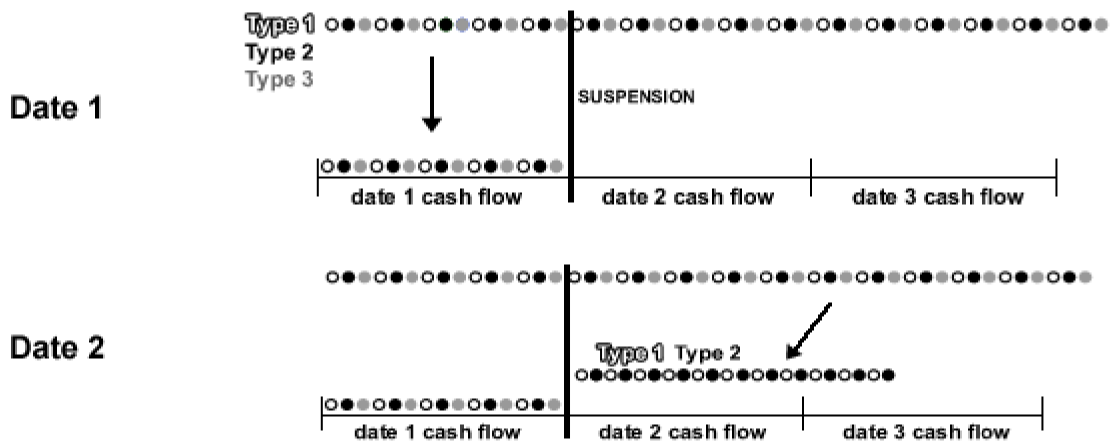

Figure 1: Runs under a suspension policy

Consider now the decision of an individual non-type 1 depositor whether or not to participate in the run at date 1, given that all others run. If the depositor remains in the fund, she will turn out to be of type 3 with probability $\frac{1}{2}$. All type 3 depositors can withdraw 1 unit at date 3 and hence get a payoff of 1. With probability $\frac{1}{2}$ she will turn out to be type 2. In this case, she will be able to withdraw 1 unit at date 2 with probability $\frac{3}{4}$. With probability $\frac{1}{4}$, she can only withdraw at date 3, in which case she receives a payoff of δ. The expected payoff for a type 2 depositor left in the fund at date 2 thus equals $\frac{3}{4}+\frac{1}{4}\,\delta$. Putting this all together, the expected payoff of *not* participating in the run at date 1 (i.e. staying in the fund until date 2 if all others run) for a non-type 1 depositor equals:

$$\frac{1}{2}\overbrace{\left(\frac{3}{4}+\frac{1}{4}\delta\right)}^{\substack{\text{expected}\\ \text{payoff type 2}}}+\frac{1}{2}\overbrace{1}^{\substack{\text{payoff}\\ \text{type 3}}}=\frac{7}{8}+\frac{1}{8}\delta$$

Consumption of a non-type 1 depositor who withdraws 1 unit at date 1 equals $1-\kappa$. It follows that participating in the run is the best response iff:

$$\overbrace{1-\kappa}^{\substack{\text{payoff of}\\ \text{withdrawing}\\ \text{early}}} \geqslant \overbrace{\frac{7}{8}+\frac{1}{8}\delta}^{\substack{\text{expected payoff of}\\ \text{staying in fund}}} \tag{3}$$

Rewriting condition (3) yields $\delta \leqslant 1-8\kappa$. If this condition is fulfilled, then payout policy (2) does not implement first-best as the unique equilibrium of the withdrawal game since the withdrawal game exhibits a run equilibrium where all depositors play 'withdraw' at date 1 irrespective of their type. It is not difficult to show that the converse is also true: if $1-8\kappa<\delta$, then the withdrawal game does not exhibit a run equilibrium and first-best is uniquely implemented. We get the following proposition whose full proof is given in appendix A:

Proposition 3.1. *The suspension policy (2) uniquely implements first-best if and only if* $\delta > 1-8\kappa$.

3.2. Fees

Consider now the following payout policy (dubbed *fee policy*) in which the fund charges a fee τ (modelled as a haircut) on withdrawals after liquid assets are depleted:

$$f_t\left(z;\left(\overline{z_t}\right)_{j=1}^{t-1}\right)=\begin{cases}1 & \text{if } z\leqslant \frac{1}{3}t-\sum\limits_{j=1}^{t-1}\overline{z}_j\\ 1-\tau & \text{otherwise}\end{cases} \qquad \text{for} \quad t=1,2,3 \tag{4}$$

As with the suspension policy, it is straightforward that the fee policy (4) does implement first-best as an equilibrium of the withdrawal game, for any fee τ. In particular, if none of the other depositors withdraw from the fund earlier than the date that corresponds to their type, then doing the same is the best response.

Note next that the fee policy (4) satisfies the fund's budget constraint for all profiles of withdrawal strategies if and only if the fee satisfies $\tau \geqslant 1-\lambda$. If this condition is fulfilled, then depositors who withdraw from the fund at a time when the fund has depleted its period cash-inflow receive at most the liquidation return λ. A fee policy as in (4) can only prevent run equilibria if $\tau \geqslant 1-\lambda$. To see this, suppose the fund sets $\tau < 1-\lambda$ and consider a hypothetical scenario where all depositors play 'withdraw' at date 1, irrespective of their type and irrespective of how much they can withdraw (that is, all depositors play $s_1(\cdot,\cdot) =$'withdraw'). The fund will then run out of assets before everybody could withdraw at date 1. A depositor who does not withdraw at date 1 consumes zero, implying that participating in the run is the best response. For unique implementation of first-best, we thus need to restrict attention to policies (4) where the fee satisfies $\tau \geqslant 1-\lambda$.

The previous paragraph shows that a fee policy (4) can only prevent runs if the fee τ is set high enough. However, if the fee is set very high, then incentive compatibility will be violated in the sense that depositors of type t will not be willing to pay the fee on date t withdrawals. For instance, if τ is very high, a type 1 depositor will prefer waiting until date 2 instead of withdrawing $1-\tau$ units at date 1. If the fee is so high that depositors never actually pay the fee in equilibrium, then the fee policy (4) becomes equivalent to the suspension policy (2). As a result, if the fee is prohibitively high, there may be a run equilibrium at date 1 for the same reason as with the suspension policy.

The discussion above shows that, in order to prevent runs (and to improve over the suspension policy), the fee τ should not be too high and not too low. Suppose now the fund sets the fee τ within $\tau \in [1-\lambda, 1-\delta]$, which is a non-empty interval iff parameters satisfy $\lambda \geqslant \delta$. If the fee is set within this interval, it satisfies two criteria: (i) the fee is high enough such that any liquidation losses which the fund incurs are borne by the depositors who withdraw and (ii) the fee is low enough such that, if given the choice, type 1 depositors prefer to withdraw at date 1 and pay the fee to waiting until date 2. The latter is the case since $1-\tau \geqslant \delta$, which means that type 1 depositors prefer consuming $1-\tau$ units at date 1 to consuming 1 unit at date 2. It follows that, in a hypothetical situation where some of the non-type 1 depositors withdraw at date 1, all type 1 depositors arriving late in line (those with indices $i_1 > \frac{1}{3}$) will pay the fee on date 1 withdrawals and leave the fund at date

1.[18] Furthermore, the type 1 depositors who leave the fund at date 1 do not impose any liquidation losses on the depositors who remain fund. Hence even if a run occurs at date 1, the fund will start 'unharmed' at date 2 - the run does not impose liquidation losses on the depositors who are left in the fund and neither does the run cause a backlog of type 1 depositors left in the fund at date 2. It follows that, no matter how many non-type 1 depositors run at date 1, all type 2 depositors left in the fund at date 2 will be able to withdraw 1 unit at date 2 and all type 3 depositors left in the fund will be able to withdraw 1 unit at date 3. This takes away any incentive for non-type 1 depositors to participate in a run at date 1 in the first place, so that the withdrawal game does not exhibit a run equilibrium. Figure 2 provides a graphical illustration.

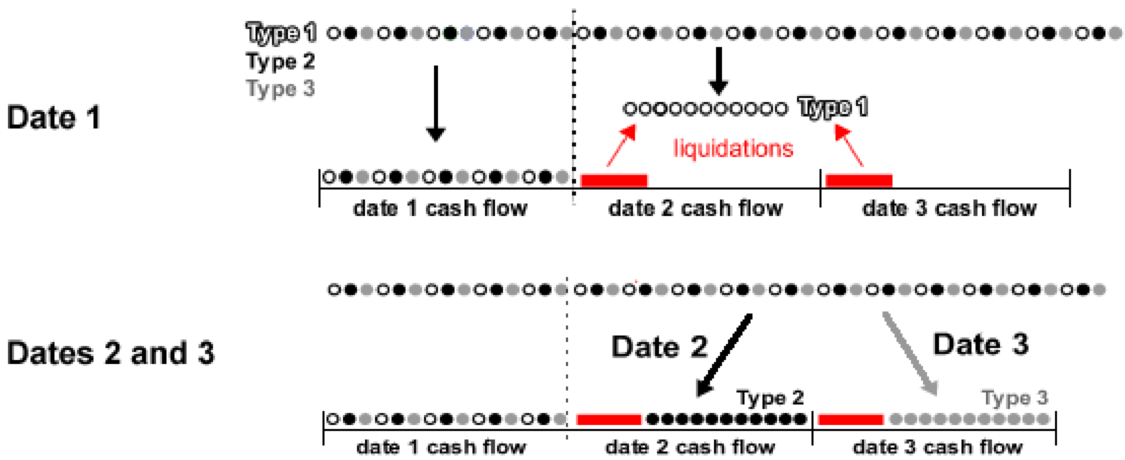

Figure 2: Run under a fee policy (off the equilibrium path)

Consider now the situation where $\lambda < \delta$, in which case the interval $[1-\lambda, 1-\delta]$ is empty. In this case, asset liquidity λ is relatively low, which calls for a high fee to make sure that liquidation losses are internalized by redeeming depositors. At the same time, liquidity preference is also low (δ is high) which means that depositors have a low willingness to pay a fee on withdrawals. To study whether a fee policy (4) can prevent runs if $\lambda < \delta$ I will again play through a scenario where all depositors run at date 1. Whether participating in the run at date 1 is the best response for non-type 1 depositors depends on the backlog of type 1 depositors left in the fund at date 2 after the run occurred at date 1. The more type 1 depositors are left in the fund at date 2, the larger the number

[18]This is obvious if $1-\tau > \lambda$ since paying the fee is a strictly dominant strategy for type 1 depositors. In equilibrium, it must also be the case if $1-\tau = \lambda$. See appendix B for the details.

of depositors that will rush to the fund at date 2 and hence the lower the probability that a depositor who turns out to be of type 2 will be able to withdraw from the fund at date 2 without paying a fee. As with the suspension policy, non-type 1 depositors' incentive to withdraw at date 1 is thus increasing in the number of type 1 depositors that will be left in the fund at date 2.

The backlog of type 1 depositors left in the fund at date 2 depends on the behavior of type 1 depositors that arrive late in line in the run at date 1. The more of the late-arriving type 1 depositors are willing to pay the fee on date 1 withdrawals, the smaller the number of type 1 depositors left in the fund at date 2. Suppose the fund sets the fee to $\tau = 1 - \lambda$, which is the lowest fee that may potentially prevent a run (see above) and consider the best response of type 1 depositors that arrive late in line in a run at date 1. Type 1 depositors arriving late in line face a trade-off between (i) withdrawing at date 1 and paying the fee τ and (ii) waiting until date 2, thereby incurring the discount cost $1 - \delta$ but (possibly) escaping the fee τ.

Since $1 - \tau = \lambda < \delta$, type 1 depositors arriving late in line will prefer to wait until date 2 if they can escape the fee at date 2 for sure. The probability that they can escape the fee at date 2 is decreasing in the number of (type 1 and 2-) depositors that will try to withdraw at date 2. If no type 1 depositors are left in the fund at date 2, then the fee can be escaped for sure. This implies that, with $\lambda < \delta$ and a fee $\tau = 1 - \lambda$, there will always be some type 1 depositors left in the fund at date 2 after a run occurred at date 1. How many of the type 1 depositors remain in the fund until date 2 depends on the distance between λ and δ: the lower λ, the higher the fee and (keeping δ fixed) the smaller the proportion of type 1 depositors (among those late in line) that will pay the fee at date 1. We get the following proposition, whose proof is given in appendix B:

Proposition 3.2.

(1) There exists a fee policy (4) that uniquely implements first-best only if a fee policy with $\tau = 1 - \lambda$ uniquely implements first-best.

(2) A fee policy (4) with a fee $\tau = 1 - \lambda$ uniquely implements first-best if and only if:

(i) $\dfrac{\lambda}{\delta} \geqslant 1$ *or*

(ii) $\dfrac{\lambda}{\delta} \in \left(\dfrac{3}{4} + \dfrac{1}{4}\,\delta\,,\, 1\right)$ *and* $1 - 2\,\kappa < \dfrac{\lambda}{\delta}$ *or*

(iii) $\dfrac{\lambda}{\delta} \leqslant \dfrac{3}{4} + \dfrac{1}{4}\,\delta$ *and* $1 - 8\,\kappa < \delta$

Condition (i) in item (2) deals with the case where $\lambda \geqslant \delta$ on which I have elaborated above. Conditions (ii) and (iii) deal with the more difficult case where $\lambda < \delta$. If λ is significantly below δ, then type 1 depositors will never pay the fee on date 1 withdrawals in equilibrium. As shown appendix B, the exact condition for this is $\lambda \leqslant \dfrac{3}{4}\delta + \frac{1}{4}\delta^2$. If this condition is fulfilled, then no depositor ever pays the fee in equilibrium, neither at date 1 nor at date 2. The fee policy (4) then becomes equivalent to the suspension policy (2) and it therefore prevents a run at date 1 if and only if the suspension policy prevents a run. Together with the result of proposition 3.1 this leads to condition (iii). Finally, condition (ii) deals with the case where λ is within the (narrow) range $\lambda \in (\frac{3}{4}\delta + \frac{1}{4}\delta^2, \delta)$. In this case, some fraction of type 1 depositors arriving late in a run at date 1 will pay the fee on date 1 withdrawals. The lower the value of λ within this interval, the smaller the fraction of late-arriving type 1 depositors that pay the fee at date 1, and hence the higher the incentive to run for non-type 1 depositors.

Note that, different to the suspension policy (2), liquidity preference δ has a non-monotonic effect on the propensity to run under the fee policy (4). On the one hand, higher liquidity preference (lower δ) makes fee policies more effective since it is easier to induce type 1 depositors to actually pay the fee if a run occurs at date 1. On the other hand, within the subset of the parameter space with $\delta > \lambda$, higher liquidity preference makes it less likely that fee policies can prevent runs. The fee which the fund needs to charge in order to make sure that those who pay the fee internalize liquidation losses is then so high that (at least some) depositors avoid paying the fee, even at the date that corresponds to their type. If liquidity preference is high, then depositors run for the same reason as with the suspension policy: if others run, non-type 1 depositors correctly anticipate that they may not be able to withdraw the next period if they need to. More precisely, they may only be able to withdraw at date 2 by paying a fee which they are not willing to pay. The higher liquidity preference, the larger is the loss in payoff for type 2 depositors if they consume only at date 3 and hence the higher the propensity to run at date 1.

Figure 3 depicts the set of parameters (shaded area) for which gates and fees prevent runs, given $\kappa = 0.03$. (Changing the cost of withdrawing early κ will make the shaded areas uniformly bigger (increase in κ) or smaller (decreasing in κ) without changing their basic shape.) The set of parameters for which fees prevent runs is a superset of the set of parameters for which gates prevent runs. This is not very surprising - instead of imposing a gate, the fund can always charge a prohibitively high redemption fee that depositors are never willing to pay. In this sense, gates are a redundant tool. However, things are different if the regulator imposes a ceiling on the redemption fee that funds are allowed to charge (which is the case in the US but not the EU). Suppose the regulator imposes an upper bound $\overline{\tau}$ on the fee. For the reasons discussed further above, a fee policy (4) then cannot prevent runs for any $\lambda < 1 - \overline{\tau}$. If there is a regulatory upper bound on the fee, suspension therefore has a role in preventing runs in cases where asset liquidity is low and liquidity preference is not very strong. The results suggests that, in the US, funds with relatively liquid assets will use fees while funds with relatively illiquid assets will use gates. In the EU regulatory framework where there is no ceiling on the redemption fee, the independent role of gates in preventing runs is unclear. The present model does not give a rational why EU MMFs should ever use gates instead of fees.

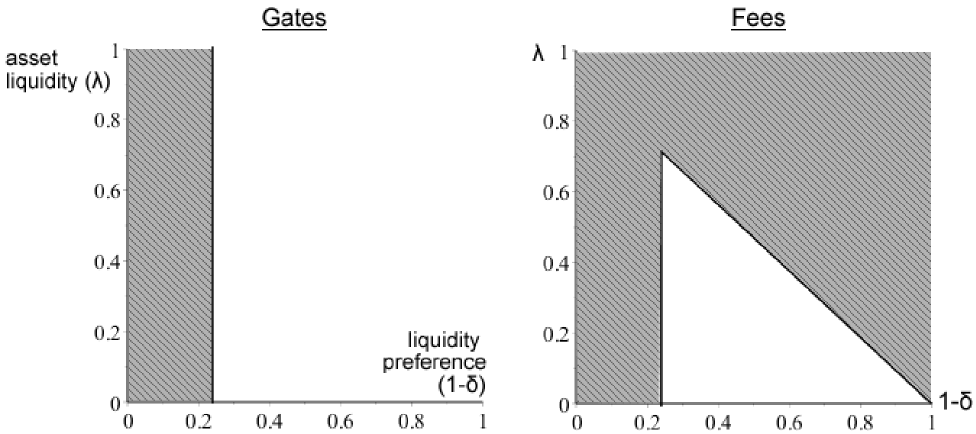

Figure 3: Set of parameters (shaded area) for which gates and fees prevent runs ($\kappa = 0.03$)

Gates and fees as in (2) and (4) that capture the tools currently available to MMFs in the US and the EU cannot prevent runs if asset liquidity λ is relatively low and liquidity preference δ is within an intermediate region. In general, the lower asset liquidity, the more difficult it becomes to prevent runs with these tools. Redemption fees whose purpose it is to make sure that redeeming depositors internalize liquidation losses run into an irresolvable conflict if asset liquidity is low. On the one hand, the fee must be relatively high to ensure that liquidation losses are fully internalized by those who withdraw in order to prevent Diamond-Dybvig type panic equilibria. On the other hand, unless liquidity preference is very strong, depositors are not willing to pay such a high redemption fee, which then gives rise to deposit-access panics. In the next subsection, I show how this conflict can be resolved by combining redemption fees with gates in a way that does not rely on asset liquidations.

3.3. Combining Gates and Fees

In this subsection, I show that a payout policy that combines suspension with fees (dubbed the *combined policy*) can always prevent runs. The payout policy has two key elements: first, the fund always suspends convertibility if liquid assets are exhausted and second, redemption fees are used as a means to implement a redistribution among depositors in a way that incentivizes depositors to remain in the fund if others run. If the fund needs to activate suspension at date 1, then (and only then) it will charge a fee on *all* withdrawals at date 2. The fund pays any fee revenue raised at date 2 to those who withdraw at date 3. Importantly, the fee on date 2 withdrawals is not related to asset liquidations. The fund never liquidates assets, so that the effectiveness of the policy does not depend on asset liquidity λ.

Abusing notation a bit, $\overline{z}_1 = \frac{1}{3} + S > \frac{1}{3}$ means that a measure $\frac{1}{3}$ of depositors withdrew at date 1 *and* the bank had to suspend convertibility (that is, a measure of depositors larger than $\frac{1}{3}$ tried to withdraw). As long as $\overline{z}_1 \leqslant \frac{1}{3}$, the payout schedule is the same as the standard suspension policy (2). If the fund had to suspend convertibility at date 1 it charges a fee $\widehat{\tau}$ on all date 2 withdrawals. Once the date 2 cash inflow is exhausted, the fund suspends convertibility:[19]

$$f_2\big(z; \tfrac{1}{3} + S\big) = \begin{cases} 1 - \widehat{\tau} & \text{if } z \leqslant \dfrac{1}{3}\dfrac{1}{1-\widehat{\tau}} \\ S & \text{otherwise} \end{cases} \quad \text{with } \widehat{\tau} \geqslant 0 \tag{5}$$

[19] If a measure z of depositors withdraw, the fund pays out an amount $z(1 - \widehat{\tau})$ of cash. The fund receives a date 2 cash inflow of $\frac{1}{3}$. Hence the date 2 cash flow is exhausted if $z(1 - \widehat{\tau}) = \frac{1}{3}$ or $z = \frac{1}{3}\frac{1}{1-\widehat{\tau}}$.

After suspending convertibility at date 1 and charging a fee on date 2 withdrawals, the fund pays everything that is left in the fund to those withdrawing at date 3:

$$f_3(z, \tfrac{1}{3} + S, \overline{z}_2) = 1 + \frac{\overbrace{\overline{z}_2\, \widehat{\tau}}^{\substack{\text{total fee revenue} \\ \text{raised at date 2}}}}{\underbrace{\tfrac{2}{3} - \overline{z}_2}_{\substack{\text{number of depositors left} \\ \text{in fund at date 3}}}} \tag{6}$$

Expression (6) shows that, if the fund suspended convertibility at date 1, then it can pay out more than 1 unit per depositor left in the fund at date 3 whenever a non-zero measure of depositors withdrew at date 2 ($\overline{z}_2 > 0$). The reason is that the fund paid out less than 1 unit for each depositor that withdrew at date 2 and the fund did not incur any liquidation losses at date 2.

It is easy to see that the combined policy satisfies the fund's resource constraint for all profiles of withdrawal strategies. Furthermore, as with the previous payout policies, it is straightforward that first-best is implemented as an equilibrium of the withdrawal game. The question is whether the withdrawal game under the combined policy exhibits run equilibria, specifically equilibria where non-type 1 depositors run at date 1. We can start with the observation that the combined policy will not be able to prevent run equilibria for the entire parameter space if the fee on date 2 withdrawals $\widehat{\tau}$ is either very high or very low.

Suppose first the fee $\widehat{\tau}$ is set very high. Then none of the depositors are willing to pay the fee on date 2 withdrawals if the fund had to activate suspension at date 1. Instead, all depositors who are left in the fund at date 2 will wait to withdraw until date 3. In this case, the combined policy would increase the propensity to run at date 1 relative to the standard suspension policy (2). If a run occurs at date 1, then *all* of the type 2 depositors left in the fund at date 2 will consume only at date 3 as a result of the prohibitively high fee charged on date 2 withdrawals. As a result, the expected payoff of a non-type 1 depositor who does not participate in a run (if others run at date 1) would be lower compared to the suspension policy (2). Furthermore, if the fee $\widehat{\tau}$ is prohibitively high, no fee revenue is generated at date 2 which can be paid to those withdrawing at date 3.

Consider next the opposite situation where the fee $\widehat{\tau}$ is set very low. In this case, the number of (type 1 and 2-) depositors who try to withdraw at date 2 after a run occurred at date 1 will be such that the fund needs to suspend convertibility at date again. This can be seen most easily for the case where the fee on date 2 withdrawals is set arbitrarily small ($\hat{\tau} \to 0$) in which case the combined policy becomes equivalent to the standard suspension policy (2). As with the suspension policy, non-type 1 depositors' incentive to participate in a run at date 1 results from the prospect of turning out to be a type 2 depositor who arrives late in line at date 2 and can only withdraw at date 3.

The main result of this subsection is that, as long as the contingent fee on date 2 withdrawals $\hat{\tau}$ is set within some (non-empty) interval, it is possible to prevent runs with the combined policy:

Proposition 3.3. *The combined policy with a redemption fee* $\hat{\tau} \in \left(\frac{1-\delta}{2}, \frac{1}{1+\delta}\right)$ *uniquely implements first-best.*

The proof of proposition 3.3 is given in appendix C. Note that the interval $\left(\frac{1-\delta}{2}, \frac{1}{1+\delta}\right)$ is non-empty for any $\delta \in (0, 1)$. Intuitively, if the fee $\hat{\tau}$ is set within the interval given in proposition 3.3, it satisfies two criteria:

(i) The fee is not 'too low' in the sense that the number of depositors who want to withdraw at date 2 is never so high that the fund needs to suspend convertibility at date 2.

(ii) The fee is not 'too high' in the sense that sufficiently many type 1 and 2 depositors left in the fund at date 2 will pay the fee on date 2 withdrawals instead of waiting until date 3. This means that enough fee revenue is generated that can be paid to those who withdraw at date 3.

To gain more intuition about the result of proposition 3.3, consider the best response of a non-type 1 depositor in a hypothetical situation where other non-type 1 depositors run at date 1. If others run, the fund suspends convertibility at date 1 and charges a fee $\hat{\tau}$ on date 2 withdrawals. The depositor knows that, if she remains in the bank and turns out to be of type 3, she will profit from the fee revenue raised at date 2. Furthermore, if she turns out to be of type 2, she will be able to withdraw at date 2 as long as she pays the fee.[20] Intuitively, the prospect of profiting from fee revenue raised by the bank together with the fact that funds can be accessed at date 2 (against a

[20] To be precise, it may well be the case that not all type 1 and 2 depositors left in the fund at date 2 will withdraw at date 2 after (off the equilibrium path) a run occurred at date 1. In this case, the number $\overline{z}_2$ of depositors who withdraw at date 2 will be such that type 2 depositors are just indifferent between withdrawing at date 2 and withdrawing at date 3. Hence all type 2 depositors receive a payoff equal to $1 - \hat{\tau}$, even if not all of them withdraw at date 2. In contrast, if the fund needed to suspend convertibility at date 2, this would imply that type 1 and 2 depositors strictly prefer withdrawing $1 - \hat{\tau}$ units at date 2 to waiting until date 3.

fee) when needed takes away the incentive to participate in the run. Since an individual non-type 1 depositor is strictly better of not running when others run, there is no equilibrium of the withdrawal game where non-type 1 depositors run on the fund at date 1.

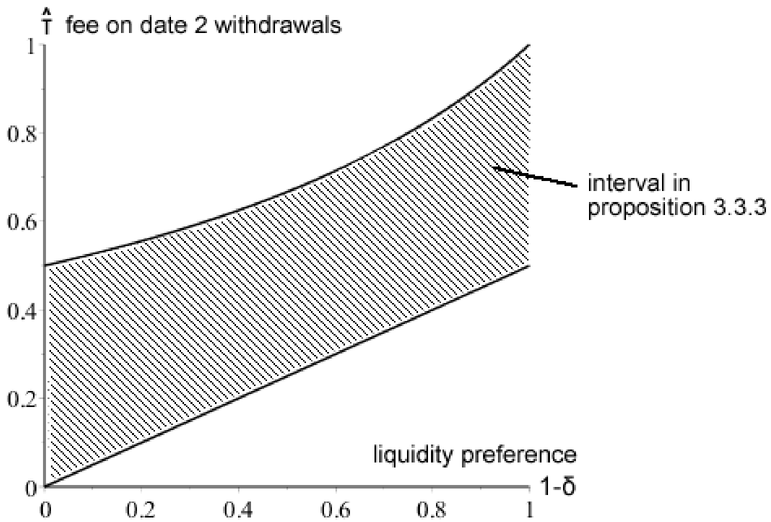

Figure 4: Set of date 2 redemption fees that prevent runs

The shaded area in figure 4 depicts the interval within which the contingent fee on date 2 withdrawals $\hat{\tau}$ eliminates run equilibria according to proposition 3.3.[21] The upper bound on $\hat{\tau}$ is related to type 1 and 2 depositors' incentive constraint; the fee $\hat{\tau}$ must be such that a high enough number of type 1 and 2 depositors are willing to pay the fee on date 2 withdrawals, should the fee be activated. The higher liquidity preference, the higher the willingness of type 1 and 2 depositors to pay a fee on date 2 withdrawals instead of waiting until date 3. For this reason, the upper bound on $\hat{\tau}$ is increasing in liquidity preference. The lower bound on $\hat{\tau}$ is related to the fact that the fund's date 2 cash-inflow is limited; if the fee is set below the lower bound, then the number of depositors who wish to withdraw at date 2 after a run occurred at date 1 may be such that the fund's date 2 cash-inflow is not sufficient to pay them all out. The higher liquidity preference, the higher is the desire by type 1 and 2 depositors to consume at date 2 rather than date 3. Higher liquidity preference therefore requires that the fund charge a higher fee on date 2 withdrawals to make sure that demand for date 2 withdrawals never exceeds the fund's date 2 cash-inflow.

[21]Note that interval in proposition 3.3 gives a sufficient, but not necessary condition to prevent run equilibria. Fees which do not satisfy condition (14) may also prevent run equilibria, depending on the cost of withdrawing early κ.

3.4. Discussion of Model Assumptions

One simplifying assumption in this paper is that depositors' preferences are linear. In the original Engineer (1989) model, ex-post preferences are of the type $u(c_1 + \delta c_2 + \delta^2 c_3)$ for type 1 depositors, $u(c_2 + \delta c_3)$ for type 2 depositors and $u(c_3)$ for type 3 depositors, where $u(\cdot)$ is a strictly increasing and strictly concave utility function. Preferences of this form have also been used in the setting with two types, e.g. in Wallace (1988).[22] Assuming non-linear preferences of the form above would not change the analysis in a fundamental way, especially since the marginal rate of substitution between consumption at different dates is still constant. However, whenever facing a choice between a deterministic and an uncertain payoff, depositors' choice would be tilted towards the deterministic option. The main effect of this is that it increases depositors' propensity to run at date 1. The decision whether or not to run at date 1 often entails a choice between a deterministic payoff resulting from withdrawing the deposit at date 1 and an uncertain payoff when remaining in the fund. For instance, with the suspension policy of subsection 3.1, depositors who remain in the fund face the risk of not being able to withdraw at date 2. With the combined policy of subsection 3.3, depositors who remain in the fund face the risk of having to pay a fee on date 2 withdrawals. This shows (not surprinsingly) that the thresholds for run equilibria to exist are sensitive to changes in the specification of depositors' preferences. The advantage of using linear preferences is that it allows for a very tractable analysis of the main trade-offs involved, generating a number of qualitative insights regarding depositors' propensity to run under fees and gates -clauses.

Another simplifying assumption is that true liquidity needs at the fund level are known. It is debatable how controversial this assumption is given the large size of most money market funds and the fact that liquidity shocks are plausibly uncorrelated among a fund's investors. The case with unknown aggregate liquidity needs has been studied extensively in settings with two types of depositors ('patient' and 'impatient').[23] Assuming that aggregate liquidity needs are unknown

[22]In Diamond and Dybvig (1983), impatient consumers attach zero value to consumption in the last date. Jacklin (1987) studies a Diamond-Dybvig setting with a non-constant marginal rate of substitution between consumption at different dates.

[23]See Green and Lin (2003), Ennis and Keister (2009b) and Andolfatto et al. (2017) for settings with a finite number of depositors and Sultanum (2014) for a setting with a continuum of depositors. Interestingly, Andolfatto et al. (2017) show how a payout policy that combines suspension with a decreasing payout schedule (that could be interpreted as charging redemption fees) uniquely implements the efficient allocation in a setting with two types and a stochastic distribution of types.

ex ante would change the analysis rather fundamentally, not least because the derivation of the benchmark allocation that is to be uniquely implemented (e.g. the best implementable allocation under a sequential service constraint) would be much more involved. An analysis of the Engineer setting under aggregate uncertainty needs to be left for future research.

Throughout the paper, I implicitly assumed that the fund can commit to its payout policy f. This assumption is important because policies that prevent run equilibria under commitment often entail measures that hurt (some) depositors after (off-equilibrium) a run occurred, as highlighted by Ennis and Keister (2009a, 2010). For instance, all payout policies studied in this paper are such that type 1 depositors who arrive late in line in a run at date 1 receive a lower payoff than the type 3 depositors left in the fund. If the fund maximizes expected utility of all depositors left in the fund at any point in time, such payout policies may not be time consistent. In the present setting, the issue of time consistency is mitigated by the fact that depositors' preferences are linear; a benevolent fund does not care per se about redistribution between depositors of different types.

4. Conclusion

This paper derives a number of qualitative results regarding the effectiveness of fees and gates-clauses in preventing runs. The paper also shows how the Engineer (1989) model can provide a rich but still sufficiently simple framework for policy analysis. The results derived in the paper suggest that the fees and gates- clauses recently added to US and EU MMF regulations will be effective in removing the first-mover advantage at some, but not all, money market funds. The tools available to MMFs under the current regulations are more likely to be effective in preventing runs on funds that are invested in assets for which there are relatively deep secondary markets allowing the fund to meet even large redemptions without having to liquidate assets at prices that are too far below fundamental value. The paper also shows how funds that are invested in relatively illiquid assets could use redemption fees and gates to remove the first-mover advantage without ever having to resort to asset liquidations. Implementing this would however require to give money market funds more flexibility how they can use fees and gates.

The focus on unique implementation of the first-best allocation implies that all types of run equilibria are treated as being equally bad. An interesting question is whether some types of runs cause higher welfare losses than others, both for fund investors and for society at large. For instance, it may be the case that Diamond-Dybvig type run equilibria that cause investors (who cannot redeem quickly enough) to incur credit losses lead to higher expected welfare losses for fund investors compared to deposit-access panics. Given that MMFs may not be able to eliminate all run equilibria with the tools currently at their disposal, they may need to choose which types of runs they want to avoid most. The same can be said for society at large. Current EU (but not US) regulations oblige MMFs to impose fees or gates under certain conditions. Presumably, the rationale behind the EU regulation is that interests of MMF owners or -investors are different from the interests of society at large when it comes to imposing restrictions on redemptions. This is often based on the notion that large assets liquidations cause negative externalities ('pecuniary externalities') that justify government intervention. Deposit-access panics do not lead to asset liquidations and may therefore be less of a concern from a societal perspective.

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Appendix

A. Proof of Proposition 3.1

First, it is useful to note that any strategy profile where all depositors' strategies satisfy the following implements first-best:

$$\begin{array}{lll} \text{Date 1:} & s_1(1,1) = \text{‘withdraw’} & s_1(not1,1) = \text{‘not withdraw’} \\ \text{Date 2:} & s_2(2,1) = \text{‘withdraw’} & s_2(3,1) = \text{‘not withdraw’} \\ \text{Date 3:} & s_3(3,1) = \text{’withdraw’} & \end{array} \tag{7}$$

If all depositors' strategies satisfy (7) then, at date 1, all type 1 depositors (and no other depositors) withdraw one unit from the fund; all type 2 depositors (and no other depositors) withdraw one unit from the fund at date 2; and finally, all type 3 depositors withdraw one unit from the fund at date 3. The following result implies that, when looking for run equilibria under suspension policy (2), we can limit attention to equilibria in which non-type 1 depositors withdraw at date 1:

Lemma A.1. *The withdrawal game under the suspension policy (2) uniquely implements first-best if and only if the withdrawal game does not exhibit an equilibrium in which non-type 1 depositors withdraw at date 1.*

Proof of Lemma A.1: The "only if" part in lemma A.1 follows directly from the definition of first-best. To prove the "if"-part, we restrict attention to strategy profiles where non-type 1 depositors play $s_1(\text{not}1, 1) =$‘not withdraw’ with probability one and then eliminate strictly dominated strategies. First, withdrawing is the strictly dominant strategy for type 1 depositors at date 1 (playing $s_1(1, 1) =$‘withdraw’ is strictly dominant). Furthermore, at date 2, withdrawing is the strictly dominant strategy for type 2 depositors (playing $s_2(2, 1) =$‘withdraw’ is strictly dominant) and not withdrawing is the strictly dominant strategy for type 3 depositors (playing $s_2(3, 1) =$‘not withdraw’ is strictly dominant). The reason for the latter is that type 3 depositors can always withdraw 1 unit at date 3. Finally, at date 3, withdrawing is the strictly dominant strategy for type 3 depositors (playing $s_3(3, 1) =$‘withdraw’ is strictly dominant). Given that we impose $s_1(\text{not}1, 1) =$‘not withdraw’, all strategy profiles that survive elimination of strictly dominated strategies therefore satisfy (7) and implement first-best. ■

It will be convenient to denote by b the fraction of type 1 depositors among the depositors left in the fund at date 2. Among the non-type 1 depositors left in the fund at date 2, $\frac{1}{2}$ will turn out to be of type 2. Hence the fraction of type 1 and 2 depositors among the depositors in the fund at date 2 equals $b + \frac{1}{2}(1-b) = \frac{1}{2}(1+b)$. Since withdrawing at date 2 is a strictly dominant strategy for type 1 and 2 depositors while not withdrawing at date 2 is a strictly dominant strategy for type 3 depositors, the fraction of depositors that try to withdraw at date 2 equals the fraction of type 1 and 2 depositors left in the fund. The fund receives one-half of its remaining cash-flow at date 2 and the other half at date 3. This implies that the fund can pay out a fraction $\frac{1}{2}\left[\frac{1}{2}(1+b)\right]^{-1} = \dfrac{1}{1+b}$ of the type 1 and 2 depositors left in the fund at date 2. Hence type 2 depositors left in the fund at date 2 can withdraw at date 2 with probability $\dfrac{1}{1+b}$ while, with probability $\dfrac{b}{1+b}$, they can only withdraw at date 3 in which case they receive a payoff of δ.

Suppose now that all depositors (type 1 and non-type 1) try to withdraw at date 1. In this case, only depositors with indices $i_1 \leqslant \frac{1}{3}$ can withdraw at date 1 while the remaining depositors are still left in the fund at date 2. The fraction of type 1, 2 and 3 depositors among those left in the fund at date 2 is then $\frac{1}{3}$ each. Hence we have $b = \frac{1}{3}$, which constitutes an upper bound on b in equilibrium. Whenever non-type 1 depositors play $s_1(not1, 1)$=‘withdraw’ with a probability of less than one, then more of the type 1 depositors can withdraw at date 1 and b will be lower. Denote $Z(b)$ as the expected payoff of a non-type 1 depositor left in the fund at date 2:

$$Z(b) = \overbrace{\frac{1}{2}}^{\substack{\text{probability of}\\ \text{being type 2}}} \overbrace{\left[\frac{1}{1+b}1 + \frac{b}{1+b}\delta\right]}^{\substack{\text{expected payoff}\\ \text{type 2}}} + \overbrace{\frac{1}{2}}^{\substack{\text{probability of}\\ \text{being type 3}}} \overbrace{1}^{\substack{\text{payoff}\\ \text{type 3}}} \tag{8}$$

Playing $s_1(\text{not1}, 1)$ =‘withdraw’ is the best response for a non-type 1 depositor iff $1-\kappa \geqslant Z(b)$. In a hypothetical situation where all other depositors play $s_1(\cdot, 1)$ =‘withdraw’ doing the same is the best response iff $1-\kappa \geqslant Z(\frac{1}{3})$. Rewriting the previous inequality gives $\delta \leqslant 1-8\kappa$. Hence if $\delta \leqslant 1-8\kappa$, then the withdrawal game exhibits an equilibrium where all depositors play $s_1(\cdot, 1)$ =‘withdraw’ and first-best is not implemented. Suppose now that $\delta > 1-8\kappa$. Then we have that $1 - \kappa < Z(\frac{1}{3})$. Since $Z(b)$ is decreasing in b, and $b = \frac{1}{3}$ is the upper bound on b in equilibrium withdrawing at date 1 is never the best response for non-type 1 depositors (playing $s_1(not, 1)$ ='withdraw' is never the best response), independent of how many other non-type 1 depositors withdraw at date 1. It follows that the withdrawal game does not exhibit an equilibrium where non-type 1 depositors withdraw at date 1 and, by lemma A.1, first-best is uniquely implemented.

B. Proof of Proposition 3.2

When looking for run equilibria under a fee $\tau \geqslant 1 - \lambda$, we can again limit attention to equilibria in which non-type 1 depositors withdraw at date 1:

Lemma B.1. *The withdrawal game under the fee policy (4) with a fee $\tau \geqslant 1-\lambda$ uniquely implements first-best if and only if the withdrawal game does not exhibit an equilibrium in which non-type 1 depositors withdraw at date 1.*

The proof of lemma B.1 is very similar to the proof of lemma A.1 and is left out. When looking for fee policies (4) that prevent run equilibria, it is without loss of generality to restrict attention to fee policies with $\tau = 1 - \lambda$. For the reasons described in the main text, this is the lowest fee that can potentially prevent runs. To see why restricting attention to a fee $\tau = 1 - \lambda$ entails no loss of generality, suppose that a fee $\tau = 1-\lambda$ does *not* prevent a run equilibrium at date 1. The only effect of increasing the fee is to decrease type 1 depositors incentive to pay the fee at date 1 in case a run occurs. Increasing τ to a lever higher than $1 - \lambda$ thus increases the number of type 1 depositors in the fund at date 2 after a run occurred at date 1, which increases non-type 1 depositors incentive to participate in a run at date 1 compared to a fee $\tau = 1 - \lambda$. For the remainder of this section, I assume the fee equals $\tau = 1 - \lambda$.

Following the notation in appendix A, I denote by b the share of type 1 depositors among the depositors left in the fund at date 2. Furthermore, I denote by $Z(b)_{[\lambda \geqslant \delta]}$ and $Z(b)_{[\lambda < \delta]}$ the expected payoff of *not* withdrawing at date 1 for a non-type 1 depositor, given that $\lambda \geqslant \delta$ and $\lambda < \delta$ respectively. As described in appendix A, the fraction of type 1 and 2 depositors among all depositors left in the fund at date 2 equals $\frac{1}{2}(1 + b)$. An individual type 1 or 2 depositor left in the fund at date

2 can withdraw 1 unit at date 2 with probability $\frac{1}{1+b}$. With probability $\frac{b}{1+b}$, the depositor will arrive late in line at date 2 and can either withdraw $1 - \tau = \lambda$ units at date 2 or wait until date 3 and then withdraw 1 unit at date 3. If $\lambda \geqslant \delta$, paying the fee on date 2 withdrawals is a dominant strategy for type 1 and 2 depositors. If $\lambda < \delta$, *not* paying the fee on date 2 withdrawals is the strictly dominant strategy for type 1 and 2 depositors. We thus have:[24]

$$
\begin{aligned}
Z(b)_{[\lambda \geqslant \delta]} &= \frac{1}{2} \overbrace{\left[\frac{1}{1+b} 1 + \frac{b}{1+b} \boldsymbol{\lambda} \right]}^{\substack{\text{expected payoff} \\ \text{type 2}}} + \frac{1}{2} \overbrace{1}^{\substack{\text{payoff} \\ \text{type 3}}} \\
Z(b)_{[\lambda < \delta]} &= \frac{1}{2} \underbrace{\left[\frac{1}{1+b} 1 + \frac{b}{1+b} \boldsymbol{\delta} \right]}_{\substack{\text{expected payoff} \\ \text{type 2}}} + \frac{1}{2} \underbrace{1}_{\substack{\text{payoff} \\ \text{type 3}}}
\end{aligned}
\tag{9}
$$

To examine whether the withdrawal game exhibits a run equilibrium at date 1, we need to look at best responses of non-type 1 depositors at date 1. Withdrawing at date 1 is the best response for non-type 1 depositors iff the payoff of withdrawing 1 unit at date 1 (and incurring the storage cost κ) is higher than the expected payoff of remaining in the fund. This is the case iff $1 - \kappa \geqslant Z(b)$. The number of type 1 depositors left in the fund (b) is itself an equilibrium quantity. In order to determine the values of b that can be part of an equilibrium, we need to look at best responses of type 1 depositors at date 1. Analogous to the expected payoff $Z(b)$ for non-type 1 depositors, it will be useful to denote by $Y(b)$ the expected payoff for a *type 1* depositor of not withdrawing at date 1, given that the fraction of type 1 depositors left in the fund at date 2 will be b. It is useful to distinguish the following three cases:

Case 1: $\lambda \geqslant \delta$

Consider first the case with strict inequality, $\lambda > \delta$. In this case, type 1 depositors strictly prefer consuming λ units at date 1 to consuming 1 unit at date 2. Hence withdrawing at date 1 (with or without paying the fee) is a strictly dominant strategy for type 1 depositors. This implies that $b = 0$ in every equilibrium of the withdrawal game with $\lambda > \delta$. Consider next the case $\lambda = \delta$. In this

[24] If $\lambda = \delta$, then type 2 depositors are indifferent between paying the fee at date 2 (and consuming λ units at date 2) or waiting until date 3 (and consuming 1 unit at date 3). In both cases the payoff will be equal to λ.

case, type 1 depositors are indifferent between consuming λ units at date 1 and consuming 1 unit at date 2. If $b > 0$, then some of the type 1 depositors left in the fund at date 2 will not be able to withdraw 1 unit at date 2. The expected payoff of remaining in the fund until date 2 for a type 1 depositor is thus strictly less than δ whenever $b > 0$. It follows that, given any $b > 0$, type 1 depositors strictly prefer to withdraw at date 1. This implies that $b = 0$ in every equilibrium of the withdrawal game with $\lambda = \delta$. Since $Z(0)_{[\lambda \geqslant \delta]} > 1 - \kappa$, there is no equilibrium of the withdrawal game where non-type 1 depositors withdraw at date 1, given $\lambda \geqslant \delta$. Combined with lemma B.1, this proves item (i) in proposition B.

Case 2: $\lambda \in \left(\frac{3}{4}\delta + \frac{1}{4}\delta^2,\, \delta\right)$

A type 1 depositor left in the fund at date 2 is able to withdraw 1 unit at date 2 with probability $\dfrac{1}{1+b}$, in which case she receives a payoff of δ. With probability $\dfrac{b}{1+b}$, the depositor will arrive late in line at date 2 and will not be able to withdraw 1 unit at date 2. Since $\lambda < \delta$, a type 1 depositor's best response, should she arrive late in line at date 2, is to wait until date 3 and withdraw 1 unit at date 3. In this case she receives a payoff of δ^2. The expected payoff of a type 1 depositor in the fund at date 2 thus equals:

$$Y(b) = \frac{1}{1+b}\,\delta + \frac{b}{1+b}\,\delta^2 \tag{10}$$

Note that $Y(b)$ is strictly decreasing in b. Paying the fee on date 1 withdrawals is type 1 depositors' best response iff $\lambda \geqslant Y(b)$. Some algebra yields that:

$$Y(\tfrac{1}{3}) < \lambda < Y(0) \quad \Leftrightarrow \quad \lambda \in \left(\frac{3}{4}\delta + \frac{1}{4}\delta^2,\, \delta\right) \tag{11}$$

Consider now a scenario where all non-type 1 depositors run at date 1, in which case $\frac{2}{3}$ of type 1 depositors will be in the last $\frac{2}{3}$ of the line and hence cannot withdraw 1 unit at date 1; if none of them pay the fee on date 1 withdrawals (all of them remain in the fund until date 2), then we have $b = \frac{1}{3}$ (see the discussion in appendix A.) If all of them pay the fee and leave the fund at date 1, then we have $b = 0$. Since $Y(b)$ is strictly decreasing in b, it follows from (11) that there is a unique $b^* \in (0, \frac{1}{3})$ such that $\lambda = Y(b^*)$. Solving $Y(b^*) = \lambda$ for b^* yields $b^* = \dfrac{\delta - \lambda}{\lambda - \delta^2}$. Note that there is no equilibrium of the withdrawal game with $b > b^*$; if $b > b^*$, then type 1 depositors strictly prefer to pay the fee and leave the fund at date 1 which would imply $b = 0$.

Going back to best responses of non-type 1 depositors at date 1, withdrawing at date 1 is non-type 1 depositors best response iff the payoff of withdrawing 1 unit at date 1 (and incurring the storage cost κ) is higher than the expected payoff of remaining in the fund until date 2. This is the case iff $1-\kappa \geqslant Z_{[\lambda<\delta]}(b)$. Since $Z_{[\lambda<\delta]}(b)$ is strictly decreasing in b, and $b \leqslant b^*$ in equilibrium, there is no equilibrium in which non-type 1 depositors withdraw at date 1 if:

$$1-\kappa < Z_{[\lambda<\delta]}(b^*) \quad \Leftrightarrow \quad 1-\kappa < \frac{1}{2}\left(1+\frac{\lambda}{\delta}\right) \tag{12}$$

Together with lemma B.1, this means that first-best is uniquely implemented whenever condition (12) is fulfilled. I will now show that the converse is also true: If condition (12) is not fulfilled, then the withdrawal game exhibits a run equilibrium where all depositors (try to) withdraw at date 1. To see this, suppose that all depositors play $s_1(\cdot, 1)$ = 'withdraw'. Suppose further that type 1 depositors play a mixed strategy where they play $s_1(1, \lambda)$ = 'withdraw' with probability $p = \dfrac{3\delta+\delta^2-4\lambda}{\delta^2+\delta-2\lambda}$ and they play $s_1(1, \lambda)$ = 'not withdraw' with probability $1-p$. Note that $p \in (0,1)$ if $\lambda \in (\frac{3}{4}\delta + \frac{1}{4}\delta^2, \delta)$. Given that all depositors run, a fraction p of the type 1 depositors with indices $i_1 > \frac{1}{3}$ will thus pay the fee and leave the fund at date 1. The fraction of type 1 depositors among all depositors left in the fund at date 2 then equals $b = \dfrac{\frac{1}{3}(1-p)}{\frac{2}{3}+\frac{1}{3}(1-p)} = b^*$.

Note first that, if condition (12) is not fulfilled, then playing $s_1(\text{not}1, 1)$ = 'withdraw' is indeed a best response for non-type 1 depositors. Second, since $\lambda = Y(b^*)$, type 1 depositors are indifferent between paying the fee at date 1 and waiting until date 2. Hence their mixing strategy constitutes a best response as well. It follows that, if condition (12) is not fulfilled, the withdrawal game exhibits a run equilibrium which completes the proof of condition (ii) in proposition 3.2.

Case 3: $\lambda \leqslant \frac{3}{4}\,\delta + \frac{1}{4}\,\delta^2$

From appendix A we know that, if all depositors run at date 1 (all depositors play $s_1(\cdot, 1)$ = 'withdraw'), and none of the type 1 depositors with indices $i_1 > \frac{1}{3}$ pay the fee at date 1 (all depositors play $s_1(1, \lambda)$ = 'not withdraw'), then we have $b = \frac{1}{3}$ which is an upper bound on b in equilibrium.

Not paying the fee on date 1 withdrawals (playing $s_1(1, \lambda) =$ 'not withdraw') is type 1 depositors' best response iff $\lambda < Y(b)$. We have that:

$$\lambda \leqslant Y\left(\tfrac{1}{3}\right) \quad \Leftrightarrow \quad \lambda \leqslant \frac{3}{4}\,\delta + \frac{1}{4}\,\delta^2 \tag{13}$$

Recall that $Y(b)$ is strictly decreasing in b. Hence for any $b < \frac{1}{3}$ we have $\lambda < Y(b)$. Whenever some type 1 depositors pay the fee on date 1 withdrawals, then we have $b < \frac{1}{3}$ so that type 1 depositors strictly prefer not to pay the fee on date 1 withdrawals. Hence there is no equilibrium of the withdrawal game in which type 1 depositors pay the fee on date 1 withdrawals. We have already established that no depositor ever pays the fee on date 2 withdrawals in equilibrium if $\lambda < \delta$. Hence no depositor ever pays the fee on withdrawals, neither at date 1 nor at date 2. The policy is then equivalent to the suspension policy in (2) and it prevents a run if and only if the suspension policy prevents a run. Together with proposition 3.1, this proves condition (iii) in proposition 3.2.

C. Proof of Proposition 3.3

We again start with the following result which shows that, when looking for run equilibria, we can limit attention to equilibria where non-type 1 depositors withdraw at date 1:

Lemma C.1. *The withdrawal game under the combined policy with some fee $\widehat{\tau} \geqslant 0$ uniquely implements first-best if and only if the withdrawal game does not exhibit an equilibrium in which non-type 1 depositors withdraw at date 1.*

The proof of lemma C.1 is very similar to the proof of lemma A.1 and is left out. Given that the fund suspended convertibility at date 1, date 3 payouts (6) can be expressed as a strictly increasing function $f_3(\overline{z}_2)$. Some algebra yields that:

$$\delta\, f_3\left(\frac{1}{3}\right) < 1 - \widehat{\tau} < \delta\, f_3\left(\frac{1}{3(1-\widehat{\tau})}\right) \quad \Leftrightarrow \quad \widehat{\tau} \in \left(\frac{1-\delta}{2}, \frac{1}{1+\delta}\right) \tag{14}$$

To proof proposition 3.3 we can proceed in four steps. In steps 1-4, we assume the fund follows the combined payout policy and sets the fee $\widehat{\tau}$ within the interval given by (14).

Step 1: There is no equilibrium of the withdrawal game where the fund suspends convertibility at both dates 1 and 2.

Proof: Note first that only type 1 and 2 depositors may withdraw at date 2 in equilibrium since not withdrawing at date 2 is the strictly dominant strategy for type 3 depositors. Suppose now the fund suspends convertibility at date 1 and then again suspends convertibility at date 2. By (5) this implies $\overline{z}_2 \geqslant \dfrac{1}{3(1-\widehat{\tau})}$. Since $1-\widehat{\tau} < \delta f_3 \left(\dfrac{1}{3(1-\widehat{\tau})}\right)$ all date 1 and 2 depositors left in the fund then strictly prefer *not* to withdraw at date 2. Hence $\overline{z}_2 = 0$ which leads to a contradiction.

Step 2: There is no equilibrium of the withdrawal game where the fund suspends convertibility at date 1 <u>and</u> the number of depositors that withdraw at date 2 satisfies $\overline{z}_2 < \frac{1}{3}$.

Proof: Note first that, in equilibrium, at least half of the depositors left in the fund at date 2 will be of either type 1 or 2, implying that the measure of type 1 and 2 depositors in the fund at date 2 is at least $\frac{1}{3}$. Suppose now the fund suspends convertibility at date 1 and the number of depositors that withdraw at date 2 satisfies $\overline{z}_2 < \frac{1}{3}$. Since $1-\widehat{\tau} > \delta f_3\left(\frac{1}{3}\right)$, all type 1 and 2 depositors then strictly prefer withdrawing at date 2 over withdrawing at date 3. Hence we have $\overline{z}_2 \geqslant \frac{1}{3}$, which leads to a contradiction.

Step 3: There is no equilibrium of the withdrawal game where the fund suspends convertibility at date 1.

Proof: Suppose there is an equilibrium in which the fund suspends convertibility at date 1. This implies that at least some non-type 1 depositors run on the fund at date 1. The expected payoff for a non-type 1 depositor of *not* running at date 1 equals $\frac{1}{2}(1-\widehat{\tau}) + \frac{1}{2} f_3(\overline{z}_2)$ which is weakly larger than 1 if $\overline{z}_2 \geqslant \frac{1}{3}$. By step 2, we have $\overline{z}_2 \geqslant \frac{1}{3}$ which implies that non-type 1 depositors are strictly better of not running at date 1 (for any $\kappa > 0$) so that we arrive at a contradiction.

Step 4: There is no equilibrium of the withdrawal game where non-type 1 depositors withdraw from the fund at date 1.

Proof: Suppose there is an equilibrium where non-type 1 depositors withdraw at date 1. Then withdrawing at date 1 is a best response for non-type 1 depositors. If withdrawing at date 1 is a best response for non-type 1 depositors, then withdrawing at date 1 is the strictly best response for type 1 depositors. Hence the measure of depositors who try to withdraw from the fund at date 1 is strictly higher than $\frac{1}{3}$ which means that the fund suspends convertibility at date 1. By step 3, there is no equilibrium in which the fund suspends convertibility at date 1, so that we arrive a contradiction.

Step 4 together with lemma C.1 completes the proof of proposition 3.3.