Estimation of conditional tail expectation at extreme levels for time series data

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Abstract

We consider the estimation of the conditional expectation $\mathbb{E}(X_h|X_0 > U_X(1/p))$, provided $\mathbb{E}|X_0| < \infty$, at extreme levels, where $(X_t)_{t\in\mathbb{Z}}$ is a strictly stationary time series, U_X its tail quantile function, h is a positive integer and $p \in (0,1)$ is such that $p \to 0$. We use the multivariate regular variation framework and consider initially the case of non-negative time series. A two-step method is used in order to propose an estimator of this risk measure: first, by introducing an estimator in the intermediate case and, then, by extrapolating outside the data by a Weissman-type construction. Under suitable assumptions, we prove the weak convergence of the estimator of this risk measure. Subsequently, we extend our approach to the case of real-valued time series by using the decomposition of the original time series into the positive and negative parts and we prove again the weak convergence of the proposed estimator under additional assumptions. The asymptotic variance of this estimator being difficult to approximate, we show the consistency of the multiplier block bootstrap in our context and use it to construct confidence interval for $\theta_{h,p}$. Finally, the finite sample properties of the estimator are evaluated with a simulation study and the methodology is illustrated on a dataset of daily precipitation measurements.

Keywords: Time series; Extreme value theory; Risk measures; Empirical process.

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