English Abstract

The current best description of high-energy physics, the Standard Model, is a quantum field theory and an incredibly successful one at that. In quantum field theories, the renormalization group is an integral part, describing how theories evolve with changes in energies. To make perturbative predictions across different energy scales, it is necessary to account for the running of the couplings of the theory, which in turn is determined by the β-functions. As such, the β-functions play an important role in both precision physics and the search for models of new physics with a controlled behavior at high energies; they are the primary focus of this thesis. Due to the challenges presented by perturbation theory, they are, however, only known to a few loop orders in general four-dimensional theories.

Using large-N_f methods, we study gauge-Yukawa theories in the presence of a large number N_f of charged fermions. By expanding quantities in these theories in powers of $1/N_f$, it is possible to overcome some of the usual challenges of the perturbative method and calculate contributions to all loop orders in closed-form. We calculate the large-N_f β-functions for all couplings of theories with a semi-simple gauge group in a manner that is applicable to any model-building effort based on large-N_f gauge-theories.

With the renormalization group causing theories to change behavior as they flow from the ultraviolet to the infrared, a natural question is if there are any inherent restrictions on possible flows. The strong A-theorem posits that there exists a function A that decreases monotonically along the flow. A is closely tied to the Weyl anomaly, which, when probed in the framework of the local renormalization group, is found to satisfy consistency conditions, the most well-known of which is Osborn's equation. We calculate the well-motivated A-function of Osborn's equation to leading order in the large-N_f expansion in gauge-fermion theories. It is shown to loose monotonicity for strong couplings in the minimal scheme contrary to the behavior observed in perturbation theory. This result indicates that there is a need for another definition of the A-function or, at least, a way to choose an appropriate scheme for the curved space counterterms.

The last part of the thesis pivots to the study of Osborn's equation as a tool for extracting Weyl consistency conditions on β-function coefficients in generic gauge-Yukawa theories. We set up a compact formalism for such theories, allowing us to parametrize the β-functions up to the 4-loop order for gauge, the 3-loop order for Yukawa, and the 2-loop order for quartic couplings in terms of 629 coefficients. The coefficients are in turn found to be constrained by a total of 297 consistency conditions. On the basis of these results, we extract the completely general 3-loop gauge β-function. Furthermore, they are demonstrated to completely determine the part of the 4-loop gauge β-function that is ambiguous in dimensional regularization due to the presence of γ_5.