Abstract

Quantum field theory is the mathematical framework that most successfully unifies special relativity and quantum mechanics. It describes the physics of elementary particles, the building blocks of our universe. The well-established quantum field theory that accommodates all particles discovered so far is the Standard Model of particle physics. Most of its theoretical predictions are in excellent agreement with all the experimental tests run so far. Nonetheless, we lack a theoretical understanding of the whole framework in non-perturbative regimes, and we are mostly restricted to make predictions in which we assume quantum fluctuations to be small. It is possible to get insight into non-perturbative physics using theoretical tools such as large-N limits and supersymmetry.

In this dissertation I will explore a particular limit of gauge theories, models that allow us to describe fundamental interactions. This limit is realised when gauge theories include a large number of fermion flavours, and is thus known as the large-N_f limit. We will explore whether certain non-perturbative features can arise within these models, such as the presence of ultraviolet completions. The novelty of this understudied limit of four-dimensional quantum field theories is that it can produce closed-form results in coupling constants for very general models resembling the Standard Model in structure. We present novel results for renormalisation group β-functions in this context.

We will then apply the large-N_f expansion to test the strong version of the a-theorem in (non-)Abelian gauge theories, which is conjectured to hold also beyond perturbation theory. This theorem formalises a key idea of the renormalisation group flow: degrees of freedom are irreversibly integrated out when flowing from the ultraviolet to the infrared. This could be used in principle to test whether the large-N_f ultraviolet completions are trustable. Interestingly, the theorem appears to be violated at the leading order in the expansion when the 't Hooft coupling becomes large. We will discuss the implications and possible resolutions of this problem. Finally, we will show that large-order methods can be efficiently applied to extend our analysis to the higher orders in the $1/N_f$ expansion, where no closed form result can be obtained.