Abstract

An online problem is a problem where an algorithm has to make irrevocable decisions without knowing the whole input instance. In the advice complexity model, the algorithm is allowed to learn the value of any function of the whole input. This value is called 'advice'. In most of this thesis, we study the trade-off between the length of the advice an algorithm receives and the quality of the solution it can output.

A large part of this thesis concerns the class AOC which contains maximization (minimization) accept/reject problems, where the following holds:

- The profit (cost) of a feasible solution is the number of accepted requests.
- A subset (superset) of an optimal solution is still feasible.

Let

$$B_c = \log\left(1 + (c-1)^{c-1}/c^c\right).$$

We show a c-competitive algorithm which works for every problem in this class and reads $B_c n + O(\log n)$ advice bits. For some problems in AOC we give a lower bound of $B_c n - O(\log n)$ advice bits for being c-competitive (we call these AOC-complete problems). We show that Online Independent Set, Online Dominating Set, Online Vertex Cover, Online Set Cover, Online Disjoint Path Allocation, and Online Cycle Finding are all AOC-complete. We show that the 'Maximum Induced Subgraph With Hereditary Property' problem is almost complete for AOC, independent of the property: A c-competitive algorithm needs at least $B_c n - O(\log^2 n)$ advice bits. For the dual minimization problem, the number of advice bits varies a lot depending on the property. For some, a c-competitive algorithm needs $B_c n + O(\log n)$ advice bits. For others, an algorithm can be 1-competitive with $O(\log n)$ advice bits. Continuing in this direction, we investigate what happens when the problems in AOC are weighted. Again, maximization and minimization problems behave quite differently. For the maximization problems, roughly the same number of advice bits is required to be c-competitive as in the unweighted case. The minimization problems, however, require many more advice bits. Here, $n - O(\log n)$ bits of advice are required to be $f(n)$-competitive for any function, f.

The main contributions in the thesis, which are not related to AOC, are:

- For the Online Search Problem, $(M/m)^{\frac{1}{2^b+1}}$ bits of advice are necessary and sufficient for a c-competitive algorithm.
- Deciding the Online Chromatic Number of a graph with a pre-coloring is PSPACE-complete.
- The greedy algorithm is online optimal for Online Independent Set when the graph has a sufficient number of isolated vertices.