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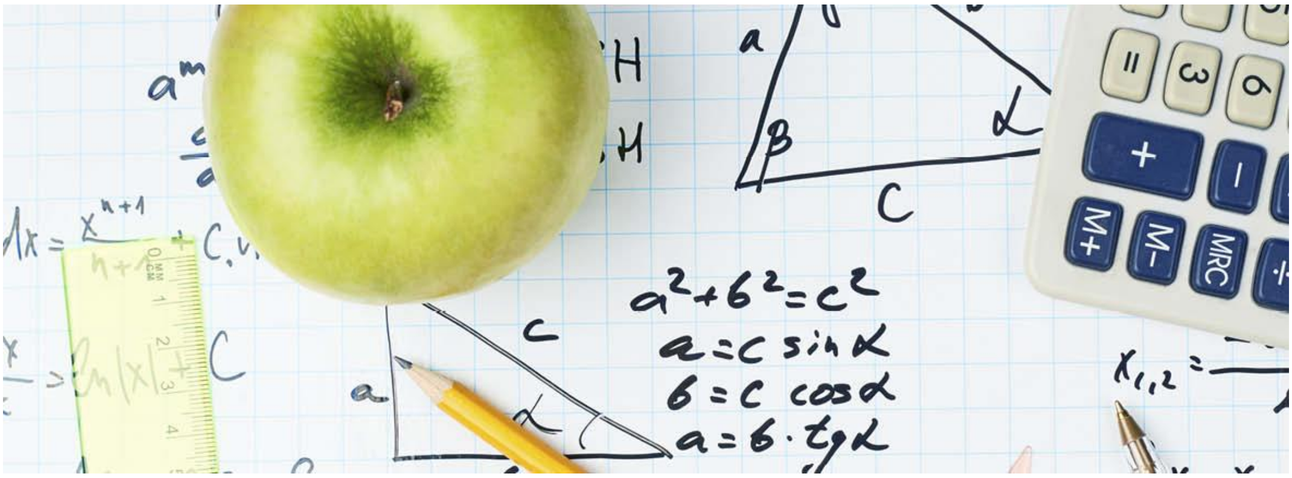

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Identification of the timing-of-events model with multiple competing exit risks from single-spell data

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Abstract

The (single-spell) identification result of the timing-of-events model by Abbring and Van den Berg (2003b) is extended to a model with several competing exit risk equations. The extended model can be used for example to simultaneously identify the different effects a benefit sanction has on the rate to find work and the rate to leave the labor force. A flexible dependence structure between competing exit risks and the duration until entry into treatment accounts for selection effects caused by unobserved characteristics of the job searcher.

Keywords: Competing risks, Timing-of-events, Mixed proportional hazard model, Program evaluation, Unobserved heterogeneity, Identification.

JEL-codes: C14, C31, C41, J64.

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1 Introduction

This note combines two popular multivariate duration models: the timing-of-events model (see Abbring and Van den Berg, 2003b) and the mixed proportional hazard (MPH) competing risks model (see Abbring and Van den Berg, 2003a). The former approach exploits the timing of events to identify the causal effect of an endogenous treatment time on the subsequent rate to exit the state of interest. Its most popular field of application is to evaluate the effect of labor market programs or benefit sanctions on the subsequent rate to find work (see, e.g. Van den Berg et al., 2004; Rosholm and Svarer, 2008). Here, endogeneity of the time to treatment is a common problem, since unobserved characteristics of the job searcher such as motivation and skill level simultaneously affect the speed of entering into a labor market program and the rate of finding work.

Besides the transition to (regular) employment, exits due to other reasons are often observed in the data such as transitions to temporary employment or exits from the labor force. It is common practice to assume that these alternative exits are independent of the exit of interest conditional on covariates and thus can be conveniently dealt with through right-censoring (see, e.g. Van den Berg et al., 2004). However, this assumption is often violated since unobserved characteristics usually have a simultaneous effect on all exit risks; e.g. motivation or skill level of a job searcher simultaneously affect his rate to find regular or temporary employment or to exit the labor force.

On this account, this note extends the timing-of-events identification result of Abbring and Van den Berg (2003b) to a more general model that accounts for the different effects of one[1] endogenous treatment on multiple competing exit risks where all equations can be dependent by way of unobserved characteristics.[2] Nonparametric identification is achieved from single-spell[3] data under similar assumptions as used by Abbring and Van den Berg

[1]The result of this note can be straightforwardly extended to a model with several treatments that are independent conditional on observed and unobserved characteristics.

[2]For an application see e.g. Arni et al. (2013) who use this extension as part of a larger model.

[3]If repeated unemployment spells (multiple spells) are observed and motivation and skill level are assumed to stay constant across repeated spells, identification is straightforward in our setting (see Abbring and Van

(2003b). For ease of exposition, the case of two competing exit risks is presented first and the straightforward generalization to more than two exit risks is addressed at the end of this note.

2 The timing-of-events model with two competing exit risks

At time $t_0 = 0$, a worker enters into unemployment. $\forall\, t \in \mathbb{R}_+$ he faces two $(J = 2)$ competing hazard rates θ_1, θ_2 to exit this state. The two exit hazards are affected by a treatment that occurs at time $S = s$ with hazard rate θ_S

$$\begin{aligned}
\theta_1(t|S,x,V_1) &= \lambda_1(t)\ \phi_1(x)\ \delta_1(t|S,x)^{\mathbb{I}(t>S)}\ V_1 \\
\theta_2(t|S,x,V_2) &= \lambda_2(t)\ \phi_2(x)\ \delta_2(t|S,x)^{\mathbb{I}(t>S)}\ V_2 \\
\theta_S(s|x,V_S) &= \lambda_S(s)\ \phi_S(x)\ V_S,
\end{aligned} \tag{1}$$

where $\mathbb{I}$ is the indicator function.

The two functions $\delta_1(t|S,x)$ and $\delta_2(t|S,x)$ capture the different effects of the treatment on the two competing exit risks. For example, a labor market program could increase the rate to find work and at the same time reduce the risk of the job searcher to exit the labor force. In addition, the arguments x, t and S capture the dependence of the treatment effects on covariates, their dynamics over time and how they change with the time when the treatment is experienced.

In the absence of any treatment effects ($\delta_1 = \delta_2 = 1$), the three hazard rates in (1) have the well known mixed proportional hazard (MPH) structure, where $\lambda_q(t)$ reflects dependence on elapsed duration, $\phi_q(x)$ the effect of observed covariates x, and V_q the effect of unobserved characteristics for $q = 1, 2, S$. Selection on unobservables is captured through the trivariate

den Berg, 2003a). Consequently, this note is relevant for empirical work where multiple-spells are not available or where the assumption of constant unobservables is considered to be too restrictive.

distribution function G from which the vector $V = (V_1\ V_2\ V_S)$ is drawn.

If the treatment never occurs ($S = \infty$), the first two equations in (1) reduce to the MPH competing risks model of Abbring and Van den Berg (2003a). Let Y_1 and Y_2 denote the two latent unemployment durations elapsed until an exit of type 1 or 2 occurs respectively. Since only the first exit is observed, instead of the full joint distribution (Y_1, Y_2) one only observes (Y, I) where $Y = \min_{j\in\{1,2\}} Y_j$ and $I = \arg\min_{j\in\{1,2\}} Y_j$.

In the special case of only one exit type ($J = 1$), model (1) reduces to the timing-of-events model by Abbring and Van den Berg (2003b). In this single-risk case, other existing exits in the data are assumed to cause independent random right-censoring of the single outcome duration of interest Y_1. Consequently, the distribution of Y_1 is assumed to be fully observed in a large dataset, which facilitates the identification of the single treatment effect function $\delta_1(t|S, x)$. In the case considered in the next section ($J = 2$), two treatment effect functions $\delta_1(t|S, x)$ and $\delta_2(t|S, x)$ need to be identified while the joint distribution of the two latent durations (Y_1, Y_2) is not fully observed by the researcher. The remainder of this note addresses the resulting identification problem.

3 Assumptions

Assumption 1 *$\phi_1 : \mathbb{X} \to (0, \infty), \phi_2 : \mathbb{X} \to (0, \infty)$, $\phi_S(x) : \mathbb{X} \to (0, \infty)$ are continuous functions with $\phi_1(x^*) = \phi_2(x^*) = \phi_S(x^*) = 1$ for some $x^* \in \mathbb{X}$. Furthermore, $(\phi_1(x), \phi_2(x), \phi_S(x); x \in \mathbb{X})$ contains a non-empty open subset of $\mathbb{R}^3_+$.*

Assumption 2 *For $q = 1, 2, S$, $\lambda_q : \mathbb{R}_+ \to (0, \infty)$ has integral $\Lambda_q(t) := \int_0^t \lambda_q(\omega)d\omega < \infty\ \forall\ t \in \mathbb{R}_+$ with $\Lambda_1(t^*) = \Lambda_2(t^*) = \Lambda_S(t^*) = 1$ for some $t^* \in (0, \infty)$.*

Assumption 3 *V is an $\mathbb{R}^3_+$-valued random vector with distribution G independent of x with $\mathbb{E}(V_1) < \infty$, $\mathbb{E}(V_2) < \infty$, $\mathbb{E}(V_S) < \infty$ and $P(V \in (0, \infty)^3) > 0$.*

Assumption 4 *For $j = 1, 2$ and $\delta_j : \{(t, s) \in \mathbb{R}^2_+ : t > s\} \times \mathbb{X} \to (0, \infty)$, the integrals*

$\Upsilon_j(t|s,x) := \int_s^t \lambda_j(\omega)\delta_j(\omega|s,x)d\omega < \infty$ *and* $\Delta_j(t|s,x) := \int_s^t \delta_j(\omega|s,x)d\omega < \infty$ *exist and are càdlàg wrt* s*.*

Assumptions 1 - 4 are very similar to the assumptions used by Abbring and Van den Berg (2003b). The differences directly result from the extension to two competing exit risks, which requires an additional dimension in the variation of covariates (Assumption 1) and the unobserved heterogeneity distribution G (Assumption 3).

4 Identification result

Let $-j = 2$ if $j = 1$ and $-j = 1$ if $j = 2$. In a large dataset we observe for $j = 1, 2$ and $\forall\ (t,s) \in (0,\infty)^2$ with $t > s$ and $\forall\ x \in \mathbb{X}$

$$Q_{Y_j}(t|x) := P(Y_j > t, Y_{-j} > Y_j, S > Y|x) \tag{2}$$

$$Q_{Y_j,S}(t,s|x) := P(Y_j > t, Y_{-j} > Y_j, S > s, Y > S|x). \tag{3}$$

Proposition 1 *Let Assumptions 1-4 hold. Then, the functions* Λ_1*,* ϕ_1*,* Λ_2*,* ϕ_2*,* Λ_S*,* ϕ_S*,* G*,* Δ_1*, and* Δ_2 *are identified from* $\{Q_{Y_1}, Q_{Y_2}, Q_{Y_1,S}, Q_{Y_2,S}\}$*.*

Proof.

(i) Define $Q_S^0(s|x) = Q_{Y_1,S}(0,s|x) + Q_{Y_2,S}(0,s|x) = P(S > s, Y > S|x)$. The joint distribution of $\min\{Y_1, Y_2, S\}$ and $\arg\min\{Y_1, Y_2, S\}$ is fully characterized by $\{Q_{Y_1}, Q_{Y_2}, Q_S^0\}$ (Tsiatis, 1975). From the identification result for MPH competing risks models with single-spell data of Abbring and Van den Berg (2003a) it follows that under Assumptions 1-3 the functions $\Lambda_1, \phi_1, \Lambda_2, \phi_2, \Lambda_S, \phi_S$, and G are identified from $\{Q_{Y_1}, Q_{Y_2}, Q_S^0\}$.

(ii) In the sequel, we focus on the identification of Δ_1 and Δ_2. Let $\mathcal{L}_G^{(j)}$ (and $\mathcal{L}_G^{(j,3)}$) denote the (cross) derivative of the trivariate Laplace transform of G wrt the j-th (and 3-rd) argument. Taking the derivative of (2) wrt t and solving for $\lambda_j(t)$ and taking the derivative of (3) wrt t, s and solving for $\lambda_j(t)\delta_j(t|s,x)$ yields for almost all $t, s > 0$ with $t > s$ and all $x \in \mathbb{X}$

$$\lambda_j(t) = \left[-\mathcal{L}_G^{(j)}\Big(\phi_1(x)\Lambda_1(t), \phi_2(x)\Lambda_2(t), \phi_S(x)\Lambda_S(t)\Big)\phi_j(x)\right]^{-1} \frac{\partial Q_{Y_j}(t|x)}{\partial t} \quad (4)$$

$$\lambda_j(t)\delta_j(t|s,x) = \Big[\mathcal{L}_G^{(j,3)}\Big(\phi_1(x)\big[\Lambda_1(s) + \Upsilon_1(t|s,x)\big], \phi_2(x)\big[\Lambda_2(s) + \Upsilon_2(t|s,x)\big], \phi_S(x)\Lambda_S(s)\Big)\phi_j(x)\phi_S(x)\lambda_S(s)\Big]^{-1} \frac{\partial^2 Q_{Y_j,S}(t,s|x)}{\partial s \partial t}. \quad (5)$$

In the following, s and x are treated as fixed. Define $\Lambda_S^s := \Lambda_S(s)$, $\lambda_S^s := \lambda_S(s)$ and suppress the dependence of ϕ_j and ϕ_S on x. Further, define $\mathcal{H}(t) := (\mathcal{H}_1(t)\ \mathcal{H}_2(t))'$. For $0 < t \leq s$ define $\mathcal{H}_j(t) := \Lambda_j(t)$, $r_j(t, \mathcal{H}(t)) := [-\mathcal{L}_G^{(j)}\Big(\phi_1\mathcal{H}_1(t), \phi_2\mathcal{H}_2(t), \phi_S\Lambda_S(t)\Big)\phi_j]^{-1}$, $\mathcal{Q}_j(t) := \frac{\partial Q_{Y_j}(t|x)}{\partial t}$, and for $t > s$ define $\mathcal{H}_j(t) := \Lambda_j(s) + \Upsilon_j(t|s,x)$, $r_j(t, \mathcal{H}(t)) := [\mathcal{L}_G^{(j,3)}\Big(\phi_1\mathcal{H}_1(t), \phi_2\mathcal{H}_2(t), \phi_S\Lambda_S^s\Big)\phi_j\phi_S\lambda_S^s]^{-1}$, $\mathcal{Q}_j(t) := \frac{\partial^2 Q_{Y_j,S}(t,s|x)}{\partial s \partial t}$.

When combined, equations (4) for $0 < t \leq s$ and (5) for $t > s$ with $j = 1, 2$ yield a system of two first order differential equations in the sense of Carathéodory (1918) (see Walter, 1998) for almost all $t \in (0, \infty)$

$$\frac{d}{dt}\mathcal{H} = f\left(t, \mathcal{H}\right), \text{ with initial conditions } \mathcal{H}(\tau) = \gamma_\tau \text{ for some } \tau \in (0, s),$$
$$\text{with } f := \left(f_1\ f_2\right)' \text{ and } f_j\left(t, \mathcal{H}\right) = \mathcal{Q}_j(t)\ r_j(t, \mathcal{H}). \quad (6)$$

Choosing a $\tau \in (0, s)$ yields the initial conditions $\mathcal{H}(\tau) = (\Lambda_1(\tau)\ \Lambda_2(\tau))' = \gamma_\tau$. $\mathcal{Q}_j(t)$ is known for almost all $t > 0$. $\phi_j, \phi_S, G, \Lambda_S$ are identified from step (i) and thus also $\mathcal{L}_G^{(j)}$, $\mathcal{L}_G^{(j,3)}$ and Λ_S^s, λ_S^s (for almost all s) are known. Hence, for all $x \in \mathbb{X}$ and almost all $s \in (0, \infty)$ f is a known function of t and $\mathcal{H}$ and it is shown in the Appendix that system (6) has a unique solution $\mathcal{H}(t)$ over $t \in (0, \infty)$.

Since by Assumption 4, $\Upsilon_j(t|s,x)$ is càdlàg wrt s, it follows that the function is identified everywhere on $\{(t,s) \in \mathbb{R}_+^2 : t > s\} \times \mathbb{X}$. By definition, the latter yields identification of Δ_j on its domain for $j = 1, 2$.

■

5 Extension to multiple competing exit risks

It is straightforward to extend Proposition 1 to a model with J competing exit risks, where J is a positive finite integer. In this case, in Assumption 3 G is extended to a J-dimensional distribution. Furthermore, Assumption 1 is extended such that $(\phi_1(x), ..., \phi_J(x), \phi_S(x); x \in \mathbb{X})$ contains a nonempty open subset in $\mathbb{R}_+^{J+1}$. For example, for $J = 3$ and four covariates with $\phi_q(x) = exp(x^\top \beta_q)\ \forall q \in \{1, 2, 3, S\}$, it would be sufficient that the 4×4 matrix $(\beta_1\ \beta_2\ \beta_3\ \beta_S)$ has full rank and $\mathbb{X}$ contains a non-empty open subset in $\mathbb{R}^4$. Thus, in most applications, where the rank condition is fulfilled, $J + 1$ continuous covariates will generate sufficient variation.

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Appendix

Let $\mathbb{M} = \mathbb{T} \times \mathbb{K}$ with $\mathbb{T} = [\tau, \tau + a]$ for some $a > 0$ and $\mathbb{K} \subset (0, \infty)^2$ be a closed ball. Simple algebra, yields for $(t, \mathcal{H}), (t, \mathcal{H}^*) \in \mathbb{M}$

$$\|f(t, \mathcal{H}) - f(t, \mathcal{H}^*)\| \leq |\mathcal{Q}_1(t) + \mathcal{Q}_2(t)| \, \|r\,(t, \mathcal{H}) - r\,(t, \mathcal{H}^*)\| \,. \tag{A.1}$$

$r_j(t, \mathcal{H})$ is continuously differentiable in $\mathcal{H}$ for fixed t. Furthermore, $\frac{\partial r_j(t,\mathcal{H})}{\partial \mathcal{H}_\kappa}$ $(\kappa = 1, 2)$ is continuous in t for $t \leq s$ and a finite constant for $t > s$. Consequently, $\exists\, \mathcal{C} < \infty$ s.t. for each $\kappa, j \in \{1, 2\}$ $\sup_{(t,\mathcal{H})\in\mathbb{M}} \left| \frac{\partial r_j(t,\mathcal{H})}{\partial \mathcal{H}_\kappa} \right| \leq \mathcal{C}$. By applying separately for each $r_j(t, \mathcal{H})$ the multivariate mean value theorem wrt to the vector $\mathcal{H}$, it can be shown that

$$\|r(t, \mathcal{H}) - r(t, \mathcal{H}^*)\| = \left\| B(t, \tilde{\mathcal{H}})(\mathcal{H} - \mathcal{H}^*) \right\| \leq \left\| B(t, \tilde{\mathcal{H}}) \right\| \|\mathcal{H} - \mathcal{H}^*\| \leq 2\mathcal{C} \, \|\mathcal{H} - \mathcal{H}^*\| \,, \tag{A.2}$$

where $B(t, \tilde{\mathcal{H}})$ is the Jacobian matrix (wrt to $\mathcal{H}$) of $r(t, \mathcal{H})$ evaluated at the mean value $\tilde{\mathcal{H}}$.

$|\mathcal{Q}_1(t) + \mathcal{Q}_2(t)|$ in (A.1) is measurable and integrable on compact sets. Combining (A.1) and (A.2) shows that f satisfies a generalized Lipschitz condition. Thus, by Walter (1998) theorem §10.XX(b), this implies that system (6) has a unique solution $\mathcal{H}$ on $(0, \infty)$.