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The implications of equal value of life and prioritarianism for the evaluation of population health

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The implications of equal value of life and prioritarianism for the evaluation of population health*

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Abstract

We analyze the implications of several principles related to the concepts of equal, or prioritarian, entitlement to continued life. These principles, when modeled as axioms for the evaluation of health distributions, and combined with some basic structural axioms, provide several characterization results of population health evaluation functions. Our analysis implies that the scope of the concepts of equal and prioritarian entitlement to continued life needs to be limited, in order to allow for morbidity (and not just mortality) concerns in the evaluation of population health.

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1 Introduction

One of the central postulates of most egalitarian doctrines is the idea that every life has equal value. The idea is at the core of a wide array of philosophical, political and social discussions. It constitutes one of the central slogans of the Bill & Melinda Gates foundation, the largest private foundation in the world, and it has received strong endorsements by numerous public figures.

The concept of equal value of life has also been scrutinized in academic research. One of its strongest defenders is John Harris, who, in a series of contributions that date back to the 80's and 90's, argued ethical concerns for a fundamental right to continued life to which all individuals are entitled to the same extent (e.g., Harris, 1985; 1987; 1996; 1997). Harris' arguments led to the conclusion that, even if some lives are not lived at perfect health, lives are in fact equally valuable, as long as they are valued by those living those lives. Such a conclusion has also been endorsed within the health economics community (e.g., Arnesen and Nord, 1999; Nord, 2001).

Another argument usually considered to defend equal value of life is the recurrent argument within political philosophy that welfare interpersonal comparisons are incommensurate, and, therefore, that it is wrong to discriminate on the basis of health states. Nevertheless, such an argument has been contested (e.g., Singer et al., 1995; McKie et al., 1997) and debated (e.g., Grimley Evans, 1997; Williams, 1997).

We provide in this paper a new perspective on the concept of equal value of life, in connection with the evaluation of population health. To do so, we consider the new axiomatic approach to the evaluation of population health, recently introduced by Hougaard, Moreno-Ternero and Østerdal (2013), and also considered by Moreno-Ternero and Østerdal (2014). In such an approach, the health of an individual in the population is defined according to the two standard dimensions (quality of life and quantity of life), but one of them (quality of life) receives a special treatment, as no restrictions are made regarding its mathematical structure. A distinguishing feature of this approach is that it does not make assumptions about individual preferences over quantity and quality of life. In doing so, we depart from the strand of the literature on population health evaluation in which the analysis relies on individual preferences on quantity and quality of life (e.g., Østerdal, 2005; Harvey and Østerdal, 2010), and also from the popular strand of the (health economics) literature in which the analysis relies on a generic individual health utility concept (e.g., Wagstaff, 1991; Bleichrodt, 1997; Dolan, 1998; Bleichrodt et al., 2004). In doing so, we circumvent basing our analysis on the concept of individual health preferences, which has faced recurrent criticisms over its conceptual foundation and elicitation procedures (e.g. Dolan 2000).

We formalize equal value of life as an axiom of social preferences for population health evaluations in the model described above. More precisely, we consider a cohort of equally old individuals and aim to evaluate the effects of alternative health care policies for such a cohort, on the grounds of the resulting distributions of health (that the policies would generate for the cohort). In such a scenario, equal value of life is formalized as the axiom stating that if two distributions of health only differ in granting an amount of extra years to one or another individual, then they are considered equally good by the social planner (as all lives are valued equally). We show that the combination of such an axiom with two other structural axioms (known as time monotonicity at perfect health, and the social zero condition) characterizes the population health evaluation function that ranks distributions according to the unweighted aggregation (across agents in the population) of lifetimes in the distribution. Such a function does not include any concern whatsoever for the quality of life at which individuals in the population experience those lifetimes, which is in contrast with some traditional forms of evaluation for health distributions, such as the so-called Quality Adjusted Life Years (e.g., Pliskin, Shepard and Weinstein, 1980), in short QALYs, and the so-called Healthy Years Equivalent (e.g., Mehrez and Gafni, 1989), in short HYEs. In other words, under the presence of some structural axioms, endorsing the principle of equal value of life in its full force drives towards dismissing morbidity concerns in the evaluation of population health.

It is worth mentioning that the result described above is closely related to a result in Hasman and Østerdal (2004), which establishes a general incompatibility between a specific form of the equal value of life principle and the weak Pareto principle, which can only be formalized in a context where individual preferences on quality and quantity of life are an ingredient of the model, which is not the case of this paper.

The idea of equal value of life, as introduced above, prevents any form of discrimination against individuals with worse quality of life, when it comes to allocate extra life years. Now, some political philosophers have endorsed going a step ahead, arguing that justice requires that a positive discrimination in favor of the worse-off be allowed. The most extreme position is advocated by Rawls (1971), with his so-called *difference principle*, for whom differences in primary goods are only morally acceptable if they maximize the level of primary goods achieved by the worst-off individual. Parfit (1997) coined the term *prioritarianism* for the view that the worse off should be given priority over the better off, but that they need not necessarily receive the extreme priority that characterizes the difference principle. A recent comprehensive endorsement of the prioritarian evaluation of outcomes and policies is provided by Adler (2012).[1] There are several ways in which the principle of prioritarianism could be formalized. In a welfarist setting, prioritarianism is usually characterized as a social welfare function with strictly convex upper contour sets (e.g., Roemer, 2004). In a non-welfarist setting of resource allocation, it can be formalized as an axiom of *no-domination* (e.g., Moreno-Ternero and Roemer, 2006; 2012). In our setting, we can formalize the principle by means of similar axioms regarding the allocation of extra life years. More precisely, we can unambiguously say that an agent is disabled with respect to another, if the latter dominates the former in both quality and quantity of life. Our *disability priority* axiom formalizes the idea that extra life years should not be valued less when awarded to a disabled agent, so defined. We also consider another weaker axiom in which the principle is restricted to (pairs of) agents at perfect health.

We show that the combination of the disability priority axiom with some other structural axioms characterizes the population health evaluation function referring to the unweighted aggregation (across agents in the population) of lifetimes in the distribution, after being submitted to a concave (increasing and continuous) function. Thus, as with the case of equal value of life, morbidity concerns are excluded from the evaluation of the distribution of health. Nevertheless, mortality concerns are allowed to be included in a more general (and egalitarian-oriented) form.

To conclude, if we consider instead the weaker axiom of priority, we characterize the family of population health evaluation functions arising upon aggregating individual HYEs, after being submitted to a concave (and increasing) function.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the structural axioms we consider for our analysis. In Section 3, we introduce the axiom of equal value of life and explore its implications. In Section 4, we move to extend the analysis to the case of prioritarian (rather than equal) value of life. We conclude in Section 5. The formalization and technical aspects of our analysis are relegated to an appendix.

[1]Priority arguments have been used by Moreno-Ternero and Roemer (2008) to reject the veil of ignorance as a tool for implementing distributive justice.

2 The preliminaries

Let us consider a cohort of equally old individuals (in brief, "population"). Imagine a policy maker who has to evaluate several alternative health policies for such a population.[2] Each policy is characterized by a given distribution of health it generates for the population. The health of each individual in the population is described by a duplet indicating the level achieved in two parameters: quality of life and quantity of life (gained). Assume that there exists a set of possible health states (formalizing quality of life) defined generally enough to encompass all possible health states for everybody in the population. We emphasize that this set is not assumed to have any particular mathematical structure. Quantity of life (gained) is simply described by the set of nonnegative real numbers. An individual in the population will then be characterized by a duplet indicating the units of time (e.g., days, months, years) that she will be obtaining (from the moment the policy is implemented), each unit being experienced at some quality level.[3] A population health distribution (or, simply, a health profile) specifies the health duplet of each individual in the population. Even though we do not impose a specific mathematical structure on the set of health states (quality levels), we assume that it contains a specific element, which we refer to as *perfect health*, and which is univocally identified, as a "superior" state, by the policy maker.

The policy maker's preferences (or social preferences) over health profiles are expressed by a preference relation, to be read as "at least as preferred as". We assume that the preference relation is a *weak order*. More precisely, we assume that it is *complete* (for each pair of health profiles, either the first is at least as preferred as the second, the second is at least as preferred as the first, or both) and *transitive* (if a first health profile is at least as preferred as a second profile, and the second profile is at least as preferred as a third one, then the first health profile is at least as preferred as the third profile).

A *population health evaluation function* is a real-valued function that represents the policy maker's preferences. That is, the function assigns a higher (or equal) number to a health profile than to another if an only if the former one is deemed at least as preferred as the latter one.[4]

[2]Think, for instance, of alternative vaccinations for any of the stages in the immunization schedule of infants, or alternative (universal) screening procedures for the early detection of some forms of cancer.

[3]The running interpretation is that agents only experience chronic health states, but it could also be interpreted that the health state reflects an average level of quality at which the associated lifespan is experienced.

[4]Note that if a population health evaluation function represents the policy maker's preferences, then any strictly increasing transformation of it would also do so.

Instead of proposing specific population health evaluation functions directly, we aim to derive them following the so-called *axiomatic approach*, a somewhat unexplored approach in the population health evaluation literature. An axiomatic study begins with the specification of a domain of problems, and the formulation of a list of desirable properties (axioms) of solutions for the domain, whereas it ends with (as complete as possible) descriptions of the families of solutions satisfying various combinations of the properties (e.g., Thomson, 2001). An axiomatic study often results in characterization theorems. They are theorems identifying a particular solution, or perhaps a family of solutions, as the only solution or family of solutions, satisfying a given list of axioms. This is precisely what we do in this paper. We list next some appealing axioms for the evaluation of population health and then derive precise measures to evaluate the health of a population. We first rely on a list of basic structural axioms. We then resort to additional independent axioms to this list that formalize the main two principles over which this work relies (namely, equal and prioritarian value of life). Ultimately, we show how the different combinations of these latter axioms with some of the former structural ones lead to characterize several population health evaluation functions.

2.1 Basic structural axioms

We now list the basic structural axioms for social preferences that we consider in this paper.

First, the axiom of **anonymity**, which represents standard formalization of the principle of impartiality in axiomatic work. It says, in our context, that the evaluation of the population health should depend only on the list of quality-quantity duplets, not on who holds them.

The second axiom, **separability**, says that if the distribution of health in a population changes only for a subgroup of agents in the population, the relative evaluation of the two distributions should only depend on that subgroup. In particular, the axiom excludes the possibility of externalities in the evaluation of a health distribution, and it underlies the use of incremental analysis in cost-effectiveness analysis (the standard currency in the economic evaluation of health care programs).

Third is the standard technical condition of **continuity**, which says that, for fixed distributions of health states, small changes in lifetimes should not lead to large changes in the evaluation of the population health distribution. The axiom is thus excluding certain forms of arbitrariness in the evaluation of population health (in particular, leximin functional forms).

The next two axioms are the closest formulation to the Pareto principle we can consider in our framework.

The first one, **perfect health superiority**, introduces some structure in the domain of health states. It simply says that replacing the health status of an agent by that of perfect health, ceteris paribus, cannot worsen the evaluation of the population health.

The second one, **time monotonicity at perfect health**, says that if each agent is at perfect health, increasing the time dimension is strictly better for the policy maker.

The next two axioms are somewhat dual to the previous two, as they convey principles referring to the bottom of the domain of health profiles.

The first one, **positive lifetime desirability**, says that the health of the population improves if any agent moves from zero lifetime to positive lifetime (for a given health state). In particular, the axiom implies that all health states are worth living.

The last basic axiom, **social zero condition**, says that if an agent gets zero lifetime, then her health state does not influence the social desirability of the health distribution.

In what follows, we refer to the set of axioms introduced above as the **basic structural axioms**.

3 Equal value of life

We start this section adding to the previous list of basic structural axioms the axiom modeling the notion of equal value of life, discussed above. In words, the axiom of **equal value of life** says that a certain amount of additional life years to individual i is socially seen as just as good as the same amount of additional life years to individual j, regardless of health states.[5] Note that, as mentioned above, our model refers to a cohort of equally-old individuals. For those individuals, the axiom states that we should be indifferent about who receives extra lifetime.[6]

Our first result exhibits the strength of the notion of equal value of life. More precisely, Theorem 1 shows that it suffices to combine it with only two of the structural axioms described above to characterize the so-called *aggregate lifetime* population health evaluation function, which evaluates population health distributions by means of the aggregate lifetime the distribution yields.

[5]Hasman and Østerdal (2004) define a similar axiom in their model.

[6]The axiom implicitly assumes that all health states are worth living, and, therefore, that this additional lifetime would be valued by all agents.

Thus, the *aggregate lifetime* population health evaluation function reflects the traditional view for the evaluation of the impact of health care only in terms of its effect on mortality. As the next result shows, this population health evaluation function is characterized by the combination of the axioms of equal value of life, time monotonicity at perfect heath, and the social zero condition.

Theorem 1 *The policy maker's preferences satisfy equal value of life, time monotonicity at perfect health, and the social zero condition if and only if they can be represented by the aggregate lifetime population health evaluation function.*

The formal proof of Theorem 1 is relegated to the Appendix. We only provide here its intuition. It is straightforward to see that the aggregate lifetime population health evaluation function satisfies the three axioms at the statement of the theorem. As for the non-trivial implication, assume that the policy maker's preferences satisfy the three mentioned axioms. By iterated application of *equal value of life*, and the transitivity of the social preferences, each health profile must be indifferent to the health profile in which one agent is endowed with the aggregate lifespan, whereas the others are endowed with zero lifespans. By iterated application of the *social zero condition*, and the transitivity of the social preferences, it is also indifferent to the profile in which the agents endowed with zero lifespans are enjoying the perfect health quality status. By *equal value of life*, as well as a new iterated application of the *social zero condition*, and the transitivity of the social preferences, the indifference prevails with respect to the profile in which the agent endowed with the aggregate lifespan of the initial profile is also enjoying the perfect health quality status. Consequently, we obtain that the social preferences only depend on the aggregate lifespan of the profile. *Time monotonicity at perfect health*, and the transitivity of the social preferences, allow to conclude.

Theorem 1 exhibits the strong logical implications of the axiom of equal value of life, as its combination with two structural axioms discards including morbidity concerns in the evaluation of population health.

A natural weakening of the equal value of life axiom arises when one restricts the attention to pairs of agents with zero lifetime. The resulting axiom, **weak equal value of life**, would thus be stating, in particular, the independence of health states to the decision of saving the life of an agent in the cohort.

It turns out, as shown in the next result, that this new axiom also characterizes the aggregate lifetime population health evaluation function, provided one resorts to the whole set of basic structural axioms.

Theorem 2 *The policy maker's preferences satisfy weak equal value of life, and the basic structural axioms, if and only if they can be represented by the aggregate lifetime population health evaluation function.*

The formal proof of Theorem 2 is relegated to the Appendix. As before, we only provide here its intuition. It is straightforward to see that the aggregate lifetime population health evaluation function satisfies all the axioms at the statement of the theorem. As for the non-trivial implication, assume that the policy maker's preferences satisfy weak equal value of life, and the basic structural axioms. Then, by Theorem 1 in Hougaard, Moreno-Ternero and Østerdal (2013), the social preferences can be represented by a separable population health evaluation function. By *weak equal value of life*, it follows that the social preferences are indeed represented by an aggregate lifetime population health evaluation, as desired.

Theorem 2 shows that the proposed weakening of the equal value of life axiom exhibits similar strong implications to its full-fledged counterpart. Nevertheless, limiting the scope of equal value of life in different ways allows for weaker implications. In particular, Hougaard, Moreno-Ternero and Østerdal (2013) show that alternative population health evaluation functions, including morbidity concerns, can be characterized when the principle of equal value of life is restricted to agents experiencing the same quality of life, and the resulting axiom is combined with the set of basic structural axioms described above.

4 Prioritarian value of life

We propose in this section an alternative to the previous analysis of the concept of equal value of life. For that matter, we formalize a *prioritarian* view for the entitlement to continued life. To do so, we begin formalizing the axiom of **disability priority**, which says that a certain amount of additional life years to an individual is socially seen at least as good as the same amount of additional life years to another (abler) individual, who is enjoying perfect health, and a higher lifetime.

It turns out that adding the previous axiom to the set of basic structural axioms, we characterize a general form of lifetime aggregation in which lifetimes are submitted to an arbitrary increasing and *concave* function. More precisely, we define the *concave aggregate lifetime* population health evaluation function as the population health evaluation function that evaluates health distributions by means of the aggregate value obtained when all the lifetimes the distribution yields are submitted to an increasing and concave function.[7]

Theorem 3 *The policy maker's preferences satisfy disability priority, and the basic structural axioms, if and only if they can be represented by the concave aggregate lifetime population health evaluation function.*

The formal proof of Theorem 3 is relegated to the Appendix. Its intuition is similar to the one of the proof of Theorem 2 described above. More precisely, if the policy maker's preferences satisfy the basic structural axioms, then they can be represented by a separable population health evaluation function. Now, by *disability priority*, we can obtain that the social preferences are indeed represented by a concave aggregate lifetime population health evaluation, as desired.

It then follows that, even though disability priority (which is, after all, a weakening of the axiom of equal value of life) allows for other more general forms of population health evaluation functions, these also involve dismissing any concern whatsoever over quality of life.

The alternative axiom we consider to model prioritarianism is the weakening of the previous one, when only applied to agents enjoying perfect health. More precisely, **disability priority at perfect health** says that, among agents at perfect health, we prioritize those with lower lifetimes (hence, disabled) when it comes to allocate extra additional life years.

As shown in the next result, if the previous axiom is added to the set of basic structural axioms, we characterize a general family of population health evaluation functions including a concern for morbidity. Before presenting the result, we need first to introduce the following concept. We define *healthy years equivalent*, in short HYEs, as the lifespans that, when awarded to each agent, after replacing their original health states by perfect health, makes the resulting health profile indifferent (according to social preferences) to the original one. We then define the *concave aggregate healthy years equivalent* population health evaluation function, as the population health evaluation function that evaluates health distributions by means of the aggregate value the distribution yields, after submitting each individual HYE to a concave function.

[7] If one dismisses the caveat regarding the monotonicity of lifetimes in the statement of the disability priority axiom, the resulting (weaker) axiom, in combination with the basic structural axioms, would also characterize the lifetime aggregation population health evaluation function, as the equal value of life axioms were shown to do.

We have the following result:

Theorem 4 *The policy maker's preferences satisfy disability priority at perfect health, and the basic structural axioms, if and only if they can be represented by the concave aggregate healthy years equivalent population health evaluation function.*

The formal proof of Theorem 4 is relegated to the Appendix. Its intuition is also similar to the ones of the previous two theorems. More precisely, if the policy maker's preferences satisfy the basic structural axioms, then they can be represented by a separable population health evaluation function. Now, by *disability priority at perfect health*, we can only obtain weaker implications in such a separable representation. Whereas *equal value of life* and *disability priority* implied that the "healthy years equivalent function" would be independent of individual health states, *disability priority at perfect health* has no implications on such a function. Its only effect on the separable structure of the population health evaluation function is, as in the case of *disability priority*, to make each of its arguments enter after being submitted to a concave function.[8]

5 Discussion

We have explored in this paper the implications of the principle of equal value of life, which conveys an equal entitlement to additional life years, in the context of the evaluation of health distributions. Our main result shows the strength of that principle as its combination with two weak structural axioms (one stating the appeal of enjoying more life years at perfect health; another indicating the irrelevance of the health status when there is no expected lifetime to experience it) leads to evaluating health distributions by the aggregate lifetime they offer, dismissing any concern whatsoever for the morbidity associated to health distributions. Nevertheless, if the scope of the principle is reduced to individuals sharing some characteristics, more general population health evaluation functions (including, not only a concern for mortality, but also a concern for morbidity) can be recovered.

Another related principle we have explored is that of prioritarian value of life, conveying the idea that disabled individuals are prioritized in the allocation of additional life years. Two axioms formalizing that principle have been considered. One turns out to exhibit similarly

[8] *Equal value of life* has a stronger effect, in such a step, making that function linear, instead of concave.

strong implications as equal value of life, at least under the presence of structural axioms. Another has weaker implications leading to characterize more general population health evaluation functions; namely, concave aggregation of healthy years equivalent.

To conclude, it is worth mentioning that our work has been set in a context without uncertainty. In other words, and following Broome (1993), we consider a formulation of the population health evaluation problem which contains no explicit element of risk, and in which we obtain characterizations of population health evaluation functions without assumptions on the policy maker's (or individuals') risk attitudes. It is left for further research to extend our analysis in that direction.

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6 Appendix

We have included in this appendix all the technical parts of our analysis.

Let $N = \{1, ..., n\}$ denote a cohort of equally old individuals (in brief, "population"). For each $i \in N$, let $h_i = (a_i, t_i) \in A \times T$ denote the health duplet of individual i, which indicates that i is endowed with $t_i \in T = [0, +\infty)$ units of time (e.g., days, months, years) each experienced at quality $a_i \in A$. A population health distribution (or, simply, a health profile) $h = [h_1, \ldots, h_n] = [(a_1, t_1), ..., (a_n, t_n)]$ specifies the health duplet of each individual in the population. We denote the set of all possible health profiles by H.[9] Even though we do not impose a specific mathematical structure on the set A, we assume that it contains a specific element, a_*, which we refer to as *perfect health* and which is univocally identified, as a "superior" state, by the policy maker.

The policy maker's preferences (or social preferences) over health profiles are expressed by a preference relation $\succsim$, to be read as "at least as preferred as". As usual, $\succ$ denotes strict preference and $\sim$ denotes indifference. We assume that the relation $\succsim$ is a weak order.[10]

A *population health evaluation function* is a real-valued function $P : H \to \mathbb{R}$. We say that P represents $\succsim$ if

$$P(h) \geq P(h') \Leftrightarrow h \succsim h',$$

for each $h, h' \in H$. Note that if P represents $\succsim$ then any strictly increasing transformation of P would also do so.

The formal definition of the basic axioms for social preferences that we employ in this paper comes next.

Anonymity: $h \sim h_\pi$ for each $h \in H$, and each permutation π of the set N.

Separability: $[h_S, h_{N \setminus S}] \succsim [h'_S, h_{N \setminus S}] \Leftrightarrow \left[h_S, h'_{N \setminus S}\right] \succsim \left[h'_S, h'_{N \setminus S}\right]$, for each $S \subseteq N$, and $h, h' \in H$.

Continuity: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}) \to (a_i, t_i) = h_i$. If $h^{(k)} \succsim h'$ for each k then $h \succsim h'$, and if $h' \succsim h^{(k)}$ for each k then $h' \succsim h$.

Perfect health superiority: $[(a_*, t_i), h_{N \setminus \{i\}}] \succsim h$, for each $h = [h_1, \ldots, h_n] \in H$ and $i \in N$.

[9] For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

[10] More precisely, we assume that $\succsim$ is complete (for each health profiles h, h', either $h \succsim h'$, or $h' \succsim h$, or both) and transitive (if $h \succsim h'$ and $h' \succsim h''$ then $h \succsim h''$).

Time monotonicity at perfect health: If $t_i \geq t'_i$, for each $i \in N$, with at least one strict inequality, then $[(a_*, t_1), \ldots, (a_*, t_n)] \succ [(a_*, t'_1), \ldots, (a_*, t'_n)]$.

Positive lifetime desirability: $h \succsim [h_{N \setminus \{i\}}, (a_i, 0)]$, for each $h = [h_1, \ldots, h_n] \in H$ and $i \in N$.

Social zero condition: For each $h \in H$ and $i \in N$ such that $t_i = 0$, and $a'_i \in A$, $h \sim [h_{N \setminus \{i\}}, (a'_i, 0)]$.

In what follows, we refer to the set of axioms introduced above as the **basic structural axioms**.

We now introduce the formal definition of the axiom of equal value of life:

Equal Value of Life: For each $h \in H$, $c > 0$, and $i, j \in N$,

$$\left[(a_i, t_i + c), (a_j, t_j), h_{N \setminus \{i,j\}}\right] \sim \left[(a_i, t_i), (a_j, t_j + c), h_{N \setminus \{i,j\}}\right].$$

Theorem 1 shows that it suffices to combine it with only two of the structural axioms described above to characterize the so-called *aggregate lifetime* population health evaluation function, which evaluates population health distributions by means of the aggregate lifetime the distribution yields. Formally,

$$P^t[h_1, \ldots, h_n] = P^t[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} t_i. \tag{1}$$

As Theorem 1 states, P^t is characterized by the combination of the axioms of equal value of life, time monotonicity at perfect heath, and the social zero condition.

Theorem 1 *The policy maker's preferences satisfy equal value of life, time monotonicity at perfect health, and the social zero condition if and only if they can be represented by the aggregate lifetime population health evaluation function.*

Proof. We focus on its non-trivial implication. Formally, assume $\succsim$ satisfies *equal value of life*, the *social zero condition* and *time monotonicity at perfect health.* Let P be a population health evaluation function representing $\succsim$ and let $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$. By iterated application of *equal value of life*, and the transitivity of $\succsim$,

$$h \sim [(a_1, t_1 + ... + t_n), (a_k, 0)_{k \neq 1}].$$

By iterated application of the *social zero condition*, and the transitivity of $\succsim$,

$$[(a_1, t_1 + ... + t_n), (a_k, 0)_{k \neq 1}] \sim [(a_1, t_1 + ... + t_n), (a_*, 0)_{k \neq 1}].$$

By *equal value of life*,

$$[(a_1, t_1 + ... + t_n), (a_*, 0)_{k\neq 1}] \sim [(a_1, 0), (a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1,2}].$$

By the *social zero condition*,

$$[(a_1, 0), (a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1,2}] \sim [(a_*, 0), (a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1,2}].$$

Finally, by *equal value of life*,

$$[(a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1}] \sim [(a_*, 0), (a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1,2}].$$

Altogether, by the transitivity of $\succsim$, we obtain,

$$h \sim [(a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1}],$$

from which we conclude that $\succsim$ depends only on $t_1 + ... + t_n$.

Let now $h' = [(a'_1, t'_1), \ldots, (a'_n, t'_n)] \in H$. By the above argument,

$$h' \sim [(a_*, t'_1 + ... + t'_n), (a_*, 0)_{k\neq 1}].$$

Thus, by *time monotonicity at perfect health*,

$$h' \sim [(a_*, t'_1 + ... + t'_n), (a_*, 0)_{k\neq 1}] \succsim [(a_*, t_1 + ... + t_n), (a_*, 0)_{k\neq 1}] \sim h.$$

if and only if

$$\sum_{i=1}^{n} t'_i \geq \sum_{i=1}^{n} t_i.$$

Thus, the transitivity of $\succsim$ concludes. □

We now consider the weakening of the equal value of life axiom that arises when one restricts the attention to pairs of agents with zero lifetime. Formally,

Weak Equal Value of Life: For each $h \in H$, $c > 0$, and $i, j \in N$,

$$\left[(a_i, c), (a_j, 0), h_{N\setminus\{i,j\}}\right] \sim \left[(a_i, 0), (a_j, c), h_{N\setminus\{i,j\}}\right].$$

It turns out, as shown in the next result, that this new axiom also characterizes the aggregate lifetime population health evaluation function, provided one resorts to the whole set of basic structural axioms.

Theorem 2 *The policy maker's preferences satisfy weak equal value of life, and the basic structural axioms, if and only if they can be represented by the aggregate lifetime population health evaluation function.*

Proof. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies *weak equal value of life* and the *basic structural axioms*. Then, by Theorem 1 in Hougaard, Moreno-Ternero and Østerdal (2013), $\succsim$ can be represented by a separable population health evaluation function, i.e.,

$$P^s[h_1, \ldots, h_n] = P^s[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(f(a_i, t_i)), \tag{2}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing and continuous function, and $f : A \times T \to T$ is a function indicating the *healthy years equivalent* for each individual, i.e.,

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and
- For each $h = [h_1, \ldots, h_n] = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Now, let $c > 0$, $h \in H$, and $i, j \in N$. By *weak equal value of life*,

$$\left[(a_i, c), (a_j, 0), h_{N \setminus \{i,j\}}\right] \sim \left[(a_i, 0), (a_j, c), h_{N \setminus \{i,j\}}\right].$$

Equivalently,

$$g(0) + g(f(a_j, c)) = g(0) + g(f(a_i, c)).$$

From here, it follows that, by the strict monotonicity of g,

$$f(a_j, c) = f(a_i, c), \qquad \text{for each } c > 0, \text{ and } a_i, a_j \in A.$$

In other words, f is constant with respect to its first variable. Without loss of generality, we can say that $f(a_i, t) = t$, for each $a_i \in A$ and $t \in T$, from where it follows that $\succsim$ is indeed represented by a population health evaluation function satisfying (1), as desired. $\square$

We now present the formal definition of our first axiom capturing a *prioritarian* view for the entitlement to continued life. Formally,

Disability Priority: For each $c > 0$, $h \in H$, and $i, j \in N$, such that $t_i \geq t_j$,

$$\left[(a_*, t_i), (a_j, t_j + c), h_{N \setminus \{i,j\}}\right] \succsim \left[(a_*, t_i + c), (a_j, t_j), h_{N \setminus \{i,j\}}\right].$$

It turns out that adding the previous axiom to the set of basic structural axioms, we characterize a general form of lifetime aggregation in which lifetimes are submitted to an arbitrary increasing and *concave* function. More precisely, we define the *concave aggregate lifetime* population health evaluation function as the population health evaluation function that evaluates population health distributions by means of the aggregate value obtained when all the lifetimes the distribution yields are submitted to an increasing and concave function. Formally,

$$P^{gt}[h_1, \ldots, h_n] = P^{gt}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(t_i), \tag{3}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing, continuous and concave function.

Theorem 3 *The policy maker's preferences satisfy disability priority, and the basic structural axioms, if and only if they can be represented by the concave aggregate lifetime population health evaluation function.*

Proof. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies *disability priority*, and the *basic structural axioms*. Then, as in the proof of the previous result, $\succsim$ can be represented by a separable population health evaluation function, as in (2).

Now, let $c > 0$, $h \in H$, and $i, j \in N$ be such that $a_i = a_*$. By *disability priority*,

$$\left[(a_*, 0), (a_j, c), h_{N\setminus\{i,j\}}\right] \succsim \left[(a_*, c), (a_j, 0), h_{N\setminus\{i,j\}}\right].$$

Equivalently,

$$g(f(a_*, 0)) + g(f(a_j, c)) \geq g(f(a_*, c)) + g(f(a_j, 0)),$$

i.e.,

$$g(f(a_j, c)) \geq g(c),$$

which, in combination with the condition $0 \leq f(a_j, c) \leq c$ (expressed in the definition of f stated above), and the fact that g is an increasing function, leads to the fact that

$$f(a_j, c) = c,$$

for each $c > 0$ and $a_j \in A$. By definition, $f(a_j, 0) = 0$, for each $a_j \in A$. Altogether, we obtain that $\succsim$ can be represented by the following population health evaluation function:

$$P^{g}[h_1, \ldots, h_n] = P^{g}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(t_i),$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing and continuous function.

Now, let $h \in H$ and $i, j \in N$, such that $t_i \geq t_j$ and $a_i = a_*$. Then, by *disability priority*,

$$\left[(a_*, t_i), (a_j, t_j + c), h_{N\setminus\{i,j\}}\right] \succsim \left[(a_*, t_i + c), (a_j, t_j), h_{N\setminus\{i,j\}}\right],$$

which translates into

$$g(t_i) + g(t_j + c) \geq g(t_i + c) + g(t_j),$$

for each $t_i, t_j \in T$, such that $t_i \geq t_j$, and $c > 0$. As g is continuous, it follows from the above condition that g is concave, which concludes the proof. □

The alternative axiom we consider to model prioritarianism is the weakening of the previous one, when only applied to agents enjoying perfect health. Formally,

Disability Priority at Perfect Health: For each $c > 0$, $h \in H$, and $i, j \in N$, such that $t_i \geq t_j$,

$$\left[(a_*, t_i), (a_*, t_j + c), h_{N\setminus\{i,j\}}\right] \succsim \left[(a_*, t_i + c), (a_*, t_j), h_{N\setminus\{i,j\}}\right].$$

As shown in the next result, if the previous axiom is added to the set of basic structural axioms, we characterize a general family of population health evaluation functions including a concern for morbidity. More precisely, we define the *(aggregate) HYE* population health evaluation function as the function evaluating population health distributions by means of the aggregation of individuals' *healthy years equivalents*. Formally,

$$P^h[h_1, \ldots, h_n] = P^h[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} f(a_i, t_i), \tag{4}$$

where $f : A \times T \to T$ is a function indicating the healthy years equivalents for each individual, i.e., for each $h = [h_1, \ldots, h_n] = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, and each $i \in N$,

$$h \sim [(a_*, f(a_i, t_i))_{i\in N}].$$

We now define the *concave aggregate HYE* population health evaluation function, as the population health evaluation function that evaluates population health distributions by means of the aggregate value the distribution yields, after submitting each individual HYE to a concave function. Formally,

$$P^{gh}[h_1, \ldots, h_n] = P^{gh}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(f(a_i, t_i)), \tag{5}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing, concave, and continuous function, and $f : A \times T \to T$ is a function indicating the HYEs for each individual, as described in (4).

We have the following result:

Theorem 4 *The policy maker's preferences satisfy disability priority at perfect health, and the basic structural axioms, if and only if they can be represented by the concave aggregate healthy years equivalents population health evaluation function.*

Proof. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies *disability priority at perfect health*, and the *basic structural axioms*. Then, as mentioned above, $\succsim$ can be represented by a separable population health evaluation function, as in (2). Now, let $c > 0$, $h \in H$, and $i, j \in N$, such that $a_i = a_j = a_*$, and $t_i \geq t_j$. By *disability priority at perfect health*,

$$\left[(a_*, t_i), (a_*, t_j + c), h_{N \setminus \{i,j\}}\right] \succsim \left[(a_*, t_i + c), (a_*, t_j), h_{N \setminus \{i,j\}}\right].$$

Equivalently,

$$g(f(a_*, t_i)) + g(f(a_*, t_j + c)) \geq g(f(a_*, t_i + c)) + g(f(a_*, t_j)),$$

i.e.,

$$g(t_j + c) - g(t_j) \geq g(t_i + c) - g(t_i), \quad \text{for each } c > 0, \text{ and } t_i \geq t_j.$$

It then follows, as argued in the previous proof, that g is concave, as desired. $\square$