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Normative foundations for equity-sensitive population health evaluation functions

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Normative foundations for equity-sensitive population health evaluation functions*

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Abstract

Standard models for the evaluation of population health, such as the so-called models of aggregate Quality Adjusted Life Years (QALYs), or aggregate Healthy Years Equivalent (HYEs), are usually criticized on equity grounds. We provide in this paper normative justifications for alternative equity-sensitive models, such as the so-called models of multiplicative QALYs, multiplicative HYEs, and generalizations of the two. Our axiomatic approach assumes social preferences over distributions of individual health states experienced in a given period of time. It conveys informational simplicity, as it does not require information about individual preferences on health.

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1 Introduction

The goal of developing methods to measure the health status of individuals and populations has long been established in the health economics literature (e.g., Torrance, 1976; 1986). It is frequently argued that the benefit a patient derives from a particular health care intervention is defined according to two dimensions: quality of life and quantity of life (e.g., Pliskin, Shepard and Weinstein, 1980). The so-called Quality Adjusted Life Years (in short, QALYs) constitute the standard currency to deal with both health dimensions in the methodology of cost-utility analyses, probably the most widely accepted methodology in the economic evaluation of health care nowadays (e.g., Drummond et al., 2005). Nevertheless, QALYs are usually criticized on equity grounds (e.g., Harris, 1987; Smith, 1987) and the importance of considering alternative (equity-sensitive) measures of health in cost-utility analyses is widely accepted (e.g., Wagstaff, 1991; Bleichrodt, 1997; Williams, 1997; Dolan, 1998; Østerdal, 2003).[1] The purpose of this paper is to present normative foundations for equity-sensitive population health evaluation functions. To do so, we follow the new axiomatic approach to the evaluation of population health, recently introduced by Hougaard, Moreno-Ternero and Østerdal (2013a) ("HMØ" in what follows). In such approach, the health of an individual in the population is defined according to the two dimensions mentioned above (quality of life and quantity of life), but one of them (quality of life) receives a special treatment, as it is assumed that it might not have a standard mathematical structure. The approach has the advantage of being informationally simple, as it does not make assumptions about individual preferences over length and quality of life, which might not be available information, either for practical or ethical reasons. This is in contrast with the more standard approach in the health economics literature, where a given relationship is assumed between quality and quantity of life at the individual level, entailing the existence of individual utility functions (e.g., Østerdal, 2005). Nevertheless, even though there is a vast literature on assessing individual preferences over health profiles (see, for instance, Dolan (2000) and literature cited therein) recurrent criticisms are made to each of the approaches in that literature.

One of the equity-sensitive population health evaluation functions, for which we provide normative foundations, is the so-called multiplicative QALYs function, which evaluates the health

[1]For discussions on the related issue of the conceptual foundations of measuring (in)equity in health and health care, the reader is referred to Wagstaff and van Doorslaer (2000), Williams and Cookson (2000) and, more recently, Fleurbaey and Shokkaert (2012) and Hougaard, Moreno-Ternero and Østerdal (2013b). See also Rosa Dias (2009, 2010).

of a population by the product of the QALYs each individual in the population is enjoying. Multiplicative forms of the QALY model have been frequently endorsed in the literature (e.g., Bleichrodt, 1997; Dolan, 1998). A multiplicative form induces an obvious concern for equity, as it penalizes uneven distribution of QALYs, whereas an aggregate form is not sensitive to such uneven distributions.[2]

QALYs can be seen as a specific computation of the so-called Healthy Year Equivalents (in short, HYEs), which refer to the socially equivalent population health distribution, to a given one, in which the health outcome of one (and only one) agent is replaced by that of full health, for some quantity of time.[3] The aggregate HYE model evaluates population health by means of the unweighted aggregation of HYEs. As such, it is subject to the same criticism of its counterpart aggregate QALY model on equity grounds. We also derive in this paper normative foundations for the multiplicative HYE model in which the health of a population is evaluated by the product of the HYEs each individual in the population is enjoying.

One might argue that, for large populations, a multiplicative evaluation function might be *too equity sensitive*. For that reason, we also derive normative foundations for two families of population health evaluation functions, each generalizing the multiplicative QALY and HYE models, respectively. In such families, individual QALYs (respectively, HYEs) are submitted to an arbitrary (but increasing) function before being aggregated. When such function is logarithmic, we recover, precisely, the multiplicative QALY (respectively, HYE) model.

It is worth mentioning that our model differs, in an important aspect, from the one used in HMØ. To wit, we assume here that the quantity-of-life dimension is always strictly positive, whereas, in HMØ, it was only assumed to be non-negative. This seemingly innocuous aspect turns out to make a difference in both analyses. In HMØ, the so-called ZERO condition, which said that if an agent gets zero lifetime then her health state does not influence the social desirability of the health distribution, played an important role in simplifying the analysis. Such condition, which is reminiscent of a widely used condition for individual utility functions on health (e.g., Bleichrodt, Wakker and Johannesson, 1997; Miyamoto et al., 1998; Østerdal 2005) is beyond empirical testing, as the concept of health, in real life, is not properly understood with zero lifetime. In the analysis of this paper, we replace this condition by another saying

[2]This is arguably the main reason why the UNDP unveiled a new methodology for the calculation of the so-called Human Development Index (e.g., Zambrano, 2013).

[3]This notion can be traced back to Mehrez and Gafni (1989) who propose it as a plausible way to reflect patient's preferences over health.

that, when quantity of life is sufficiently small, quality of life becomes almost insignificant.

The normative foundations we propose for the population health evaluation functions described above are obtained by means of the axiomatic method, a somewhat unexplored method in the health economics literature, in contrast to many other subfields in economics.[4] An axiomatic study begins with the specification of a domain of problems, and the formulation of a list of desirable properties (axioms) of solutions for the domain, whereas it ends with (as complete as possible) descriptions of the families of solutions satisfying various combinations of the properties (e.g., Thomson, 2001). An axiomatic study often results in characterization theorems. They are theorems identifying a particular solution, or perhaps a family of solutions, as the only solution or family of solutions, satisfying a given list of axioms. This is precisely what we do in this paper. We list some appealing axioms for the evaluation of population health and then derive precise measures to evaluate the health of a population. We first rely on a list of structural axioms, whose combination characterizes the most general family of population health evaluation functions described above. We then show that adding three additional independent axioms to this list we can characterize each of the remaining families we have mentioned above.

The main advantage of the axiomatic method is to move the debate from hypothetical specific solutions of a given problem to the principles (axioms) those solutions should satisfy. This opens the possibility of exploring the positive appeal of different solutions by focussing on testing empirically the principles that characterize each of them. As we mention later in the discussion section, this is certainly one possibility for future research arising from this work.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the axioms we consider. In Section 3, we introduce and characterize the population health evaluation functions described above. We discuss the results and some further insights in Section 4. For a smooth passage, we defer the proofs and provide them in an appendix.

[4]Besides HMØ, another notable recent exception within the health economics literature is Canning (2013). Some instances of path-breaking contributions using the axiomatic method in other areas within economics are Nash (1950), Arrow (1951), or Sen (1970). More recent instances of uses of the axiomatic method, regarding somewhat related topics to this work, are Moreno-Ternero and Roemer (2006), or Bossert and D'Ambrosio (2013).

2 The preliminaries

Let us conceptualize a policy maker with preferences defined over distributions of health for a population of fixed size $n \geq 3$. We identify the population (society) with the set $N = \{1, ..., n\}$. The health of each individual in the population will be described by a duplet indicating the level achieved in two parameters: quality of life and quantity of life.[5] Assume that there exists a set of possible health states, A, defined generally enough to encompass all possible health states for everybody in the population. We emphasize that A is an abstract set without any particular mathematical structure.[6] Quantity of life will simply be described by the set of strictly positive real numbers, $T = (0, +\infty)$.[7] Formally, let $h_i = (a_i, t_i) \in A \times T$ denote the health duplet of individual i. A population health distribution (or, simply, a health profile) $h = [h_1, \ldots, h_n] = [(a_1, t_1), ..., (a_n, t_n)]$ specifies the health duplet of each individual in society. We denote the set of all possible health profiles by H.[8] Even though we do not impose a specific mathematical structure on the set A, we assume that it contains a specific element, a_*, which we refer to as *perfect health* and which is univocally identified, as a "superior" state, by the policy maker.

The policy maker's preferences (or social preferences) over health profiles are expressed by a preference relation $\succsim$, to be read as "at least as preferred as". As usual, $\succ$ denotes strict preference and $\sim$ denotes indifference. We assume that the relation $\succsim$ is a weak order.[9]

A *population health evaluation function* (PHEF) is a real-valued function $P : H \to \mathbb{R}$. We say that P represents $\succsim$ if

$$P(h) \geq P(h') \Leftrightarrow h \succsim h',$$

for each $h, h' \in H$. Note that if P represents $\succsim$ then any strictly increasing transformation of P would also do so.[10]

[5] It is worth mentioning that an "individual" could also be understood as the representative agent for a certain group.

[6] A could for instance refer to the resulting multidimensional health states after combining the levels of each dimension of a categorical measure, such as EQ-5D, in all possible ways.

[7] The model introduced in HMØ, differs from this one in allowing zero lifetimes.

[8] For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

[9] More precisely, we assume that $\succsim$ is complete (for each health profiles h, h', either $h \succsim h'$, or $h' \succsim h$, or both) and transitive (if $h \succsim h'$ and $h' \succsim h''$ then $h \succsim h''$).

[10] It is worth mentioning that our analysis does not deal with uncertainty. Following Broome (1993), we consider a formulation of the population health evaluation problem which contains no explicit element of risk, and in which we obtain characterizations of population health evaluation functions without assumptions on the policy maker's (or individuals') risk attitudes.

2.1 Structural axioms

We now list several structural axioms for social preferences that we endorse for population health evaluation functions. The first five were introduced in HMØ, and, therefore, the reader is referred to that paper for further details about them.

- **ANON**: $h \sim h_\pi$ for each $h \in H$, and each $\pi \in \Pi^N$.
- **SEP**: $\left[h_S, h_{N\setminus S}\right] \succsim \left[h'_S, h_{N\setminus S}\right] \Leftrightarrow \left[h_S, h'_{N\setminus S}\right] \succsim \left[h'_S, h'_{N\setminus S}\right]$, for each $S \subseteq N$, and $h, h' \in H$.
- **CONT**: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}) \rightarrow (a_i, t_i) = h_i$. If $h^{(k)} \succsim h'$ for each k then $h \succsim h'$, and if $h' \succsim h^{(k)}$ for each k then $h' \succsim h$.
- **PHS**: $[(a_*, t_i), h_{N\setminus\{i\}}] \succsim h$, for each $h = [h_1, \ldots, h_n] \in H$ and $i \in N$.
- **TMPH**: If $t_i \geq t'_i$, for each $i \in N$, with at least one strict inequality, then $[(a_*, t_1), \ldots, (a_*, t_n)] \succ [(a_*, t'_1), \ldots, (a_*, t'_n)]$.

In words, *Anonymity* says that the evaluation of the population health should depend only on the list of quality-quantity duplets, not on who holds them. *Separability* says that if the distribution of health in a population changes only for a subgroup of agents in the population, the relative evaluation of the two distributions should only depend on that subgroup. *Continuity* says that, for fixed distributions of health states, small changes in lifetimes should not lead to large changes in the evaluation of the population health distribution. *Perfect health superiority* says that replacing the health status of an agent by that of perfect health, ceteris paribus, cannot worsen the evaluation of the population health. *Time monotonicity at perfect health* says that if each agent is at perfect health, increasing the time dimension is strictly better for society.

The last structural axiom we consider, *insignificant health at negligible lifetimes*, says that quality of life improvements become almost insignificant when lifetimes are negligible. More precisely, it says that any health profile will dominate the resulting profile after improving the quality of life of (only) one agent to perfect health, provided the corresponding lifetime at which such agent will enjoy it is sufficiently small. Formally,

IHNL: For each $h \in H$, and each $i \in N$, there exists $\varepsilon > 0$ such that $h \succ [(a_*, s), h_{N\setminus\{i\}}]$, for each $0 < s < \varepsilon$.

The previous axiom replaces the pair of axioms in HMØ, made of the so-called *zero condition* (described at the introduction), and the notion of *positive lifetime desirability* (society improves if any agent moves from zero lifetime to positive lifetime, for a given health state), none of which can be formalized in the current model, which does not allow for zero lifetimes.

In what follows, we refer to the set of axioms introduced above as our *core structural axioms* (in short, **CORE**).

2.2 Alternative axioms

We now introduce three alternative axioms that will be combined, independently, to the list of core structural axioms presented above. The three axioms convey a specific concern for relative comparisons of lifetimes, but each of them formalizes such concern in a different way.

More precisely, the first one, known as *relative lifetime comparisons*, says that an additional proportion of life years to individual i is socially seen as just as good as an additional proportion of life years to individual j, regardless of health states.[11] Formally,

RLC: For each $h \in H$, $c > 0$, and $i, j \in N$, $\left[(a_i, ct_i), h_{N\setminus\{i\}}\right] \sim \left[(a_j, ct_j), h_{N\setminus\{j\}}\right]$.

Now, we could restrict the scope of the previous axiom only to the case in which all agents enjoy perfect health, giving rise to the axiom of *relative lifetime comparisons at perfect health.* Formally,

RLCPH: For each $h \in H$, $c > 0$, and $i, j \in N$, such that $a_i = a_j = a_*$,

$$\left[(a_*, ct_i), (a_*, t_j), h_{N\setminus\{i,j\}}\right] \sim \left[(a_*, t_i), (a_*, ct_j), h_{N\setminus\{i,j\}}\right].$$

Finally, we consider *common duplets relative lifetime comparisons*, which states that if we have two health profiles with common duplets then the preference between them is independent of a scaling of the life year component. Formally,

CDRLC: For each $h = [(a,t)_{i\in N}], h' = [(a',t')_{i\in N}] \in H$, such that $h \succsim h'$, and $c > 0$, $[(a, ct)_{i\in N}] \succsim [(a', ct')_{i\in N}]$.

[11] This axiom was first formalized in a health context by Østerdal (2005).

3 The results

We show in this section that some specific equity-oriented PHEFs, defined next, can be characterized by some combinations of the axioms described in the previous section.

First, we introduce the PHEF in which individual Quality Adjusted Life Years (QALYs) are multiplied to evaluate the health distribution of the population. More precisely,

$$P^{mq}[h_1,\ldots,h_n] = P^{mq}[(a_1,t_1),\ldots,(a_n,t_n)] = \prod_{i=1}^{n} \left(q(a_i)t_i\right), \tag{1}$$

where $q : A \to [0,1]$ is an arbitrary function satisfying $0 < q(a_i) \leq q(a_*) = 1$, for all $a_i \in A$.

Alternatively, we could consider the more general PHEF in which Healthy Year Equivalents (HYEs), instead of QALYs, are multiplied to evaluate the health distribution of the population. Formally,

$$P^{mh}[h_1,\ldots,h_n] = P^{mh}[(a_1,t_1),\ldots,(a_n,t_n)] = \prod_{i=1}^{n} f(a_i,t_i), \tag{2}$$

where $f : A \times T \to T$ is a function indicating the HYEs for each individual, i.e.,

- f is continuous with respect to its second variable,
- $0 < f(a_i,t_i) \leq t_i$, for each $(a_i,t_i) \in A \times T$, and
- For each $h = [h_1,\ldots,h_n] = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$,

$$h \sim [(a_*, f(a_i,t_i))_{i\in N}].$$

It is worth mentioning that the multiplicative QALY PHEF can therefore be seen as a specific instance of the multiplicative HYE PHEF, in which $f(a_i,t_i) = q(a_i)t_i$, for each $(a_i,t_i) \in A \times T$.

At the risk of stressing the obvious, note that the previous two families endorse a concern for the equity of the distribution of QALYs or HYEs (more specifically, a concern for the existence of agents with poor outcomes), which is absent in their counterpart families that evaluate a health distribution with the (unweighted) aggregation of the QALYs or HYEs in the population.

As we mentioned in Section 2, PHEFs are "immune" to monotonic transformations. More precisely, if P represents $\succsim$ then any strictly increasing transformation of P would also do so. Thus, it is straightforward to see that the following are equivalent representations of families (1) and (2):

$$P^{mq}[h_1, \ldots, h_n] = P^{mq}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} \ln\left(q(a_i)t_i\right),$$

where q is constructed as in (1).

$$P^{mh}[h_1, \ldots, h_n] = P^{ph}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} \ln\left(f(a_i, t_i)\right),$$

where f is constructed as in (2).

A natural generalization of the above families would be obtained when QALYs (or HYEs) are submitted to an arbitrary (but increasing) function before being aggregated. Formally,

$$P^{gq}[h_1, \ldots, h_n] = P^{gq}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(q(a_i)t_i), \tag{3}$$

where $g : \mathbb{R}_{++} \to \mathbb{R}$ is a strictly increasing and continuous function, and q is constructed as in (1).

$$P^{gh}[h_1, \ldots, h_n] = P^{gh}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(f(a_i, t_i)), \tag{4}$$

where $g : \mathbb{R}_{++} \to \mathbb{R}$ is a strictly increasing and continuous function, and f is constructed as in (2).

We are now ready to state the formal results of our paper. The first result says that the multiplicative QALY PHEF is characterized when relative lifetime comparisons is added to the core structural axioms.[12] Formally,

Theorem 1 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (1).

2. $\succsim$ *satisfies CORE and RLC.*

Theorem 2 shows that the multiplicative HYE PHEF is characterized when, instead of relative lifetime comparisons, only its weakening to perfect health is added to the set of core axioms. Formally,

Theorem 2 *The following statements are equivalent:*

[12] The same functional form was characterized by Østerdal (2005) in a model in which individual QALY functions (representing individual preferences) are given.

1. $\succsim$ is represented by a PHEF satisfying (2).

2. $\succsim$ satisfies CORE and RLCPH.

Similarly, Theorem 3 shows that the *generalized* QALY PHEF, P^{gq}, is characterized when common duplets relative lifetime comparisons is the added axiom to the set of core axioms. Formally,

Theorem 3 *The following statements are equivalent:*

1. $\succsim$ is represented by a PHEF satisfying (3).

2. $\succsim$ satisfies CORE and CDRLC.

Finally, the *generalized* HYE PHEF, P^{gh}, the most general family among those described above, is precisely characterized by the set of core axioms.[13] Formally,

Theorem 4 *The following statements are equivalent:*

1. $\succsim$ is represented by a PHEF satisfying (4).

2. $\succsim$ satisfies CORE.

One might argue that families (3) and (4), characterized in Theorems 3 and 4, respectively, do not necessarily include a concern for the equality of the distribution (as it indeed happens for the "logarithmic members" characterized in Theorems 1 and 2). The following results exhibit the implications of adding a concern for inequality aversion to both families. More precisely, as shown in their statements, the addition of a *Pigou-Dalton transfer at perfect health* axiom (to the axioms used in their corresponding characterizations) stating that a health profile in which two agents at perfect health have different time spans is dominated by the subsequent profile in which those agents keep the same perfect health status, but share a time span equal to the average of the former two, implies that QALYs (HYEs) enter into the PHEF in a (strictly) concave way. Formally,

PDTPH: For each $h = [(a_*, t_k)_{k \in N}] \in H$, and $i, j \in N$, such that $t_i \neq t_j$,

$$\left[\left(a_*, \frac{t_i + t_j}{2}\right), \left(a_*, \frac{t_i + t_j}{2}\right), h_{N \setminus \{i,j\}}\right] \succ h.$$

[13]Theorem 4 is the counterpart of Theorem 1 in HMØ.

Corollary 1 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (3) *with* $g(\cdot)$ *strictly concave.*

2. $\succsim$ *satisfies CORE, CDRLC and PDTPH.*

Corollary 2 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (4) *with* $g(\cdot)$ *strictly concave.*

2. $\succsim$ *satisfies CORE and PDTPH.*

4 Discussion

We have presented in this paper normative foundations for several equity-sensitive population health evaluation functions. All of them share the common ground given by our core structural axioms. The normative appeal of those core axioms seems to be strong, but we have not tested their positive appeal via experiments or questionnaires, and that could certainly be a plausible line for future research. Beyond those core structural axioms, the population health evaluation functions we single out differ from each other on the specific form of *relative lifetime comparisons* they allow. More precisely, if a policy maker is interested in the principle saying that an additional proportion of life years to an individual is as just as good as an additional proportion of life years to another individual, regardless of health states, then the multiplicative QALY function should be the one adopted to evaluate the health distribution of a population. If, instead, the policy maker is only interested in imposing such principle when all agents enjoy perfect health, then the multiplicative HYE function should be the one adopted. Finally, if the principle only refers to comparisons of health profiles with common duplets then the function emerging is the aggregation of an increasing transformation of individual QALYs. If the increasing transformation is concave, then the function conveys a concern for the equity of the distribution modeled by a standard Pigou-Dalton transfer axiom.

As mentioned in the introduction, the importance of developing equity-sensitive forms of evaluating a distribution of health has long been established within the health economics literature. One focal contribution within such literature aiming to address the issue is the so-called fair innings notion (e.g., Harris, 1985; Williams, 1997). Essentially, the notion reflects the feeling that everyone is entitled to some 'normal' span of health. In some sense, one could consider that the multiplicative QALY and HYE models characterized in this paper are implementing a variant of the fair innings notion: they both aim to give a fair number (actually, the *average*) of quality-adjusted life years, or healthy years equivalents, to each person. Nevertheless, one might also argue that the fair innings notion is captured by Williams (1997) upon endorsing a *Bergsonian* functional form to evaluate the health distribution of a population. In the parlance of this paper, that would amount to consider the subfamilies arising from (3) and (4) after imposing that g is, not only a strictly increasing, continuous, and concave function (as in Corollaries 1 and 2), but also a power function.[14] Such families were characterized in HMØ. The characterizations presented therein could be adapted to the framework analyzed here, provided the pair of axioms in HMØ, made of the *zero condition* and *positive lifetime desirability*, is replaced by the axiom of *insignificant health at negligible lifetimes* considered here.

5 Appendix. Proofs of theorems

In order to prove the results stated above, we need first the following lemma, which is interesting on its own.[15] Formally,

Lemma 1 *If $\succsim$ satisfies CORE then, for each $h \in H$ and $i \in N$, there exists $t_i^* \in T$ such that $h \sim [(a_*, t_i^*), h_{N\setminus\{i\}}]$.*

Proof. Suppose, by contradiction, that such t_i^* in the statement of the lemma does not exist. Then, $T = A \cup B$, where,

$$A = \{s \in T | h \succ [(a_*, s), h_{N\setminus\{i\}}]\},$$

and

$$B = \{s \in T | [(a_*, s), h_{N\setminus\{i\}}] \succ h\}.$$

By IHNL, $A \neq \varnothing$. By PHS and TMON, $B \neq \varnothing$. By CONT, A and B are open sets relative to T. As $A \cap B = \varnothing$, it would follow that T is not a connected set, a contradiction. ■

We are now ready to prove the most general result of our analysis and, from there, the remaining results.

Proof of Theorem 4. We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies CORE. Let $h \in H$. By Lemma 1, for each $i \in N$, there exists $t_i^* \in T$ such that $h \sim [(a_*, t_i^*), h_{N\setminus\{i\}}]$. By SEP, t_i^* only depends on (a_i, t_i) (and, thus, is independent of the

[14] Formally, there exists $\gamma \in (0, 1)$ such that $g(x) = x^\gamma$, for each $x \in \mathbb{R}_{++}$.

[15] This lemma is the counterpart of Lemma 1 in HMØ.

remaining duplets of the profile). Thus, for each $i = 1, \ldots, n$, let $f_i : A \times T \to T$ be defined such that $f_i(a_i, t_i) = t_i^*$, for each $(a_i, t_i) \in A \times T$. By ANON, $f_i(\cdot, \cdot) \equiv f_j(\cdot, \cdot) \equiv f(\cdot, \cdot)$, for each $i, j \in N$. By TMON and PHS, $0 < f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$ and, by CONT, f is a continuous function with respect to its second variable. Furthermore,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

which implicitly says that social preferences only depend on the profile of healthy years equivalents, and, by CONT, they do so continuously. It also follows that the range of f is a connected subset of $\mathbb{R}$. By Theorem 3 in Debreu (1960), there exists a strictly increasing and continuous function $g : \mathbb{R}_+ \to \mathbb{R}$ such that

$$h \succsim h' \iff \sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a_i', t_i')\right),$$

which concludes the proof. ■

Proof of Theorem 1 We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies CORE and RLC. Then, by Theorem 4, $\succsim$ can be represented by a PHEF satisfying (4).

By iterated application of RLC, and the transitivity of $\succsim$

$$g(f(a_1, t_1)) + ... + g(f(a_n, t_n)) = g\left(f\left(a_1, \prod_1^n t_i\right)\right) + g(f(a_2, 1)) + ... + g(f(a_n, 1)).$$

For a fixed common health state $\bar{a}$, $g(f(\bar{a}, \cdot))$ therefore satisfies the following functional equation:

$$g(f(\bar{a}, t_1)) + g(f(\bar{a}, t_2)) = g(f(\bar{a}, t_1 t_2)) + g(f(\bar{a}, 1)),$$

for all $t_1, t_2 > 0$. Let $r : A \times T \to T$ be the function such that $r(x, y) = g(f(x, \exp(y)))$ for each $(x, y) \in A \times T$. Thus, for each fixed common health state $\bar{a} \in A$ and any $t_1, t_2 \in T$, we have

$$r(\bar{a}, t_1 + t_2) + r(\bar{a}, 0) = r(\bar{a}, t_1) + r(\bar{a}, t_2), \tag{5}$$

which is precisely one of Cauchy's canonical functional equations. As r is continuous, it follows that the unique solutions to such equation are the linear functions (e.g., Aczel, 2006; page 43). More precisely, there exist two functions $\alpha : A \to \mathbb{R}$ and $\beta : A \to \mathbb{R}$ such that

$$g(f(\bar{a}, t)) = r(\bar{a}, \ln t) = \alpha(\bar{a}) \ln t + \beta(\bar{a}),$$

for each $t \in T$.

Now, by RLC, it follows that, for each $\bar{a}, \bar{a}' \in A$,

$$\begin{aligned} g(f(\bar{a}, t_1)) + g(f(\bar{a}', t_2)) &= g(f(\bar{a}, t_1 t_2)) + g(f(\bar{a}', 1)) \\ &= g(f(\bar{a}, 1)) + g(f(\bar{a}', t_1 t_2)). \end{aligned}$$

Thus, $\alpha(\bar{a}) = \alpha(\bar{a}') = \alpha$, and, therefore,

$$P((a_1, t_1), ..., (a_n, t_n)) = \alpha \left(\sum_{i=1}^{n} \ln(t_i) \right) + \sum_{i=1}^{n} \beta(a_i).$$

To conclude, let $q : A \to \mathbb{R}$ be such that $q(a) = \exp\left(\frac{\beta(a)-\beta(a_*)}{\alpha}\right)$, for each $a \in A$. By PHS, it follows that $0 < q(a) \leq q(a_*) = 1$, for all $a \in A$. Now, as the PHEF is uniquely determined, up to strictly increasing transformations, we can consider the monotonic transformation of P, $P' = \exp\left(\frac{P - n\beta(a_*)}{\alpha}\right)$. Then,

$$\begin{aligned} P'((a_1, t_1), ..., (a_n, t_n)) &= \exp((P((a_1, t_1), ..., (a_n, t_n)) - n\beta(a_*))/\alpha) \\ &= \exp\left(\sum_{i=1}^{n} \ln t_i + \sum_{i=1}^{n} \left(\frac{\beta(a) - \beta(a_*)}{\alpha} \right) \right) \\ &= \prod_{i=1}^{n} q(a_i) t_i, \end{aligned}$$

as desired. ■

Proof of Theorem 2 We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies CORE and RLCPH. Then, by Theorem 4, $\succsim$ can be represented by a PHEF satisfying (4). Let $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \ldots, (a'_n, t'_n)] \in H$. Then, by iterated application of RLCPH, and the transitivity of $\succsim$,

$$h \succsim h' \iff [(a_*, \prod_{i \in N} f(a_i, t_i)), (a_*, 1)_{k \in N \setminus \{i\}}] \succsim [(a_*, \prod_{i \in N} f(a'_i, t'_i)), (a_*, 1)_{k \in N \setminus \{i\}}].$$

By TMON, and the transitivity of $\succsim$,

$$h \succsim h' \iff \prod_{i \in N} f(a_i, t_i) \geq \prod_{i \in N} f(a'_i, t'_i),$$

as desired. ■

Proof of Theorem 3 We focus on its non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies CORE and CDRLC. Then, by Theorem 4, $\succsim$ can be represented by a PHEF satisfying (4). We now make two further claims.

Claim 1. We claim that for each $(a,t),(a',t') \in A\times T$, and $c>0$,

$$f(a,t)\geq f(a',t') \iff f(a,ct)\geq f(a',ct').$$

To prove the claim, let $h=[(a,t),\ldots,(a,t)], h'=[(a',t'),\ldots,(a',t')]\in H$ and $c>0$. Denote $h_c=[(a,ct),\ldots,(a,ct)]$ and $h'_c=[(a',ct'),\ldots,(a',ct')]$. By (4),

$$h\succsim h' \iff f(a,t)\geq f(a',t'),$$

and

$$h_c\succsim h'_c \iff f(a,ct)\geq f(a',ct').$$

By CDRLC, the claim follows.

Claim 2. Let $q:A\to\mathbb{R}$ be such that $q(a)=f(a,1)$, for each $a\in A$. We claim that

$$f(a,t)\geq f(a',t') \iff q(a)t\geq q(a')t',$$

for each $(a,t),(a',t')\in A\times T$.

In order to prove the claim note that, by definition, $f(a_i,1)=f(a_*,q(a_i))$. By Claim 1,

$$f(a,t)=f(a',t') \iff f(a,ct)=f(a',ct').$$

Thus, $f(a,t)\geq f(a',t') \iff f(a_*,q(a)t)\geq f(a_*,q(a')t') \iff q(a)t\geq q(a')t'$, as desired.

By Claim 2, it follows that $f(\cdot,\cdot)$ is a monotonic transformation of the function $\tau: A\times T\to\mathbb{R}$ defined by $\tau(a,t)=q(a)t$, for each $(a,t)\in A\times T$. Then, by the above, P^{gq} represents $\succsim$, as desired. ■

Proof of Corollary 1 As before, we focus on the non-trivial implication, i.e., $2\to 1$. Formally, assume $\succsim$ satisfies CORE, CDRLC, and PDTPH. Then, by Theorem 3, $\succsim$ can be represented by a PHEF satisfying (3).

Let $i,j\in N$ and consider the two health profiles $h=[(a_*,t_k)_{k\in N}]$, where $t_i\neq t_j$, and $h'=\left[\left(a_*,\frac{t_i+t_j}{2}\right),\left(a_*,\frac{t_i+t_j}{2}\right),h_{N\setminus\{i,j\}}\right]$, By PDTPH, $h'\succ h$, which, by (3), means that

$$2g\left(q(a_*)\frac{t_i+t_j}{2}\right)+\sum_{k\in N\setminus\{i,j\}}g(q(a_*)t_k)>g(q(a_*)t_i)+g(q(a_*)t_j)+\sum_{k\in N\setminus\{i,j\}}g(q(a_*)t_k).$$

Or, equivalently (as $q(a_*)=1$),

$$g\left(\frac{t_i+t_j}{2}\right)>\frac{g(t_i)}{2}+\frac{g(t_j)}{2},$$

from where it follows that g is strictly concave, as desired. $\blacksquare$

Proof of Corollary 2 As before, we focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume $\succsim$ satisfies CORE and PDTPH. Then, by Theorem 4, $\succsim$ can be represented by a PHEF satisfying (4).

Let $i, j \in N$ and consider the two health profiles $h = [(a_*, t_k)_{k \in N}]$, where $t_i \neq t_j$, and $h' = \left[\left(a_*, \frac{t_i+t_j}{2}\right), \left(a_*, \frac{t_i+t_j}{2}\right), h_{N \setminus \{i,j\}}\right]$, By PDTPH, $h' \succ h$, which, by (4), means that

$$2g\left(f\left(a_*, \frac{t_i+t_j}{2}\right)\right) + \sum_{k \in N \setminus \{i,j\}} g(f(a_*, t_k)) > g(f(a_*, t_i)) + g(f(a_*, t_j)) + \sum_{k \in N \setminus \{i,j\}} g(f(a_*, t_k)).$$

Or, equivalently (as $f(a_*, t) = t$, for each $t \in T$),

$$g\left(\frac{t_i+t_j}{2}\right) > \frac{g(t_i)}{2} + \frac{g(t_j)}{2},$$

from where it follows that g is strictly concave, as desired. $\blacksquare$

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