Baryons from holographic instantons

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My group at Henan University

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Outline

Physics at low energies

- At low energies, we have nucleons and electrons
- EM is quite well understood: Maxwell theory
- Nuclei are still understood only at the phenomenological level, the exact relation to quarks and gluons remains elusive
- E.g. the proof of Yang-Mills theory being gapped in the vacuum is an unsolved Millennium Problem, with a 1 million dollars prize from Clay Math Institute
- The symmetries relevant for the strong interactions are $G = \mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$ with $N_f = 2, 3$ depending on whether the energies are big enough to include strangeness (kaons or s)
- QCD with $N_f = 2, 3$ has an extra $\mathrm{U}(1)$ symmetry in the chiral limit, presenting the puzzle: if it's manifest, all hadronic states should appear in doublets (which they don't); or the symmetry should be spontaneously broken (but there's no Goldstone boson); what's the solution?
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BPST Instantons

The Belavin-Polyakov-Schwartz-Tyupkin [PLB59, 85 (1975)] instanton can easiest be understood as the solution to the self-dual equation

$$\begin{aligned} S &= \frac{1}{2e^2}\int \mathrm{d}^4x\ \mathrm{tr}\, F_{\mu\nu}F_{\mu\nu} \\ &= \frac{1}{4e^2}\int \mathrm{d}^4x\ \left[\mathrm{tr}\left(F_{\mu\nu}\mp\widetilde{F}_{\mu\nu}\right)^2 \pm 2\,\mathrm{tr}\, F_{\mu\nu}\widetilde{F}_{\mu\nu}\right] \\ &= \frac{1}{4e^2}\int \mathrm{d}^4x\ \left[\mathrm{tr}\left(F_{\mu\nu}\mp\widetilde{F}_{\mu\nu}\right)^2 \pm \epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\,\partial_\mu\left(A_\nu F_{\rho\sigma} + \frac{\mathrm{i}2}{3}A_\nu A_\rho A_\sigma\right)\right], \end{aligned}$$

with $\widetilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. The self-dual equation implies the full second-order equation of motion

$$D_\mu F^{\mu\nu} = \pm D_\mu \widetilde{F}^{\mu\nu} = 0.$$

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Instanton solutions

The instanton solution can be found on Euclidean $\mathbb{R}^4$ for $G = \mathrm{SU}(2)$ using the Ansatz

$$A_\mu = \frac{\mathrm{i}}{2}\sigma_{\mu\nu}\partial_\nu \log\rho,$$

with the 't Hooft symbols

$$\sigma_{i4} = \sigma_i, \qquad \sigma_{ij} = \epsilon_{ijk}\sigma_k, \qquad i,j,k = 1,2,3,$$

and the 't Hooft Ansatz

$$\rho = 1 + \sum_{I=1}^{N} \frac{\lambda_I^2}{|x - a_I|^2},$$

which encodes 5 parameters or moduli per instanton (in total N instantons). The $N = 1$ solution is the BPST instanton solution.

Instanton moduli

For $N = 1$ (a single instanton), this is the complete number of moduli according to the Atiyah-Hitchin-Singer [Proc.Natl.Acad.Sci.USA 74, 2662 (1977)] index theorem

$$\dim \mathcal{M}_{N,\mathrm{SU}(2)} = 8N.$$

which agrees with the BPST solution, since a_1 are spatial translations in $\mathbb{R}^4$, λ_1^2 is the size of the instanton and 3 global rotations in $\mathrm{SU}(2)$ correspond to the instanton's orientation in the gauge group

$$A_\mu \to g A_\mu g^{-1}.$$

Instanton moduli

For $N = 2$, $5 + 5 < 16$ but Jackiw-Nohl-Rebbi [PRD15, 1642 (1980)] found that

$$\rho = \sum_{I=0}^{2} \frac{\lambda_I^2}{|x - a_I|^2},$$

is the complete number of parameters for $N = 2$ instantons, since $3 \times 5 + 3 = 18$, but only the ratio of sizes is physical due to the derivative of the logarithm and a further parameter is simply a gauge transformation.

For $N > 2$, the Atiyah-Drinfeld-Hitchin-Manin (ADHM) construction is the only way to parametrize the entire $8N$ moduli. The reason that this construction is possible is due to the integrability properties of Yang-Mills theory.

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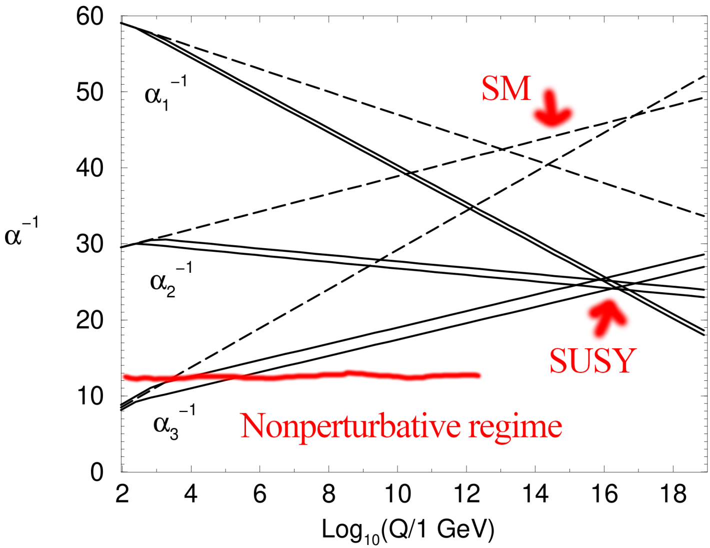
Gauge couplings – strong coupling at low energies



[Martin, Adv.Ser.Direct.HEP21, 1-153 (2010)]

Dualities?

- Since perturbation theory fails miserably at low energies for QCD, a duality would be the perfect candidate.
- An old example of a duality, is the EM duality

$$\begin{pmatrix} F \\ \widetilde{F} \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} F \\ \widetilde{F} \end{pmatrix}$$

  under which the Maxwell's equations are invariant and so is the Hamiltonian
- However, the Lagrangian is not invariant since the SO(2) rotation rotates a tensor into a pseudo-tensor

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Outline

Maldacena – AdS/CFT

- In 1997 Maldacena [Adv.Theor.Math.Phys.2,231 (1998)] proposes the AdS/CFT duality between string theory in AdS_5 space and the conformal $\mathcal{N} = 4$ super-Yang-Mills field theory on the other side
- In particular, the D3-brane in the large N_c (large number of branes) is described in type IIB string theory by $\mathrm{AdS}_5 \times S^5$.
- Chiral primary operators of $\mathcal{N} = 4$ super-Yang-Mills theory are mapped to Kaluza-Klein modes of type IIB supergravity on $\mathrm{AdS}_5 \times S^5$ [Witten, Adv.Theor.Math.Phys.2, 253 (1998)]
- Most importantly, the 't Hooft coupling $\lambda = g^2 N_c$ is mapped to $1/\alpha'$, so that strongly coupled gauge theory corresponds to weakly coupled string theory
- The symmetry group $\mathrm{SO}(2, d)$ of AdS_{d+1} acts as the conformal group on the boundary space M_d, which can be shown to be a compactification of d-Minkowski space

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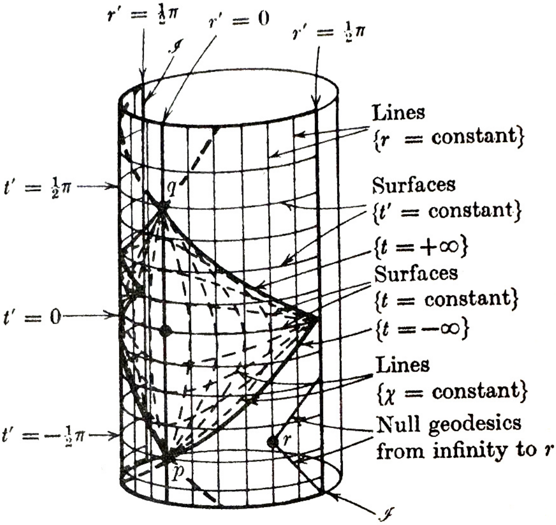
AdS space



[Hawking-Ellis, 1973]

Outline

1. Introduction

2. Holography
   - Holographic QCD
   - Holographic Nuclei

Sakai-Sugimoto – AdS/QCD

The Sakai-Sugimoto (SS) model [Prog.Theor.Phys.113, 843 (2005)] builds on the work of Witten [Adv.Theor.Math.Phys.2, 253 (1998)] where N_f D8- and $\overline{\text{D8}}$-branes are intersecting N_c D4-branes

	0	1	2	3	(4)	5	6	7	8	9
D4	○	○	○	○	○					
D8-$\overline{\text{D8}}$	○	○	○	○		○	○	○	○	○

- The SS model is in type IIA string theory, but is T-dual to a D3/D7 model by [Sugimoto-Takahashi, JHEP04, 051 (2004)], except for SUSY-breaking anti-periodic boundary conditions on the S^1 for the fermions on D4.
- Chiral symmetry is explicit by the two 8-branes, when they stretch
- Chiral symmetry breaking is string geometric as the 8-branes touch and merge – the low-energy supergravity geometry is that of a cigar-shaped space, which is AdS$_5$-like

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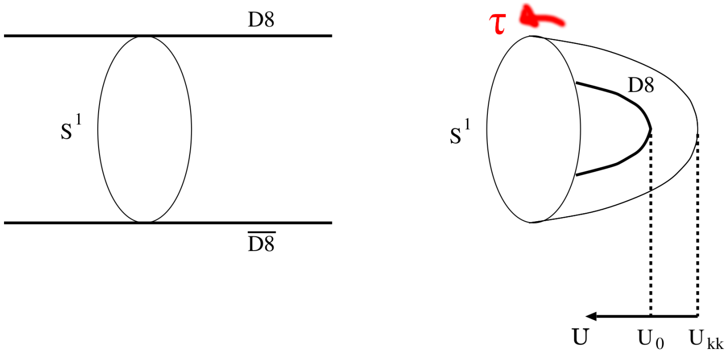
Sakai-Sugimoto – chiral symmetry breaking



[Sakai-Sugimoto, Prog.Theor.Phys.113, 843 (2005)]

- Notice that the confined geometry ends at U_{KK}

Sakai-Sugimoto – AdS/QCD

- The 't Hooft limit is considered $N_c \gg N_f$, so that the 8-branes can be considered in the probe branes embedded in the D4-background (color d.o.f.)

$$\mathrm{d}s^2 = \left(\frac{u}{R}\right)^{3/2} (\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + f(u)\mathrm{d}\tau^2) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{\mathrm{d}u^2}{f(u)} + u^2 \mathrm{d}\Omega_4^2\right)$$

$$e^\phi = g_s \left(\frac{u}{R}\right)^{3/4}, \qquad F_4 = \mathrm{d}C_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \qquad f(u) = 1 - \frac{u_{\mathrm{KK}}^3}{u^3}.$$

- The flavor d.o.f. are described by the DBI action and the Chern-Simons term at level N_c – both scale as N_c
- The leading order approximation to the DBI action is the 5-dimensional Yang-Mills term, hence the Sakai-Sugimoto model:

$$S = -\kappa\,\mathrm{tr}\int_{\mathrm{AdS}_5} \mathcal{F}\wedge *\mathcal{F} + N_c \int_{\mathrm{AdS}_5} \omega_5,$$

with YM coefficient $\kappa = \frac{\lambda N_c}{216\pi^3}$, and 't Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N_c$ (fixed),

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- The flavor d.o.f. are described by the DBI action and the Chern-Simons term at level N_c – both scale as N_c
- The leading order approximation to the DBI action is the 5-dimensional Yang-Mills term, hence the Sakai-Sugimoto model:

$$S = -\kappa\, \mathrm{tr} \int_{\mathrm{AdS}_5} \mathcal{F} \wedge *\mathcal{F} + N_c \int_{\mathrm{AdS}_5} \omega_5,$$

with YM coefficient $\kappa = \frac{\lambda N_c}{216\pi^3}$, and 't Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N_c$ (fixed),

$$g = h(z)k(z)\mathrm{d}x^\mu \mathrm{d}x_\mu + h^2(z)\mathrm{d}z^2, \qquad k(z) = h^{-3}(z) = 1 + z^2.$$

Sakai-Sugimoto – Mesons and baryons

- The scale of the theory (glueballs) is

$$R_\tau = \frac{4\pi}{3}\frac{R^{3/2}}{u_{\mathrm{KK}}^{1/2}}, \qquad M_{\mathrm{KK}} = \frac{3}{2}\frac{u_{\mathrm{KK}}^{1/2}}{R^{3/2}},$$

- And in terms of string theory

$$R^3 = \frac{1}{2}\frac{g_{\mathrm{YM}}^2 N_c l_s^2}{M_{\mathrm{KK}}}, \qquad u_{\mathrm{KK}} = \frac{2}{9} g_{\mathrm{YM}}^2 N_c M_{\mathrm{KK}} l_s^2, \qquad g_s = \frac{1}{2\pi}\frac{g_{\mathrm{YM}}^2}{M_{\mathrm{KK}} l_s}.$$

- The flavor fields can be expanded as

$$\mathcal{A}_\mu = \sum_n v_\mu^{2n-1}(x)\psi_{2n-1}(z) + \sum_n a_\mu^{2n}(x)\psi_{2n}(z), \qquad \mathcal{A}_z = \Pi(x)\phi_0(z),$$

with profile functions

$$\text{pions} : \phi_0(z) = \frac{1}{\sqrt{\pi\kappa}}\frac{1}{k(z)}, \qquad \text{vectors} : -h^{-1}(z)\partial_z(k(z)\partial_z\psi_n) = \lambda_n\psi_n,$$

with vector meson masses $M_n \sim \sqrt{\lambda_n}$

- Fitting the pion decay constant and the rho meson mass, one obtains

$$M_{\mathrm{KK}} = 949\,\mathrm{MeV}, \qquad \lambda = 16.63$$

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with the left-invariant chiral current $L_\mu = U^{-1}\partial_\mu U$, and the constants determined by string theory

$$A = \frac{9u_{\mathrm{KK}}}{4\pi}, \qquad B = \frac{R^3 b}{2\pi^4}, \qquad b \sim 15.25$$

- Notice that the Skyrme coupling is determined by the model

$$e^2 = \frac{27\pi^7}{2b}\frac{1}{\lambda N_c} \sim (7.32\cdots)^2$$

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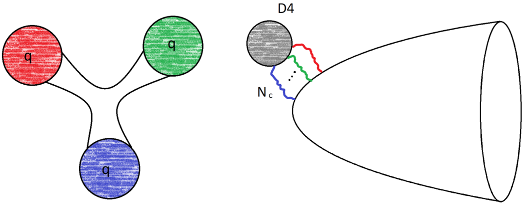
The baryon is:

- the coupling of N_c strings from the D4-branes to the 8-branes



- the instanton in an (x^1, x^2, x^2, z) slice of the AdS$_5$-like geometry
- the Skyrmion in the pion effective theory, which is a soliton of 1 dimension less than the instanton, but same S^3 target space

$$\pi_3(S^3) = \mathbb{Z} \ni k = B$$

- The 3rd homotopy group is due to the mappings being from $\sim \partial\mathbb{R}^4 \simeq S^3$ in the instanton case, and from $\mathbb{R}^3 \cup \{\infty\} \simeq S^3$ in the Skyrmion case
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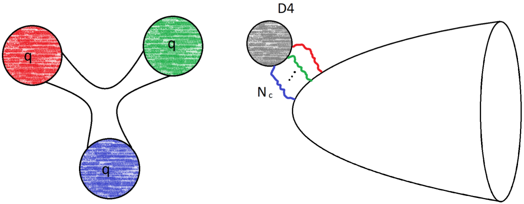

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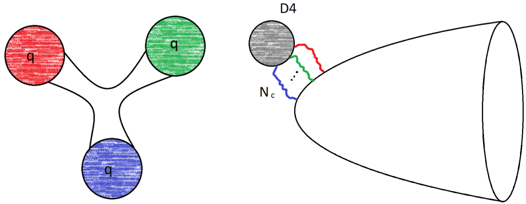

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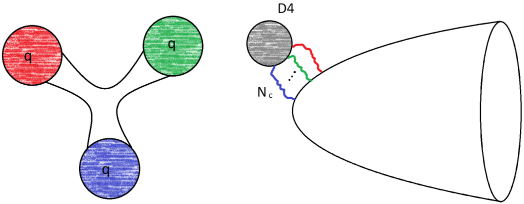

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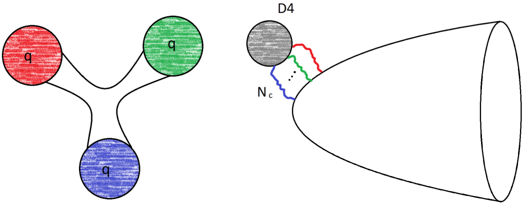

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The instanton in SS

- In the large-λ limit, the curved-space instanton is well approximated by the flat-space BPST instanton solution in the non-Abelian fields [Hata-Sakai-Sugimoto-Yamato, Prog.Theor.Phys.117, 1157 (2007)]

$$A_M = -\mathrm{i}f(\xi)g\partial_M g^{-1}, \qquad f(\xi) = \frac{\xi^2}{\xi^2+\rho^2},$$
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- The Abelian electric field is new, this field acts as a size stabilization against gravitational collapse

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- Minimization of the pseudo-moduli Z, ρ gives

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Instantons in string theory

- The instanton is in fact expected on general grounds from string theory
- The simplest description is the D0-D4-brane system in type IIA or its T-dual D(-1)-D3-brane system in type IIB [Polchinsky II, 1998]
- The orthogonality of the branes means there are no forces between the D0-branes – they are BPS objects
- The Higgs branch condition in the D4-branes gives rise to the self-dual equation

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Hashimoto-Sakai-Sugimoto

[Hashimoto-Sakai-Sugimoto, Prog.Theor.Phys.120, 1093 (2008)] computed baryon observables and showed general improvements over the Skyrme model computations by [Adkins-Nappi-Witten, NPB228, 552 (1983)]

	our model	Skyrmion [14]	experiment
$\langle r^2\rangle^{1/2}_{I=0}$	0.742 fm	0.59 fm	0.806 fm
$\langle r^2\rangle^{1/2}_{M,I=0}$	0.742 fm	0.92 fm	0.814 fm
$\langle r^2\rangle_{E,\mathrm{p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.875\ \mathrm{fm})^2$
$\langle r^2\rangle_{E,\mathrm{n}}$	0	$-\infty$	$-0.116\ \mathrm{fm}^2$
$\langle r^2\rangle_{M,\mathrm{p}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.855\ \mathrm{fm})^2$
$\langle r^2\rangle_{M,\mathrm{n}}$	$(0.742\ \mathrm{fm})^2$	∞	$(0.873\ \mathrm{fm})^2$
$\langle r^2\rangle^{1/2}_{A}$	0.537 fm	–	0.674 fm
μ_p	2.18	1.87	2.79
μ_n	−1.34	−1.31	−1.91
$\left\lvert\frac{\mu_p}{\mu_n}\right\rvert$	1.63	1.43	1.46
g_A	0.734	0.61	1.27
$g_{\pi NN}$	7.46	8.9	13.2
$g_{\rho NN}$	5.80	–	$4.2 \sim 6.5$

Approximations made

A lot of approximations has been made under the way of constructing this version of holographic QCD (many are similar in other HQCD models):

- Large-N_c: Nature is only $N_c = 3$
- Large-λ, this is necessary for using the BPST solution and corresponding analytic basis of ψ_n
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- The SS model is not asymptotically free – it is only applicable as a model for low-energy QCD
- The SS has no quark masses (see next)
- The single instanton is simple, but multi-instantons are also quite complicated! How to get large nuclei or neutron star equations of state (EOS)

Approximations made

A lot of approximations has been made under the way of constructing this version of holographic QCD (many are similar in other HQCD models):

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Quark and pion masses in SS

- The issue with the quark mass in SS, is that it involves both the left- and right-handed fermions

$$m_q(\psi_L^\dagger\psi_R + \psi_R^\dagger\psi_L),$$

which must be nonlocal in the bulk!

- [Aharony-Kutasov, PRD78, 026005 (2008)] solved this problem with a Wilson line

$$S_{\rm AK} = c\int \mathrm{d}^4x\ \mathrm{tr}\,P\left[M\left(e^{\mathrm{i}\varphi}+e^{-\mathrm{i}\varphi}-2\mathbf{1}\right)\right], \qquad \varphi = -\int \mathrm{d}z\,A_z, \tag{1}$$

- It can be interpreted also as an effect from world-sheet instantons
- Inclusion of the AK action (quark masses) deforms the size of the instanton and the mass of the instanton
- It also induces a pion mass (k_0 of ϕ_0 becomes nonvanishing) and in turn yields the Gell-Man-Oakes-Renner relation

$$4mc = f_\pi^2 m_\pi^2.$$

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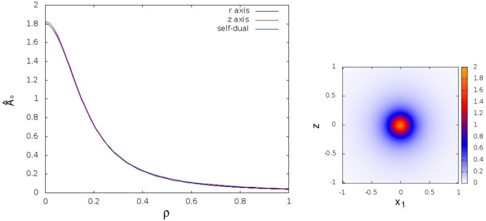
The instanton tail

- Although most of the instanton's energy is situated near $z = 0$ (for $Z = 0$), the tail of the flat instanton is incorrect at long distances, leading to contradicting and erroneous results in the literature: e.g. exponentially suppressed EM form factors
- This problem was addressed in [Bolognesi-Sutcliffe, JHEP01, 078 (2014)], where the non-commutativity of λ and large radius is discussed in detail



- The resolution is the discovery of a new scale, after which numerical solutions or nonlinear analysis is required

$$\begin{aligned} 0 < \;& \rho \lesssim L/\sqrt{\Lambda}, && \text{flat and nonlinear} \\ L/\sqrt{\Lambda} \lesssim \;& \rho \lesssim L, && \text{flat and linear} \\ L \lesssim \;& \rho \lesssim L\log\Lambda, && \text{curved and linear} \\ L\log\Lambda \lesssim \;& \rho && \text{curved and nonlinear.} \end{aligned}$$

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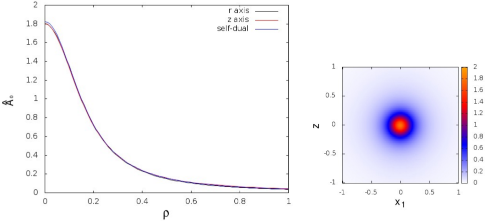

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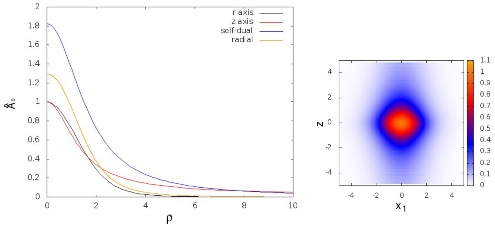

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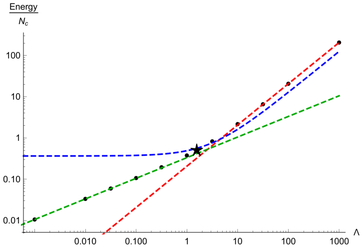
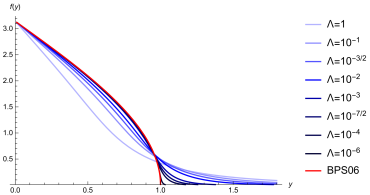
Large and small 't Hooft coupling

- [Bartolini-Bolognesi-Proto, PRD97, 014024 (2018)] studied the behavior of the SS model in the limits of small and large 't Hooft coupling
- The large-λ limit essentially converges to the results given by the BPST instanton. In the case of taking into account the correction for the quark mass, the energy is given by [Hashimoto-Hirayama-Hong, PRD81, 045016 (2010)] – in this limit the instantons are point-like
- The small λ-limit corresponds to the so-called BPS-Skyrme model, which is a field theory incarnation of a liquid-drop model





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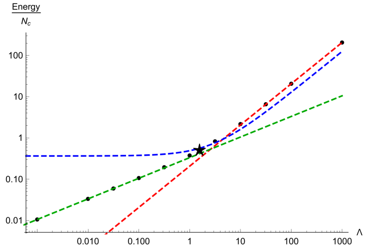

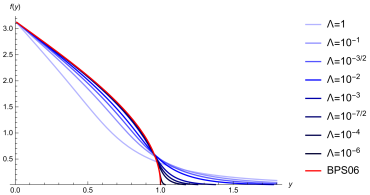

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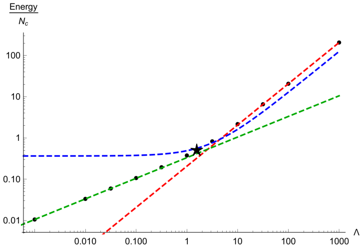

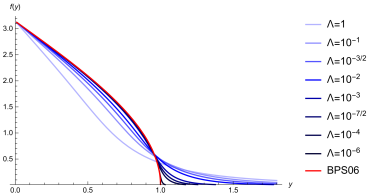

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- The Sakai-Sugimoto model with the tower of vector mesons integrated out is

$$S = \frac{\lambda N_c}{216\pi^4}\int \mathrm{d}^4x\ \mathrm{tr}\, L_\mu^2 + \frac{\lambda N_c}{216\pi^3}\int \mathrm{d}^4x\ \mathrm{tr}[L_\mu, L_\nu]^2 + \frac{51 N_c}{8960\lambda}\int \mathrm{d}^4x\ (\mathrm{tr}\, L_\mu L_\nu L_\rho \epsilon^{\mu\nu\rho\sigma})^2 + 4mc\int \mathrm{d}^4x\ (\sigma - 1).$$

- The large-λ limit gives the Skyrme model, as expected (to leading order)
- The small-λ limit makes the volume-preserving diffeomorphism invariant sextic term the dominant one
- Upon scaling the size of the soliton, one obtains the sextic term + mass term, which is exactly the BPS-Skyrme model of [Adam-Sanchez-Guillen-Wereszczynski, PLB691, 105 (2010)]

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Outline

1. Introduction

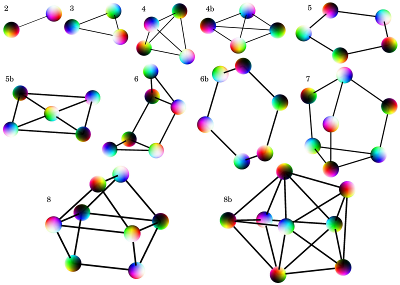
2. Holography
    - Holographic QCD
    - Holographic Nuclei

Nucleon-nucleon potential

- [Baldino-Bolognesi-Gudnason-Koksal, PRD96, 034008 (2017)] and [Baldino-Bartolini-Bolognesi-Gudnason, PRD103, 126015 (2021)] obtain the nucleon-nucleon potential from the SS model

$$V(r, B^\dagger C) = \frac{4\pi N_c}{\Lambda}\left(\sum_{n=1}^{\infty}\left(\frac{1}{c_{2n-1}}\frac{e^{-k_{2n-1}r}}{r} + \frac{6}{5}\frac{1}{c_{2n-1}}M_{ij}(B^\dagger C)P_{ij}(r_i, k_{2n-1})\frac{e^{-k_{2n-1}r}}{r^3}\right.\right.$$
$$\left.\left. - \frac{6}{5}\frac{1}{d_{2n}}\frac{e^{-k_{2n}r}}{r^3}M_{ij}(B^\dagger C)P_{ij}(r_i, k_{2n})\right) - \frac{6}{5\pi}\frac{e^{-k_0 r}}{r^3}M_{ij}(B^\dagger C)P_{ij}(r_i, k_0)\right) .$$

- Point-like nucleons in the following configurations minimize the 2-body potential



Baryons from holographic instantons

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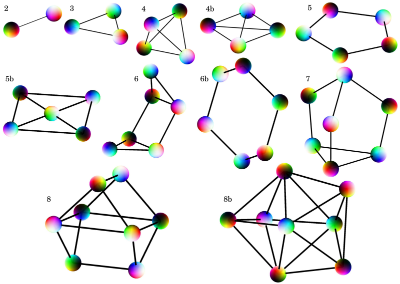

Hashimoto – matrix model for nuclei

- Instead of studying the instanton in the SS model as the baryon, one can write down the matrix model corresponding to description of the zeromodes of the D8-branes wrapping the S^4
- The matrix model is very similar to the ADHM construction of instantons
- However, with the addition of a 1-dimensional Chern-Simons term and an important mass deformation – not present in ADHM
- The symmetry of the fields is $\mathrm{U}(k) \times \mathrm{SU}(N_f) \times \mathrm{SO}(3)$
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The Sutcliffe model

- The Sutcliffe model is a flat-holography model of YM on $\mathbb{R}^5$
- The kink function is introduced for the pion profile

$$\psi_+(z) = \frac{1}{\sqrt{2}\pi^{\frac{1}{4}}} \int_{-\infty}^{z} \psi_0(\xi) \mathrm{d}\xi = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right),$$

and the gauge fields are as usual expanded as

$$A_i = -\partial_i U U^{-1} \psi_+(z) + \sum_{n=0}^{K-1} V_i^n(\mathbf{x}) \psi_n(z),$$

- Notice that no electric field or CS is present, since there is no curvature to fight against
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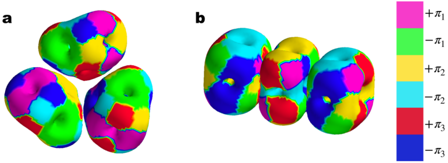

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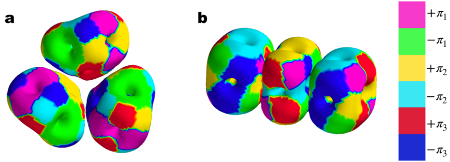

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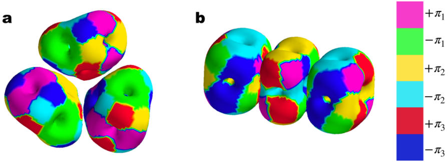


Baryons from holographic instantons

The Sutcliffe model

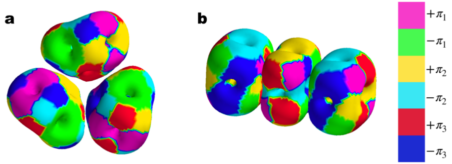
- The Sutcliffe model is a flat-holography model of YM on $\mathbb{R}^5$
- The kink function is introduced for the pion profile

$$\psi_+(z) = \frac{1}{\sqrt{2}\pi^{\frac{1}{4}}} \int_{-\infty}^{z} \psi_0(\xi) \mathrm{d}\xi = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right),$$

and the gauge fields are as usual expanded as

$$A_i = -\partial_i U U^{-1} \psi_+(z) + \sum_{n=0}^{K-1} V_i^n(\mathbf{x}) \psi_n(z),$$

- Notice that no electric field or CS is present, since there is no curvature to fight against
- The KK scale appears due to truncation of the infinite modes to K modes



Homogeneous Ansatz

- The homogeneous Ansatz is an approximation where the single instantons are sitting so densely, that they lose their individual identity
- Unfortunately, it was proved in 2007 by [Rozali-Shieh-Van Raamsdonk-Wu, JHEP01, 053 (2008)] that it is impossible for a continuous gauge field configuration to have a nonvanishing instanton number in the homogeneous Ansatz
- However, it can be done by introducing a discontinuous gauge field, which acts as a source of baryons – usually put at $z = 0$ [Li-Schmitt-Wang, PRD92, 026006 (2015)]
- The homogeneous Ansatz for SS (for $\mathrm{SU}(2)$) is

$$\mathcal{A}_0 = \frac{1}{2}\widehat{a}_0(z), \qquad \mathcal{A}_i = -\frac{1}{2}H(z)\tau^i, \qquad \mathcal{A}_z = 0,$$

with the discontinuous BCs:

$$\widehat{a}_0'(0) = 0, \qquad H(0^\pm) = \pm(4\pi^2 d)^{\frac{1}{3}}.$$

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Finite isospin density

- Following [Adkins-Nappi-Witten, NPB228, 552 (1983)], we can turn on isospin, by rotating the baryons in $\mathrm{SU}(2)$ [Bartolini-Gudnason, 2209.14309]

$$\mathcal{A}_0 = G(z) a \boldsymbol{\chi} \cdot \boldsymbol{\tau} a^{-1} + \frac{1}{2} \widehat{a}_0(z), \qquad \mathcal{A}_i = -\frac{1}{2}\left(H(z) a \tau^i a^{-1} + L(z) \chi^i\right),$$
$$\mathcal{A}_z = 0,$$

with boundary angular isospin velocity $a\boldsymbol{\chi} \cdot \boldsymbol{\tau} a^{-1} = -2i\dot{a}a^{-1}$ and the discontinuous BCs:

$$\widehat{a}_0'(0) = 0, \qquad H(0^{\pm}) = \pm(4\pi^2 d)^{\frac{1}{3}}.$$

- What are the suitable BCs for G and L?
- Guessing suitable boundary conditions based on parity, one would get

$$G'(0) = 0, \qquad L(0) = 0. \tag{2}$$

- Unfortunately, scrutinizing a bit, it turns out that $G'(0)$ does not minimize the entire action!

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Taking a closer look at the Chern-Simons term

- SS Model

$$S = -\kappa \,\mathrm{tr} \int_{\mathrm{AdS}_5} \mathcal{F} \wedge *\mathcal{F} + N_c \int_{\mathrm{AdS}_5} \omega_5 , \qquad \kappa = \frac{\lambda N_c}{216\pi^3},$$

or in more details

$$S = -\frac{\kappa}{2}\,\mathrm{tr}\int \mathrm{d}^5 x \left[h(z)\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + 2k(z)\mathcal{F}_{\mu z}\mathcal{F}^{\mu z}\right]$$
$$+ \frac{N_c}{24\pi^2}\,\mathrm{tr}\int \left(\mathcal{A}\wedge\mathcal{F}^2 - \frac{\mathrm{i}}{2}\mathcal{A}^3\wedge\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right)$$

- The CS term can be integrated by parts:

$$+\frac{N_c}{24\pi^2}\,\mathrm{tr}\int \left[3\widehat{A}\wedge F^2 + \widehat{A}\wedge\widehat{F}^2 + \mathrm{d}\left(\widehat{A}\wedge\left(2F\wedge A - \frac{\mathrm{i}}{2}A^3\right)\right)\right].$$

with $\mathcal{F} = F^a \frac{\tau^a}{2} + \widehat{F}\frac{1}{2}$.

- After integrating by part, the entire CS term is proportional to $\widehat{A}$, but this depends on what is chosen as the boundary term (ambiguity problem)

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Discarding the total derivative of CS

- Let's discard the total derivative in the CS
- and use the homogeneous Ansatz:

$$\mathcal{A}_0 = \frac{1}{2}\widehat{a}_0(z), \qquad \mathcal{A}_i = -\frac{1}{2}H(z)\tau^i, \qquad \mathcal{A}_z = 0,$$

- The full variation of the action:

$$\delta S = \int \mathrm{d}^4x\mathrm{d}z \left[(\mathrm{E.o.M.})_H\,\delta H + (\mathrm{E.o.M.})_{\widehat{a}_0}\,\delta\widehat{a}_0\right] + \delta S_{\mathrm{boundary}},$$

$$\delta S_{\mathrm{boundary}} = 2\kappa\int \mathrm{d}^4x\left[k(z)\widehat{a}_0'\delta\widehat{a}_0 - 3\left(k(z)H' + \frac{N_c}{16\pi^2\kappa}\widehat{a}_0 H^2\right)\delta H\right]_{z=0}^{z=\infty}.$$

- We arrive at the conditions:

$$\widehat{a}_0'(0)\delta\widehat{a}_0(0) = 0,$$

$$\left(H'(0) + \frac{N_c}{16\pi^2\kappa}\widehat{a}_0(0)H^2(0)\right)\delta H(0) = 0.$$

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- Looking at the baryon current at the conformal boundary

$$d_B = \frac{2}{N_c}\kappa\left[k(z)\widehat{a}_0'\right]_{-\infty}^{\infty} = \frac{4}{N_c}\kappa\left[k(z)\widehat{a}_0'\right]_{z=\infty} = d,$$

- we find the same baryon density d as given by the topological integral

$$\begin{aligned} d &= \frac{1}{32\pi^2}\int \mathrm{d}z\epsilon^{MNPQ}\,\mathrm{tr}\,F_{MN}F_{PQ} \\ &= -\frac{1}{8\pi^2}\int \mathrm{d}z\partial_z\left(H^3\right) \\ &= -\frac{1}{8\pi^2}\left[H^3\right]_{z=0^+}^{z=+\infty} - \frac{1}{8\pi^2}\left[H^3\right]_{z=-\infty}^{z=0^-} \end{aligned}$$

which is consistent.

- For the chemical potential

$$\mu_B d_B = -\frac{N_c}{8\pi^2}\mu\int_0^\infty \mathrm{d}z\;\partial_z(H^3) = \frac{N_c}{2}\mu d \qquad \Rightarrow \qquad \mu_B = \frac{N_c}{2}\mu.$$

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Including the total derivative of CS

- Let's now include the total derivative in the CS
- The full variation of the action now gives the conditions:

$$\left(\widehat{a}_0'(0) + \frac{3N_c}{64\pi^2\kappa}H^3(0)\right)\delta\widehat{a}_0(0) = 0,$$

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$$\widehat{a}_0' = -\frac{1}{k(z)}\frac{N_c}{16\pi^2\kappa}\left(H^3(z) - \frac{H^3(0)}{4}\right).$$

- Looking at the baryon current at the conformal boundary, we now find:

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  which differs from the topological density by a factor of $1/4$.
- The chemical potential is unchanged.

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$$\widehat{a}_0' = -\frac{1}{k(z)}\frac{N_c}{16\pi^2\kappa}\left(H^3(z) - \frac{H^3(0)}{4}\right).$$

- Looking at the baryon current at the conformal boundary, we now find:

$$d_B = \frac{2}{N_c}\kappa\left[k(z)\widehat{a}_0'\right]_{-\infty}^{\infty} = \frac{4}{N_c}\kappa\left[k(z)\widehat{a}_0'\right]_{z=\infty} = \frac{d}{4},$$

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Proposal: Matching the currents and the topological data

- We propose to choose which total-derivative term to discard in the CS action, by matching the physical currents with the topological degrees calculated in the bulk.
- This fixes the CS term to:

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\,{\rm tr}\int\left[3\widehat{A}\wedge F^2 + \widehat{A}\wedge\widehat{F}^2\right].$$

- Using this CS term, the variation of the fields G, L:

$$\left[G'(0) + \frac{N_c}{32\pi^2\kappa}H^2(0)L(0)\right]\delta G(0) = 0,$$
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Thermodynamic equilibrium condition

- Thermodynamic equilibrium condition can be found from

$$\widehat{a}_0(z) = \mu - \int_z^\infty \mathrm{d}z'\, \widehat{a}_0'(z'), \qquad \widehat{a}_0' = -\frac{N_c}{16\pi^2\kappa}\frac{1}{k(z)}\left(H^3(z) - H^3(0)\right),$$

Solving for μ:

$$\mu(d) = -\frac{4\kappa}{N_c}\left(\frac{4\pi^2}{d^2}\right)^{\frac{1}{3}} H'(0) + \frac{N_c d}{4\kappa}\int_0^\infty \mathrm{d}z\,\frac{1}{k(z)}\left(1 - \frac{H^3(z)}{H^3(0)}\right),$$

- The trick is that $H(z)$ can be solved without knowing μ, which it is simply an overall shift in $\widehat{a}_0$, that the EOMs do not observe.
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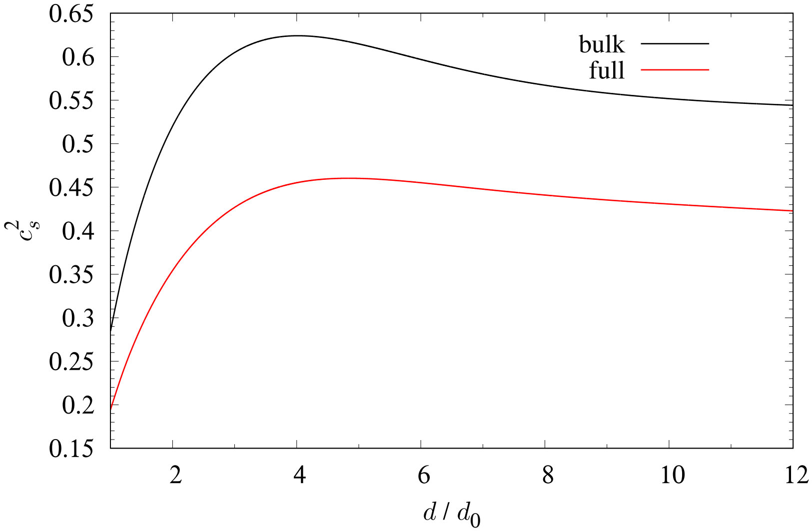

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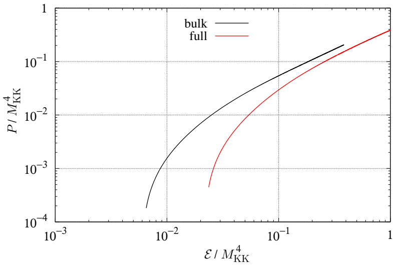

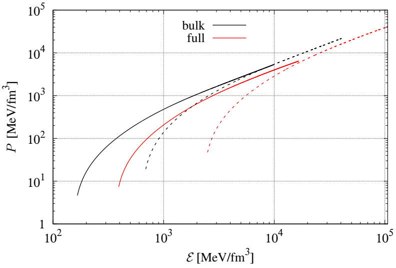

$$d_0^{\mathrm{bulk}} = 0.436 \left(\frac{M_{\mathrm{KK}}}{949\,\mathrm{MeV}}\right)^3 \mathrm{fm}^{-3},$$

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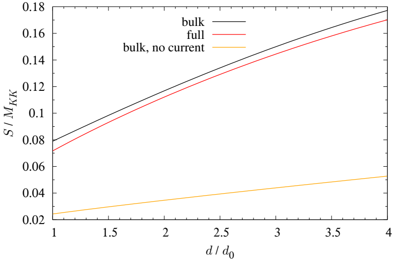

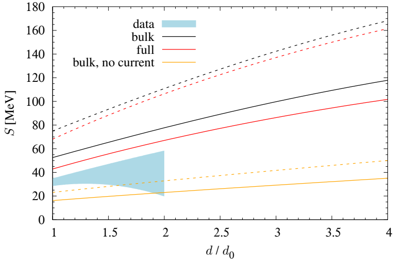

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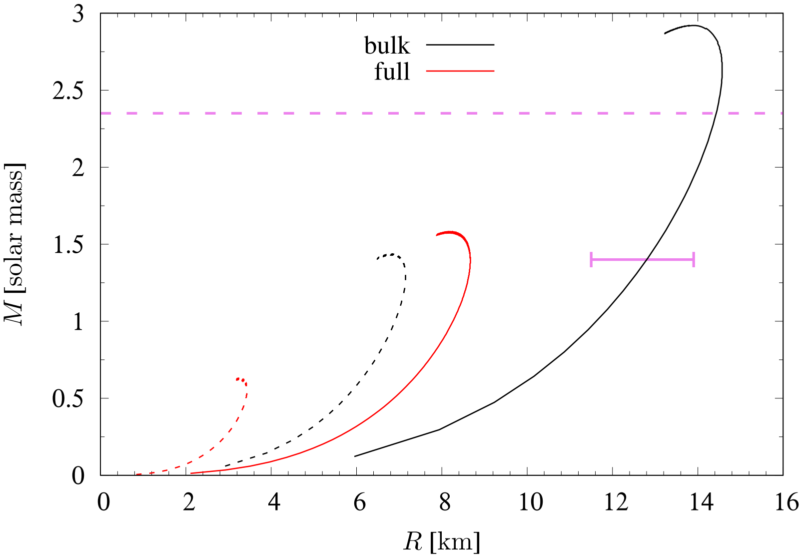

Equivalence between quantization and chemical potential for isospin

- Start with Ansatz:

$$A_0 = Ga\chi \cdot \tau a^{-1} \quad A_i = -\frac{H}{2} a\tau^i a^{-1}, \quad A_z = 0.$$

- Perform a gauge transformation:

$$A_0 \to \widetilde{A}_0 = Gba\chi \cdot \tau a^{-1} b^{-1} - \mathrm{i} b \partial_0 b^{-1},$$
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$$\widetilde{A}_0(z \to z_{\mathrm{UV}}) = \frac{1}{2}\boldsymbol{\chi} \cdot \boldsymbol{\tau}.$$

The boundary value of the field A_0 is dual to an isospin chemical potential.

- Choosing $\chi = (0, 0, \mu_I)$ and defining:

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WSS and Hard-wall on the same footing

- If we fold the SS model [Gorsky-Gudnason-Krikun, PRD91, 126008 (2015)], then we can multiply the action by 2 and integrate over $z \in [0, \infty)$

$$S \to 2 \int_0^\infty \mathrm{d}z \int \mathrm{d}^4 x \ \mathrm{tr}\, \mathcal{F} \wedge * \mathcal{F} + \ldots$$

- In the hard-wall model in the baryonic phase, where the scalar only determines the onset of the baryons, we can identify

$$L_i = -R_i = -\frac{1}{2} H(z) \tau^i, \qquad L_0 = R_0 = \frac{1}{2} \widehat{a}_0(z).$$

- With these identifications, the SS and Hard-wall model are equivalent upon identifying

$$\kappa = M_5, \tag{3}$$

and changing the warp factors and integration range (in hard-wall $z \in [0, L]$

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Symmetry energy

- Using the Ansatz

$$\mathcal{A}_0 = G a \boldsymbol{\chi} \cdot \boldsymbol{\tau} a^{-1} + \tfrac{1}{2}\widehat{a}_0, \quad \mathcal{A}_i = -\tfrac{1}{2}\left(H a \tau^i a^{-1} + L \chi^i\right), \quad \mathcal{A}_z = 0,$$

- The action is

$$S_{\rm YM} = -\kappa \int \mathrm{d}^4x \int \mathrm{d}z \Big[-8hH^2 \left(G + \frac{1}{2}\right)^2 \chi \cdot \chi + 3hH^4 + k\left[(L')^2 - 4(G')^2 + 8(KH)^2\right] \chi \cdot \chi + 3k(H')^2 - k(\hat{a}_0')^2\Big],$$

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Symmetry energy

- Quantizing the isospin Hamiltonian

$$H = \frac{1}{2} V \Lambda \boldsymbol{\chi} \cdot \boldsymbol{\chi} + VU$$
$$= 2V\Lambda \dot{a}_m^2 + VU$$
$$= \frac{\pi_m^2}{8V\Lambda} + VU$$
$$= \frac{I(I+1)}{2V\Lambda} + VU,$$

with $\pi_m = \frac{\partial H}{\partial \dot{a}_m} = 4V\Lambda\dot{a}_m$

- The inertia and potential energy are

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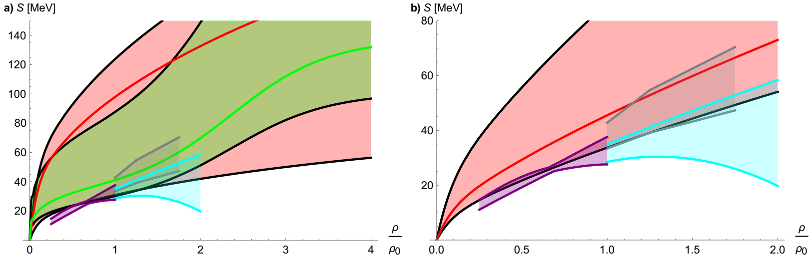
Symmetry energy with homogeneous Ansatz in WSS (and HW)



a) WSS: $M_{\rm KK} \in [300, 1200]$ MeV, HW: $\mathsf{L}^{-1} \in [110, 320]$ MeV,
b) WSS: $M_{\rm KK} \in [370, 949]$ MeV

$$S(\rho) = \frac{\rho}{8\Lambda}, \qquad \Lambda = 2\kappa \int \mathrm{d}z \left[2hH^2(2G+1)^2 + k((L')^2 + 4(G')^2)\right].$$

[Bartolini-Gudnason, 2209.14309]

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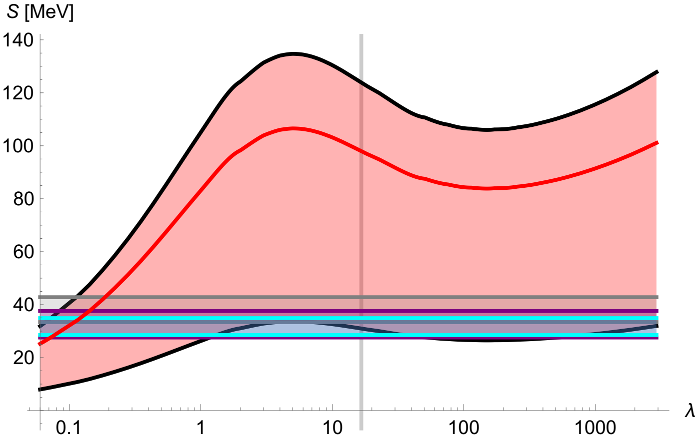

Baryon spectra in the soliton picture

- In bottom-up holography, we can stick in a field in a certain (e.g. tachyon) background and read off observables.
- With the SS soliton, the excited nucleon and nuclei spectra becomes more complicated
- In principle we could quantize all low-energy modes to obtain nuclear spectra
- See vibrations website: http://www1.maths.leeds.ac.uk/pure/geometry/SkyrmionVibrations/
- Many local vacua makes the semi-classical quantization even more involved
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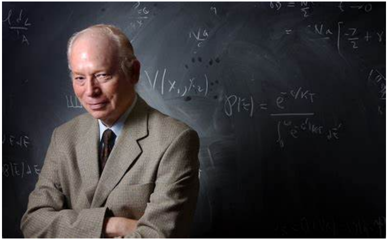
Which is the correct (dual) description of low-energy QCD?

- Weinberg says: if you get the symmetries right, then the theory is the right theory



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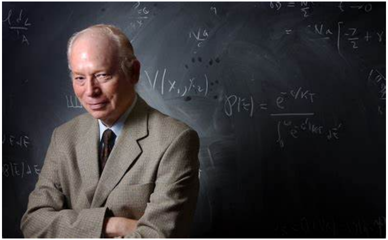

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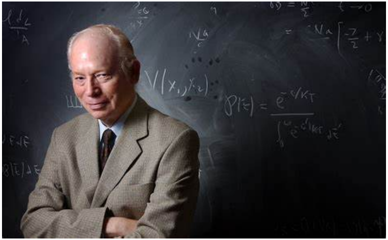

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- In [Bijnens-Gudnason-Yu-Zhang, JHEP05, 061 (2023)] we have determined the pure pion terms up to operator dimension 16 (and also for any other spacetime dimension ≤ 12 as well as other $\mathrm{O}(N)$ groups)

n_d	# EFT terms (in 3+1 dim)			
2	1	0	0	0
3	0	0	0	0
4	2	0	0	0
5	0	0	0	0
6	5	0	1	0
7	0	0	0	0
8	16	2	4	0
9	0	0	0	0
10	55	14	27	11
11	0	0	0	0
12	253	115	160	99
13	0	0	0	0
14	1260	806	1021	779
15	0	0	0	0
16	7140	5564	6379	5426

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- So the problem is to determine the many many LECs
- One way is to assume hidden local symmetry, where

$$SU(N_f)_V \subset SU(N_f)_L \times SU(N_f)_R \tag{4}$$

  becomes a local (gauged) group
- This is essentially already built-in in the SS model
- However, the symmetry group of SS is larger and hence gives numerically slightly different results, with respect to HLS
- One could contemplate that a different background gives rise to HLS
- Notice that SS, HW, HLS and the Sutcliffe model all determine all the LECs (up to the scale and 't Hooft coupling and their analogues in the other models), but all give different LECs
- Determining which background is correct amounts to determining with high precision the correct values of the LECs

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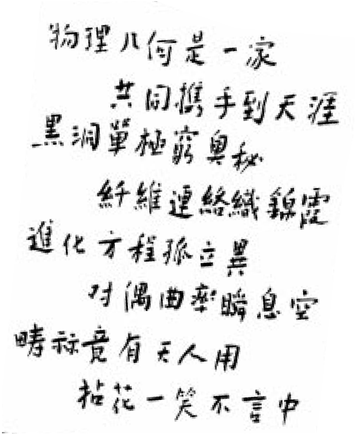
Chern says physics is geometry



Photography by Peg Skorpinski.

Shiing Shen Chern

Chern says physics is geometry

物理几何是一家
　　共同携手到天涯
黑洞单极穷奥秘
　　纤维连络织锦霞
进化方程孤立异
　　对偶曲率瞬息空
畴祘竟有天人用
　　拈花一笑不言中

Photography by Peg Skorpinski.

Shiing Shen Chern

Chern says physics is geometry

Physics and geometry are one family.
Together and holding hands they roam
to the limits of outer space.
Black hole and monopole exhaust
the secret of myths;
Fiber and connections weave to interlace
the roseate clouds.
Evolution equations describe solitons;
Dual curvatures defines instantons.
Surprisingly, Math. has earned
its rightful place
for man and in the sky;
Fondling flowers with a smile — just wish
nothing is said!
– Shiing-Shen Chern

A. Jackson and D. Kotschick, Notices of the AMS, 45 7 (1998)

Perhaps QCD is just geometry?!

Outlook/open problems

- How to systematically study and converge to the correct bulk geometry describing real world QCD?
- Using deep learning? [Hashimoto-Sugishita-Tanaka-Tomiya, PRD98, 046019 (2018)]
- How to systematically improve holographic QCD?
  - $1/N_c$ corrections?
  - $1/\lambda$ corrections? (notice that the liquid-drop-like theory term appears as $1/\lambda$ after integrating out the vectors)
  - Beyond the 8-branes being probe branes?
  - α' corrections?
- How to improve the finite/large density description, instead of using a homogeneous Ansatz?
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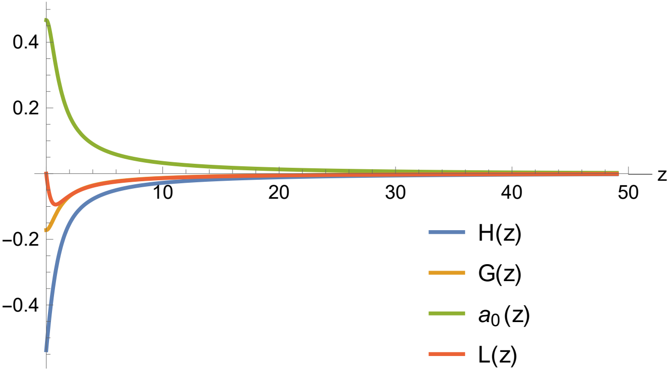
Mange tak!
Grazie mille!
非常感谢！

Backup slides

Solution at $\lambda = 16.63$



Full hydro simulation of merger