PBHs and Primordial non-gaussianity: from their abundance to gravitational waves.

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Primordial Black Holes: outline presentation

REVIEW:
A. M. Green and B. J. Kavanagh.–
arXiv:2007.10722

PART 1) ABUNDANCE of PBH: The role of NGs

arXiv:2211.01728 (*PRD*) G.Ferrante, G.Franciolini, A.J.I., A.Urbano
arXiv:2402.11033 A.Ianniccari, A.J.I., A. Kehagias, D. Perrone, A. Riotto

PART 2) GRAVITY WAVES: PBH as possible explanation of PTA

arXiv:2306.17149 (*PRL*) G.Franciolini, A.J.I., V. Vaskonen, H. Veermae
arXiv:2402.11033 A.Ianniccari, A.J.I., A. Kehagias, D. Perrone, A. Riotto

PART 3) GRAVITATIONAL WAVES: are PBHs the end of the story?

arXiv:2308.08546 (*PRD*) J, Ellis, M. Fairbairn, G. Franciolini, G.Hütsi,
A.J.I., M. Lewicki. M. Raidal, J. Urrutia, V. Vaskonen, H. Veermae

Black Holes: Astro vs Primordial

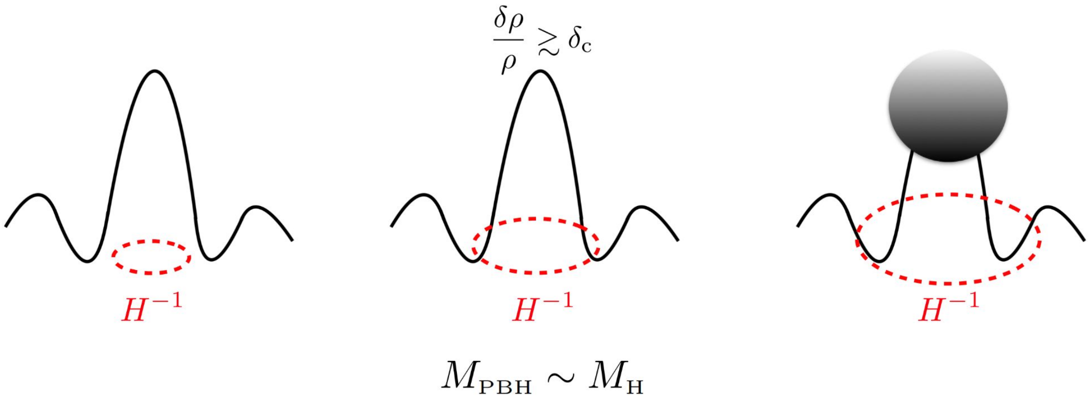
Astro BH: forms from the gravitational collapse of a star.

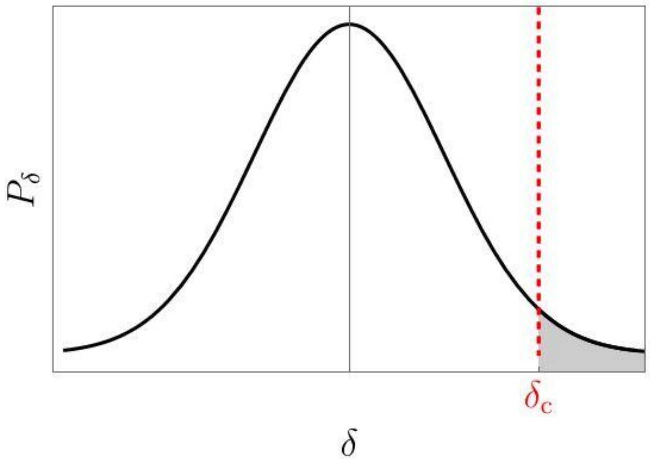
$$M > \mathcal{O}(1)\, M_\odot$$

PBH: (standard scenario) gravitational collapse of large over densities in the primordial density contrast field δ.

$$M > 10^{-18}\, M_\odot$$

PBHs from collapse of large overdensities





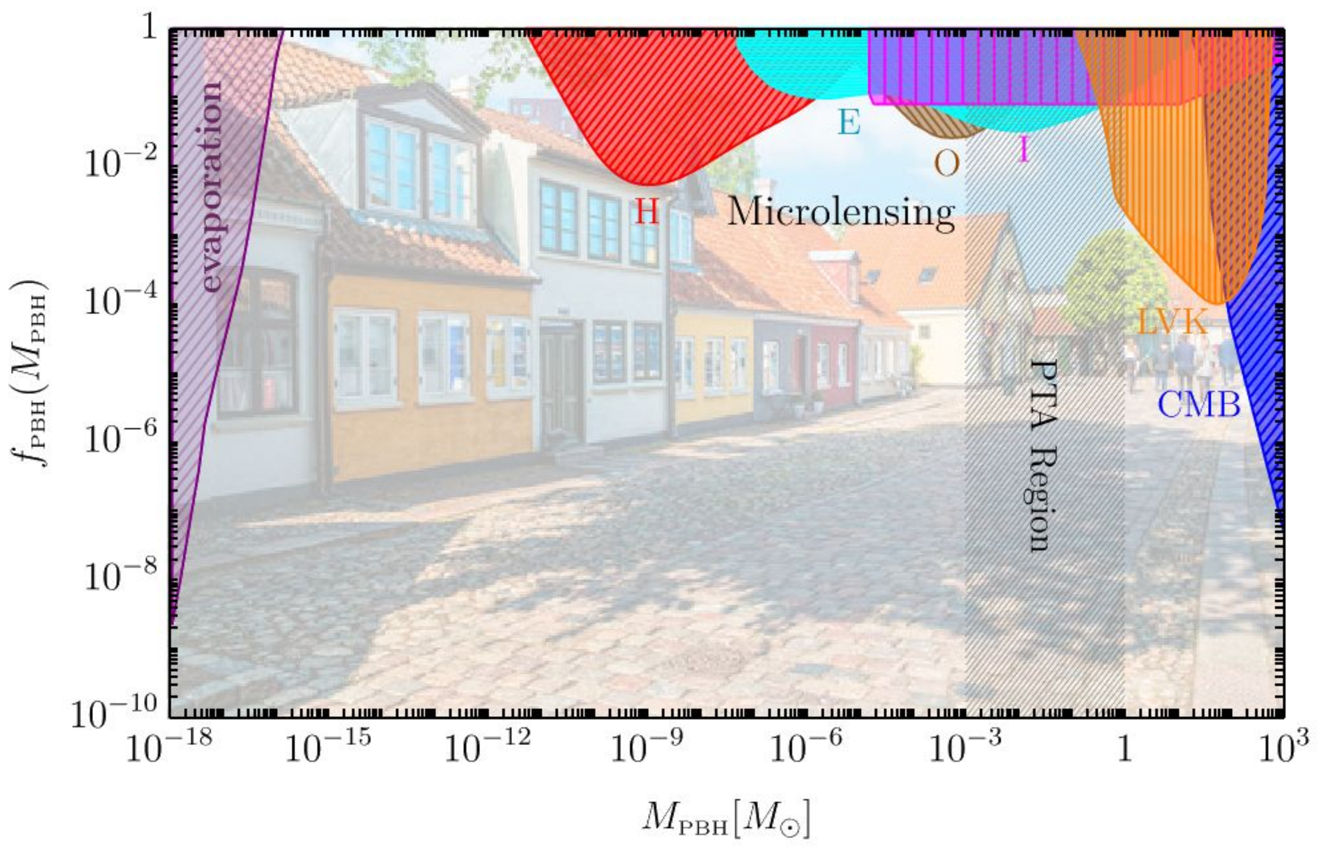
$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^\gamma \, \mathrm{P}_\delta(\delta) d\delta$$

$$\Omega_{PBH} = \int d \log M_H \left(\frac{M_{eq}}{M_H}\right)^{1/2} \beta$$

Primordial Black Holes as DM candidates

**LVK to appear*

Mass Distribution

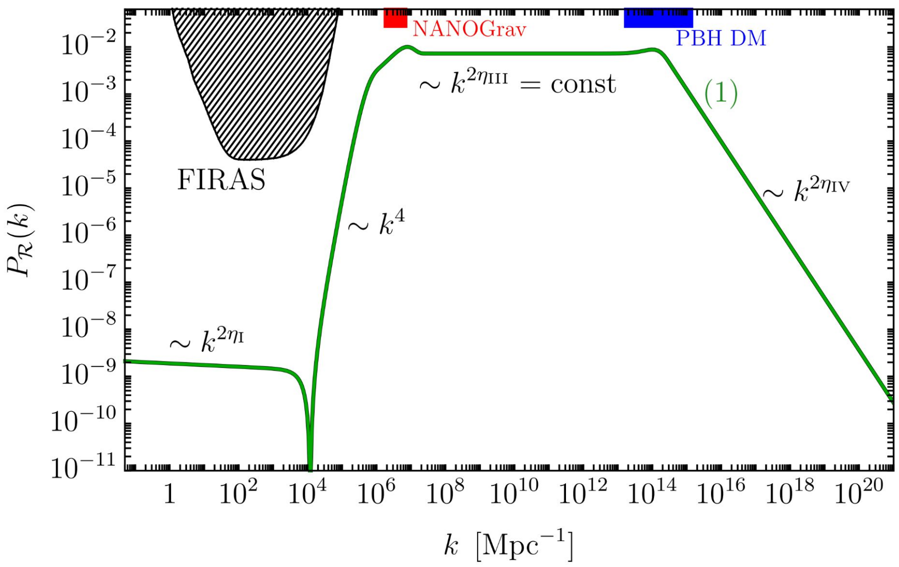


Models for PBH formation

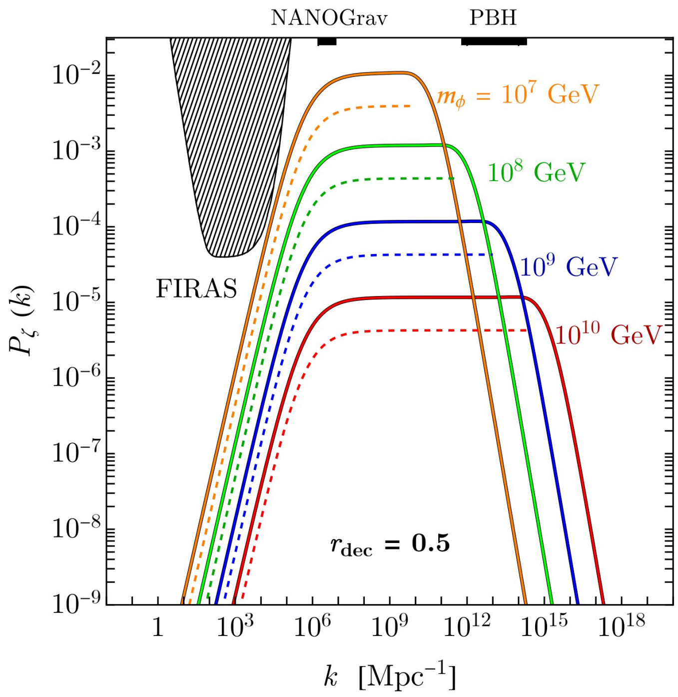
What we can compute during inflation is the curvature perturbation field ζ (or *R*).
In order to get a sizeable amount of DM *Pζ*≃10^-2 or 10^-3

PBH as Dark Matter:
- USR models
- Curvaton field
- Hybrid inflation
- And etc etc…

$$M_H \simeq 17 M_\odot \left(\frac{g_\star}{10.75}\right)^{-1/6} \left(\frac{k/\kappa}{10^6\text{Mpc}^{-1}}\right)^{-2}$$



arXiv:2305.03491 (Minor corrections PRD)
G.Franciolini, A.J.I., M. Taoso, A.Urbano



arXiv:2305.13382 (Published on JCAP)
G.Ferrante, G.Franciolini, A.J.I., A.Urbano

Abundance of PBHs: The role of Non-Gaussianities (NG).

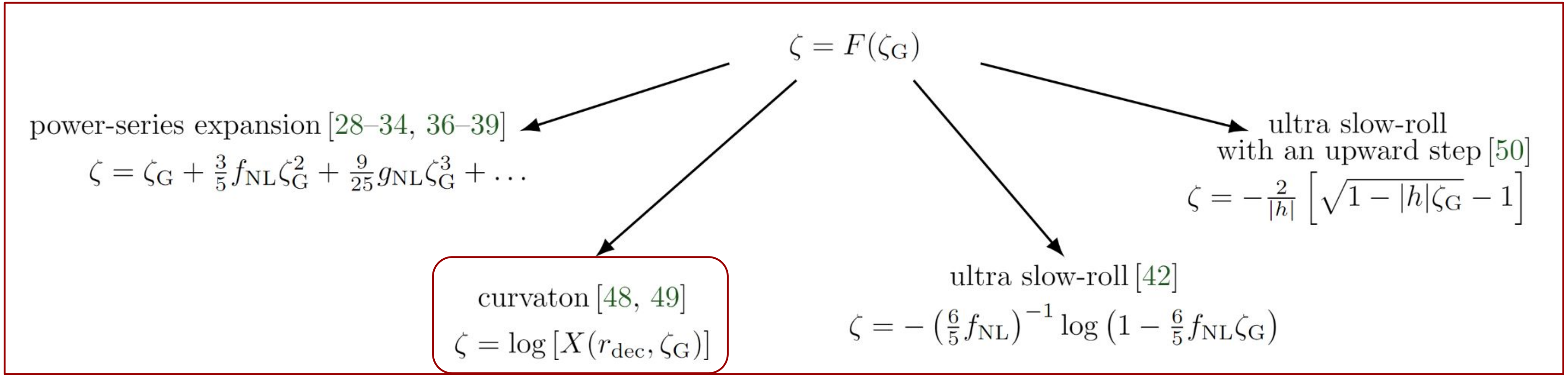
An exact formalism for the computation of PBHs mass fraction abundance NGs in the curvature perturbation field ζ:

NON-LINEARITIES (NL)

$$\delta(\vec{x},t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\partial_i\zeta(\vec{x})\partial_i\zeta(\vec{x})\right]$$

T. Harada, C. M. Yoo, T. Nakama and Y. Koga,.– arXiv:1503.03934

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$



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PRIMORDIAL NG

$$\zeta = F(\zeta_G)$$

NG PBH mass fraction adopting threshold statistics on the compaction function

$$\beta_{\rm NG} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C}-\mathcal{C}_{\rm th})^{\gamma} \mathrm{P}_{\rm G}(\mathcal{C}_{\rm G},\zeta_{\rm G})\, d\mathcal{C}_{\rm G}\, d\zeta_{\rm G}\,, \tag{56}$$

$$\mathrm{P}_{\rm G}(\mathcal{C}_{\rm G},\zeta_{\rm G}) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{cr}^2}} \exp\left(-\frac{\zeta_{\rm G}^2}{2\sigma_r^2}\right) \exp\left[-\frac{1}{2(1-\gamma_{cr}^2)}\left(\frac{\mathcal{C}_{\rm G}}{\sigma_c} - \frac{\gamma_{cr}\zeta_{\rm G}}{\sigma_r}\right)^2\right], \tag{57}$$

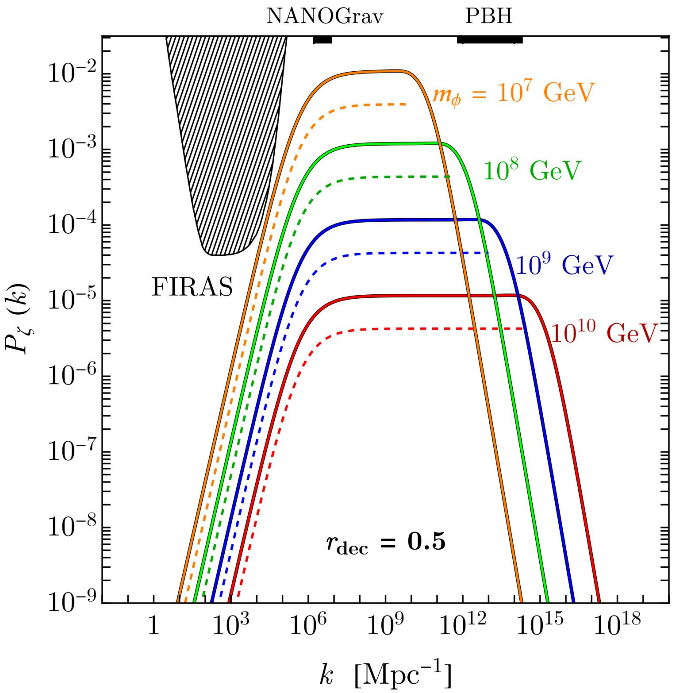
$$\mathcal{D} = \{\mathcal{C}_{\rm G},\ \zeta_{\rm G} \in \mathbb{R}\ :\ \ \mathcal{C}(\mathcal{C}_{\rm G},\zeta_{\rm G}) > \mathcal{C}_{\rm th}\ \wedge\ \mathcal{C}_1(\mathcal{C}_{\rm G},\zeta_{\rm G}) < 2\Phi\}\,, \tag{58}$$

arXiv:2211.01728 (PRD) G.Ferrante, G.Franciolini, A.J.I., A.Urbano

Later on confirmed also by D.Wands et al arXiv:2211.08348

IMPORTANT: PBH abundance depends on the amplitude of the power spectrum and the amount of NGs.

Mathematical formulation



$$\langle \mathcal{C}_{\rm G}\mathcal{C}_{\rm G}\rangle = \sigma_c^2 = \frac{4\Phi^2}{9}\int_0^\infty \frac{dk}{k}(kr_m)^4 W^2(k,r_m)T^2(k,r_m)P_\zeta(k)\,,$$

$$\langle \mathcal{C}_{\rm G}\zeta_{\rm G}\rangle = \sigma_{cr}^2 = \frac{2\Phi}{3}\int_0^\infty \frac{dk}{k}(kr_m)^2 W(k,r_m)W_s(k,r_m)T^2(k,r_m)P_\zeta(k)\,,$$

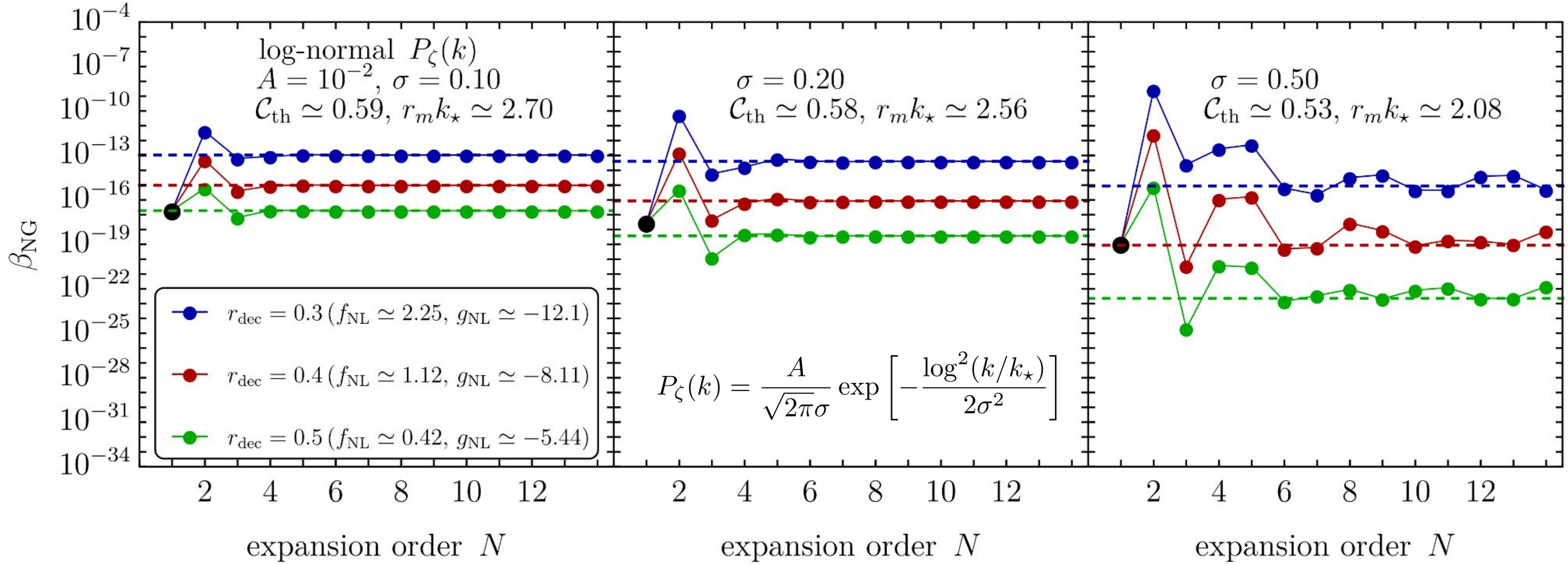
$$\langle \zeta_{\rm G}\zeta_{\rm G}\rangle = \sigma_r^2 = \int_0^\infty \frac{dk}{k}W_s^2(k,r_m)T^2(k,r_m)P_\zeta(k)\,,$$

Application to the curvaton model (1)

- - - - $\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$

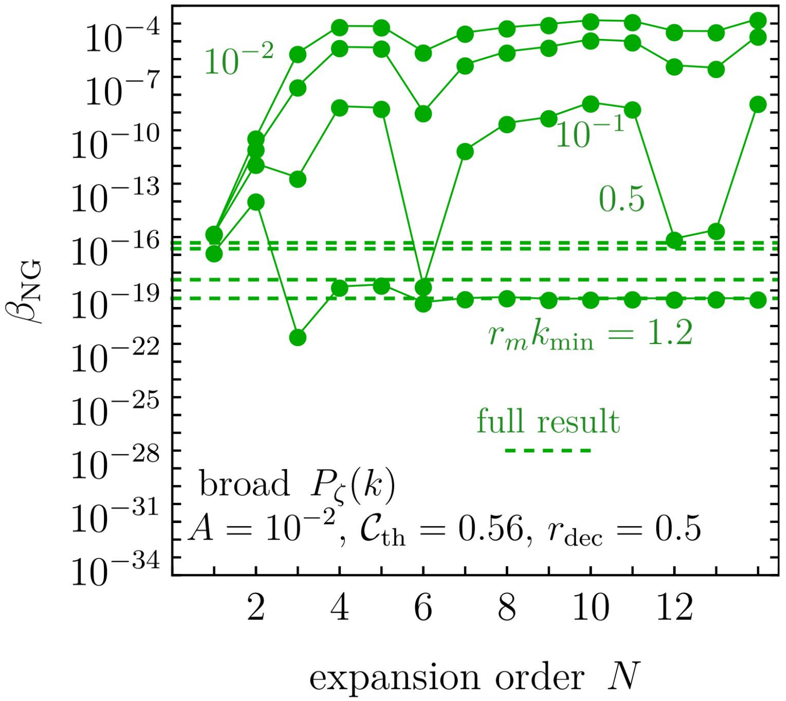
● $\zeta_N = \sum_{n=1}^{N} c_n(r_{\rm dec})\zeta_{\rm G}^n$

Failure of the perturbative approach (Narrow)



Application to the curvaton model (1)

Failure of the perturbative approach (Broad)

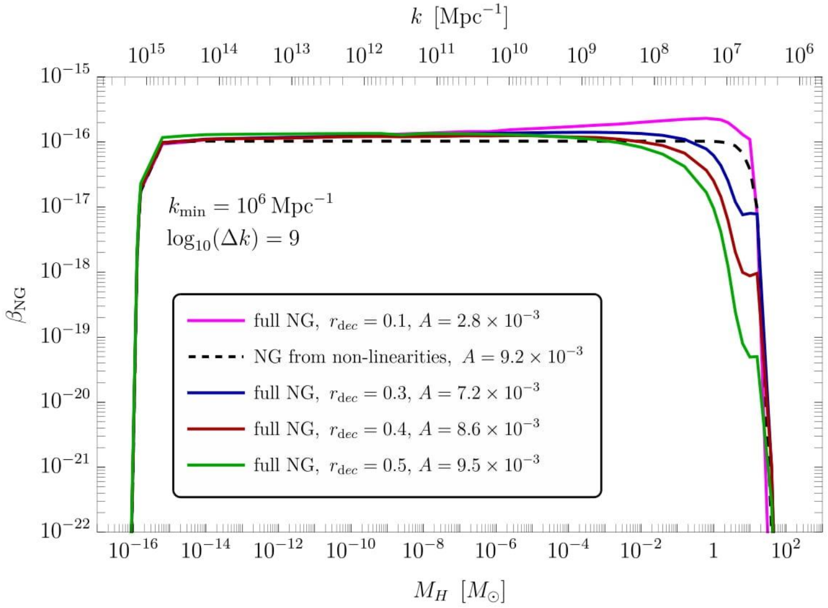


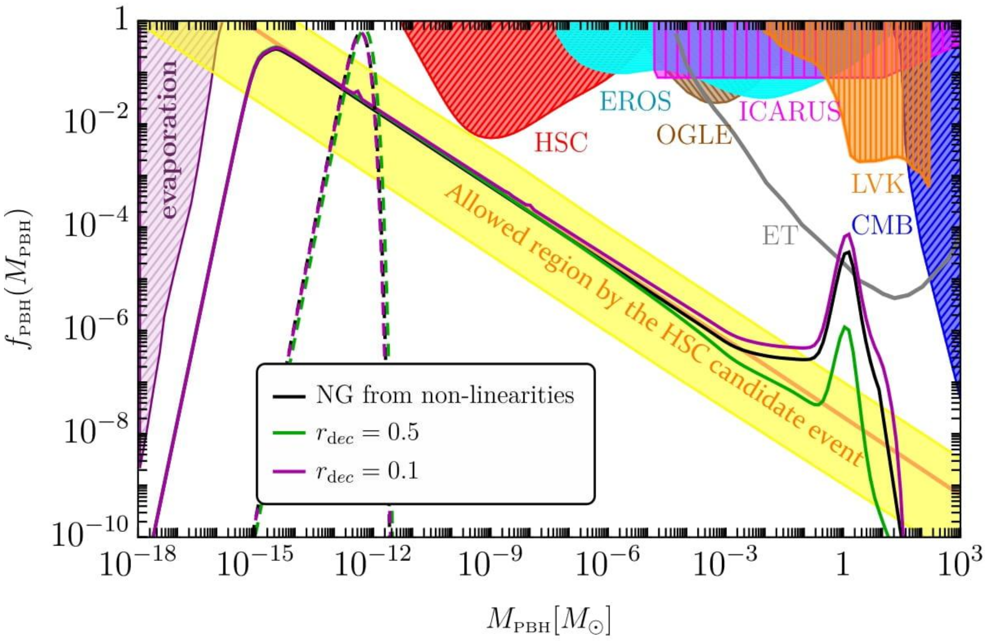
$$P_\zeta(k) = A\,\Theta(k - k_{\rm min})\,\Theta(k_{\rm max} - k)$$

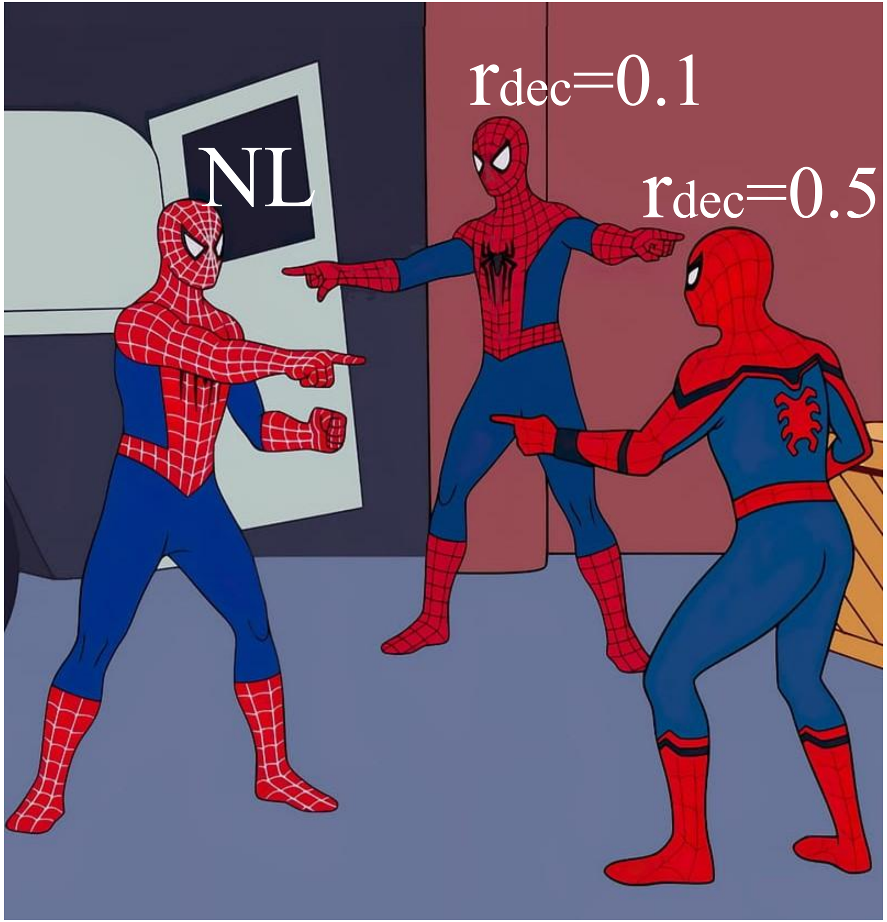
For a broad Power spectrum the power-series expansion is simply wrong and one is forced to use the full result NG.

Application to the curvaton model (2)

Breaking of M_H -Independence







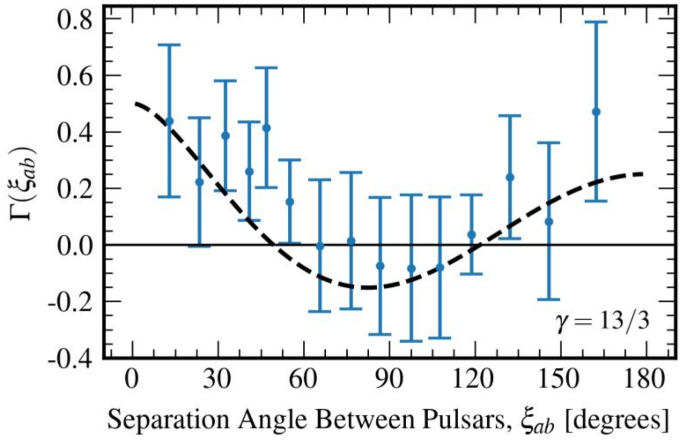
We need another observable: The induced Gravitational waves

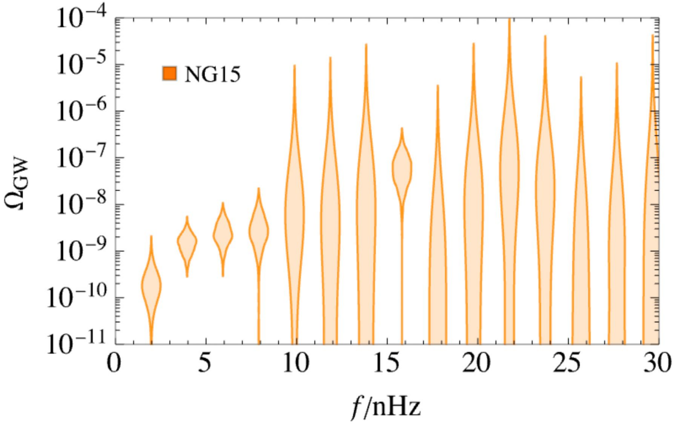
PBH and SGWB

SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

$$h^2\Omega_{\rm GW}(k) = \frac{h^2\Omega_r}{24}\left(\frac{g_*}{g_*^0}\right)\left(\frac{g_{*s}}{g_{*s}^0}\right)^{-\frac{4}{3}}\mathcal{P}_h(k) \qquad \mathcal{P}_h(k) \propto \mathcal{P}_\zeta^2(k)$$

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.





NANOGrav
– arXiv:2306.16213
– arXiv:2306.16219

PBH and SGWB

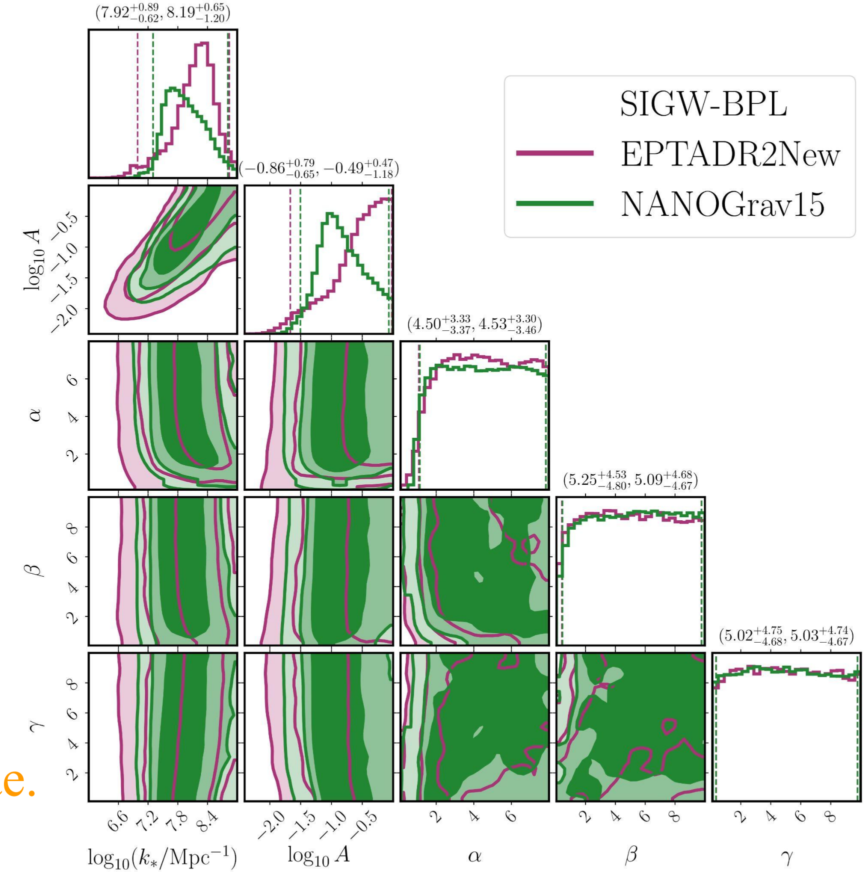
SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

Log-likelihood analysis

Fitting the posterior distributions

$$\mathcal{P}_\zeta^{\rm BPL}(k) = A \frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma} + \alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$

$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{1}{2\Delta^2} \ln^2(k/k_*)\right)$$



arXiv:2306.17149 (PRL)

G.Franciolini, A.J.I., V. Vaskonen, H. Veermae.

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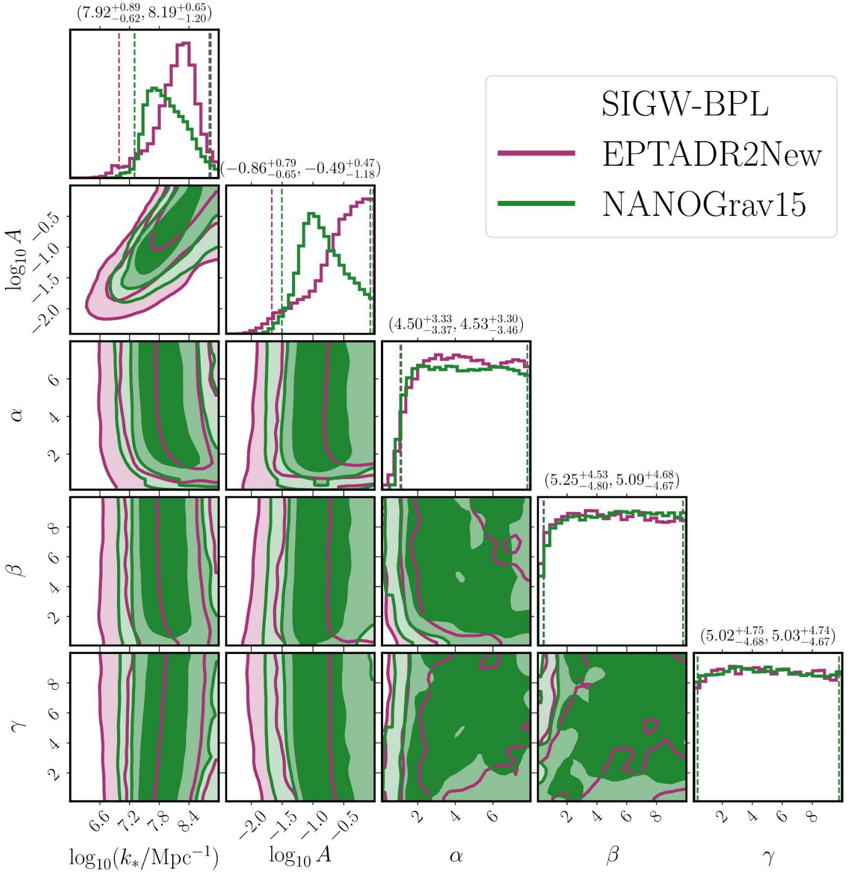

Results:

The causality tail is not good:

$$\Omega_{\rm GW}(k \ll k_*) \propto k^3(1+\tilde{A}\ln^2(k/\tilde{k}))$$

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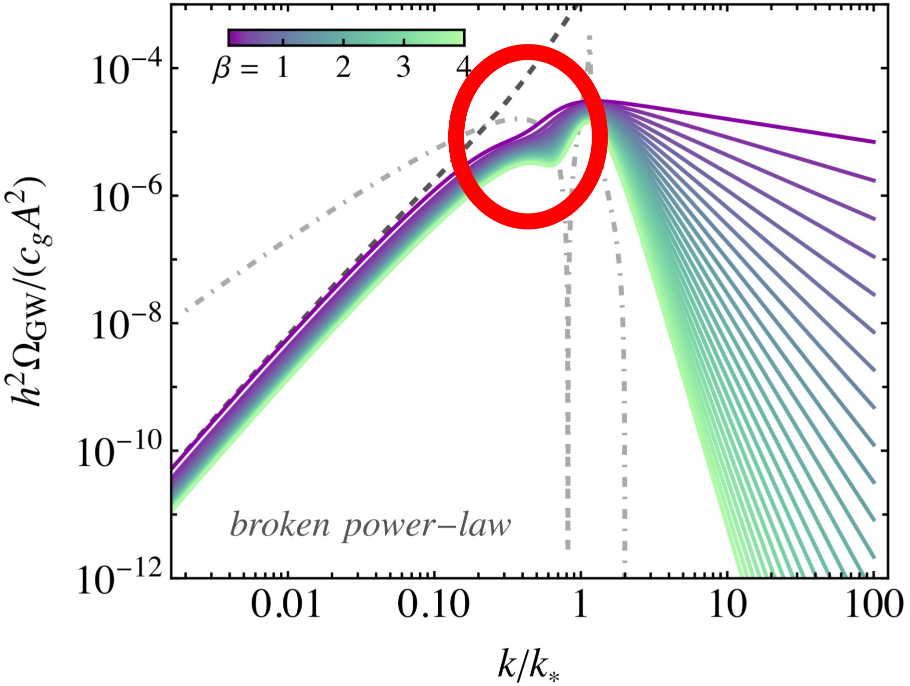

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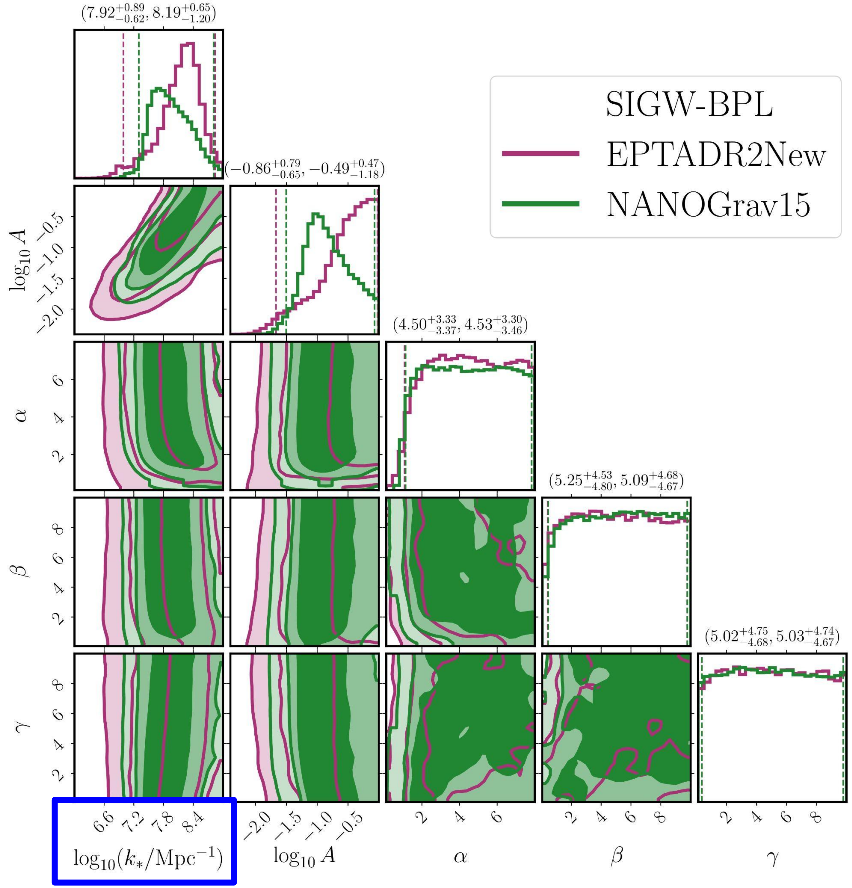
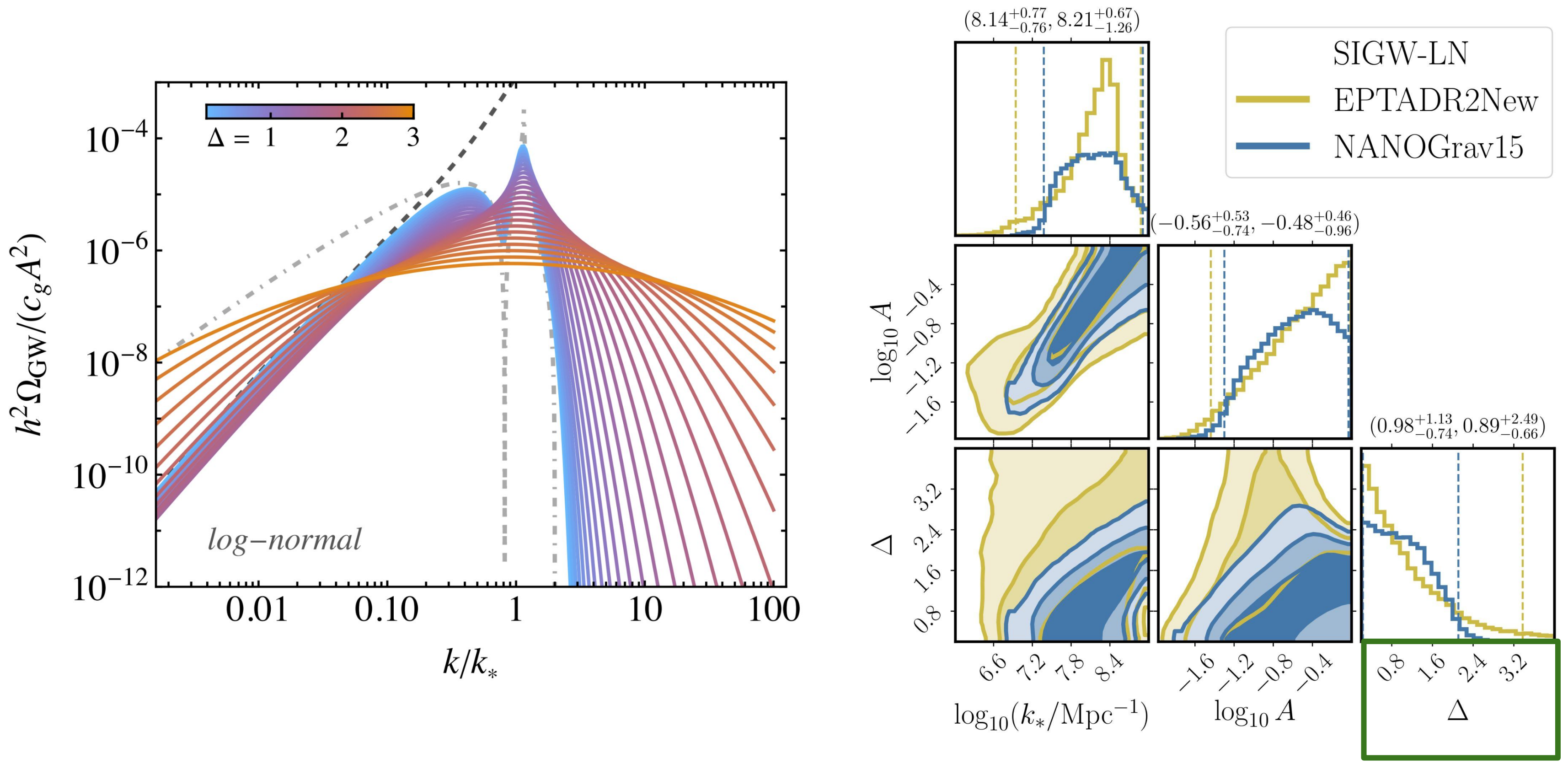
$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta}\,\exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$

Results:

Position of the peak at higher frequencies.

Broad spectrum does not fit so well.





$$\mathcal{P}_\zeta^{\rm LN}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left(-\frac{1}{2\Delta^2}\ln^2(k/k_*)\right)$$

Improvement respect to NANOGrav analysis.

NANOGrav collaboration
arXiv:2306.16219

Power spectrum <> Abundance <> GWs

- Non-Gaussianities in the abundance.
- Dependency of the PBH formation parameters on the PS shape.
- QCD impact on threshold.

NGs in the abundance: Cases under consideration

NON-LINEARITIES (NL)

$$\delta(\vec{x},t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})}\left[\nabla^2\zeta(\vec{x}) + \frac{1}{2}\partial_i\zeta(\vec{x})\partial_i\zeta(\vec{x})\right]$$

$$\delta(\vec{x},t) = -\frac{4}{9a^2H^2}\nabla^2\zeta(\vec{x})$$

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$

$$\zeta = \log\left[X(r_{\rm dec}, \zeta_{\rm G})\right]$$

curvaton case

$$\zeta = -\frac{2}{\beta}\log\left(1 - \frac{\beta}{2}\zeta_{\rm G}\right)$$

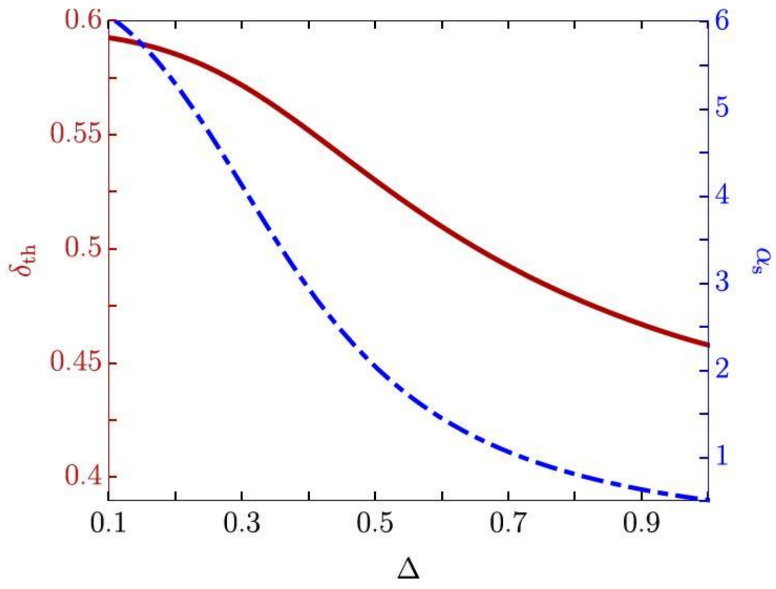
Inflection-point (USR) case

$$\zeta = \zeta_{\rm G} + \frac{3}{5}f_{\rm NL}\zeta_{\rm G}^2$$

Quadratic approx.

Abundance of PBHs:
Shape dependencies

I. Musco,V. De Luca, G. Franciolini, A.Riotto.– arXiv:2011.03014



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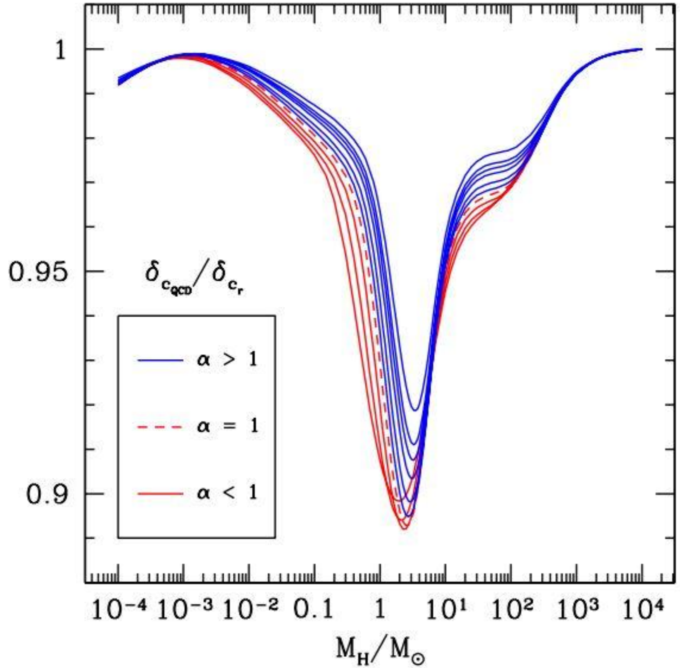
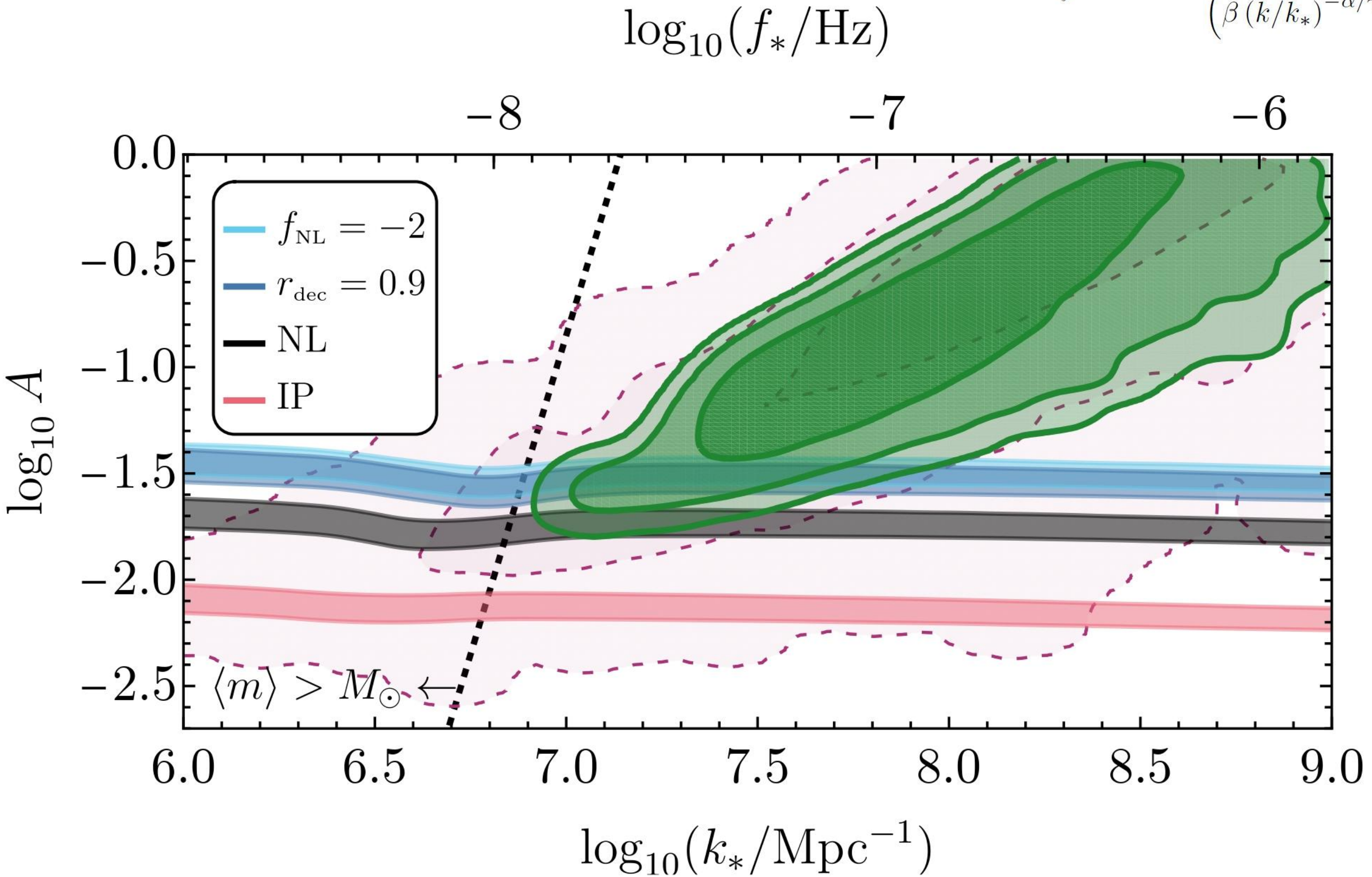
QCD phase transitions

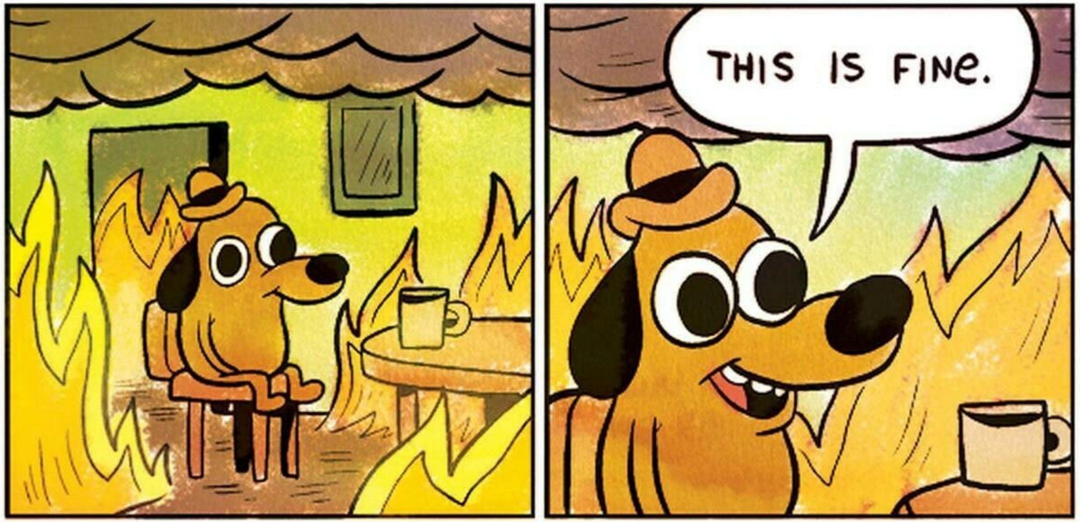
I. Musco, K.Jedamzik, S.Young.– arXiv:2303.07980

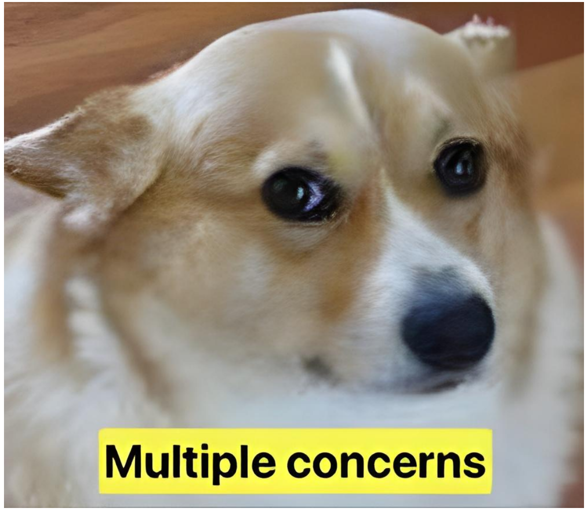


Tension between NANOGrav and PBHs

$$\mathcal{P}_\zeta^{\mathrm{BPL}}(k) = A \frac{(\alpha+\beta)^\gamma}{\left(\beta\,(k/k_*)^{-\alpha/\gamma} + \alpha\,(k/k_*)^{\beta/\gamma}\right)^\gamma}$$







A potential issue

Threshold values maybe are not correct? When we compute the threshold using the average value for the compaction, non-linear effects not included in the linear transfer function lead to different super-horizon threshold conditions.

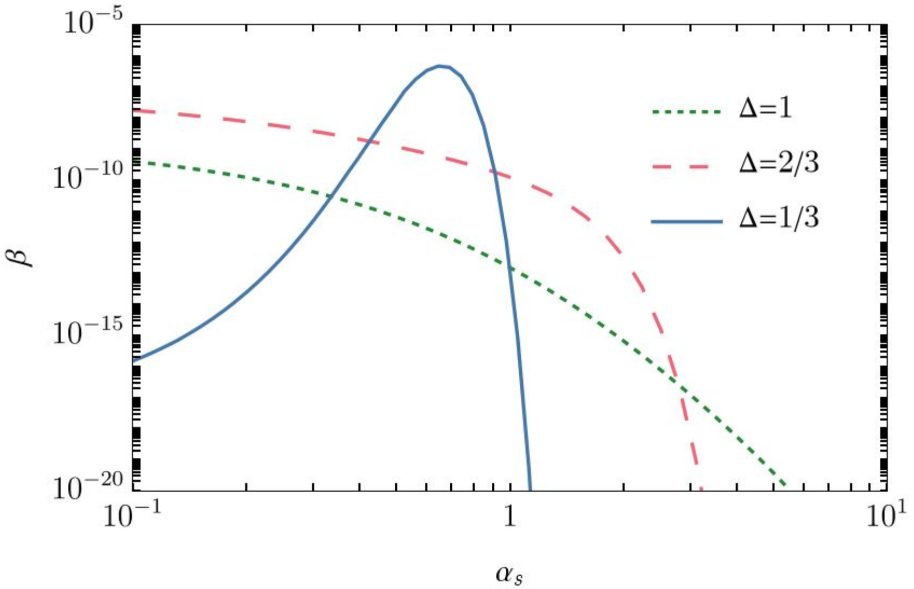
V. De Luca, A. Kehagias, A. Riotto.– arXiv:2307.13633

Going beyond the average profile

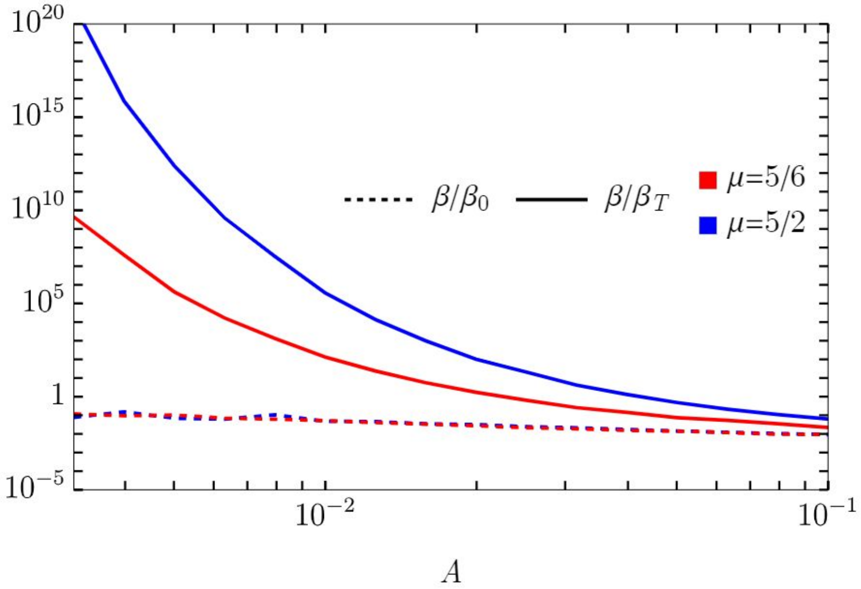
arXiv:2402.11033 A.Ianniccari, A.J.I., A. Kehagias, D. Perrone, A. Riotto

In realistic cases, threshold is determined by the broadest possible compaction function.

$$\mathcal{P}_{\rm g}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left[-\ln^2(k/k_\star)/2\Delta^2\right]$$



$$\zeta(\mathbf{x}) = -\mu_\star \ln\left(1 - \frac{\zeta_{\rm g}(\mathbf{x})}{\mu_\star}\right)$$

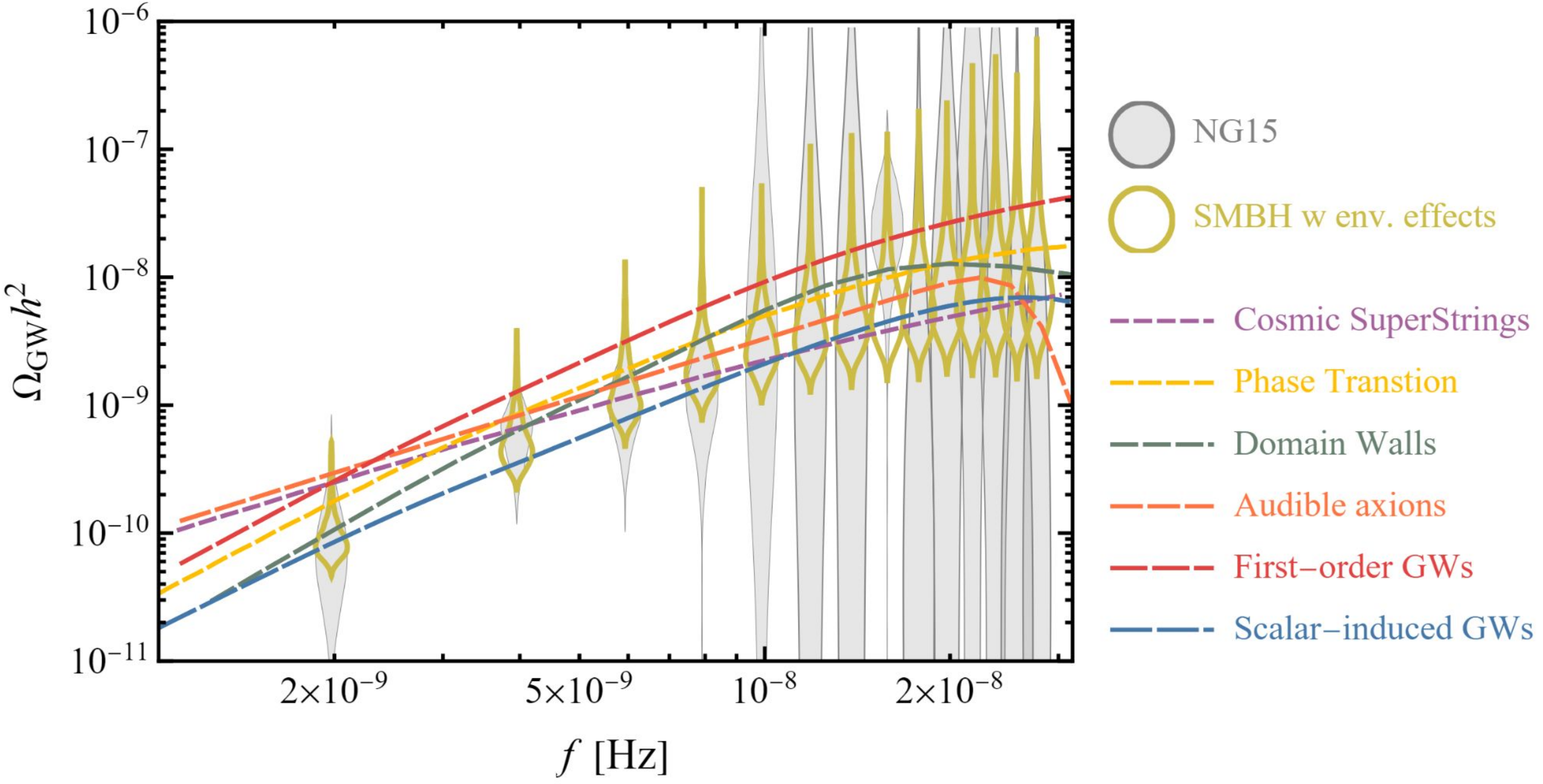


We need to reduce the amplitude of a factor O(2-3), so PTA-PBH tension is exacerbates

Are PBHs the end of the story?

arXiv:2308.08546 (PRD)
J. Ellis et al

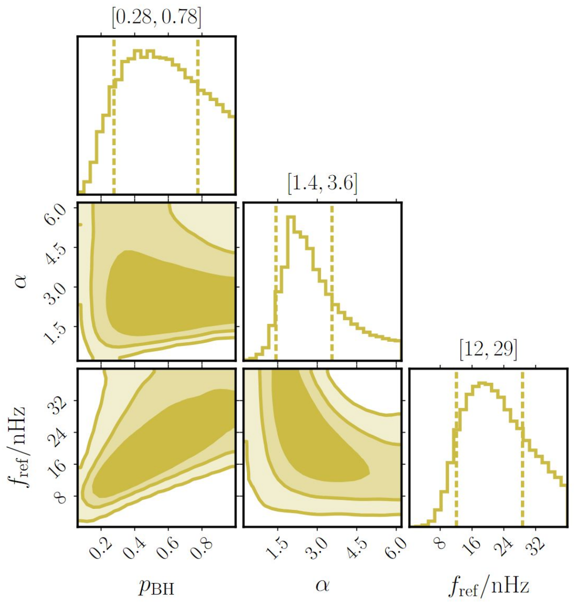
All the PTA possible sources for NANOGrav: Astro vs Cosmo

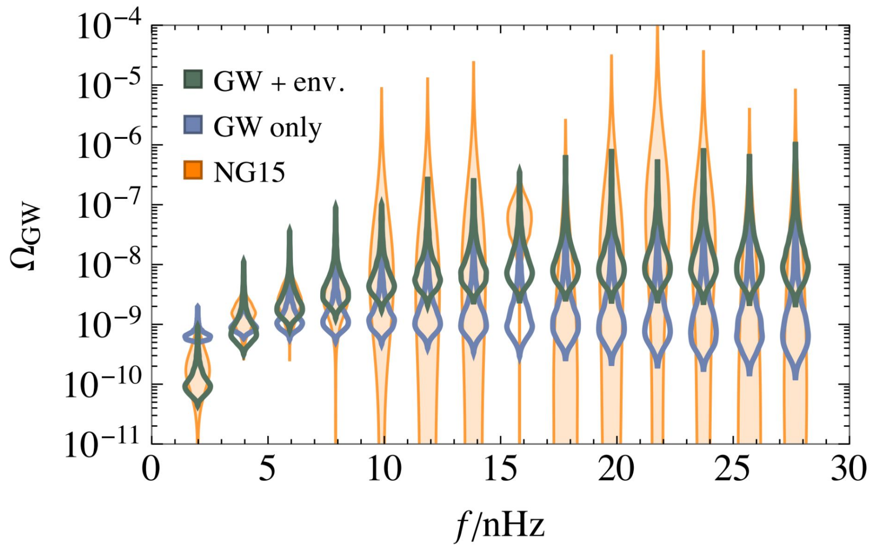


SMBH: Extra mechanism to lose energy with a different time scale

$$t_{\rm GW} \equiv |E|/\dot{E}_{\rm GW} = 4\tau \,, \qquad t_{\rm env} \equiv |E|/\dot{E}_{\rm env}$$

$$\frac{t_{\rm env}}{t_{\rm GW}} = \left(\frac{f_r}{f_{\rm GW}}\right)^{\alpha}, \quad f_{\rm GW} = f_{\rm ref}\left(\frac{\mathcal{M}}{10^9 M_\odot}\right)^{-\beta}$$





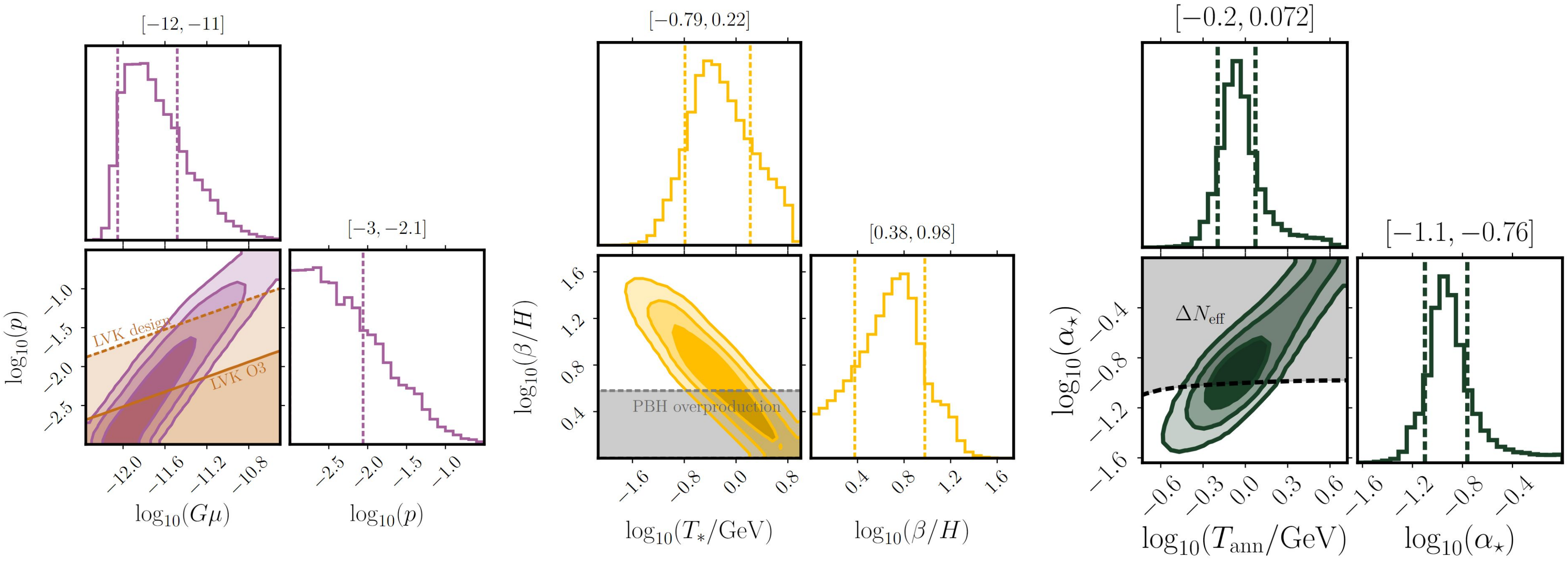
The interactions with the environments reduce the period over which the binaries emit GWs.

See also arXiv:2306.17021 J.Ellis, J.Urrutia et al

Cosmic Superstring: From evolution of a network of cosmic strings. See also arXiv:2306.17147 J.Ellis, M.Lewicki, C.Lin and V.Vaskonen

Phase Transition: From bubbles collisions and motion by inhomogeneities in the fluid.

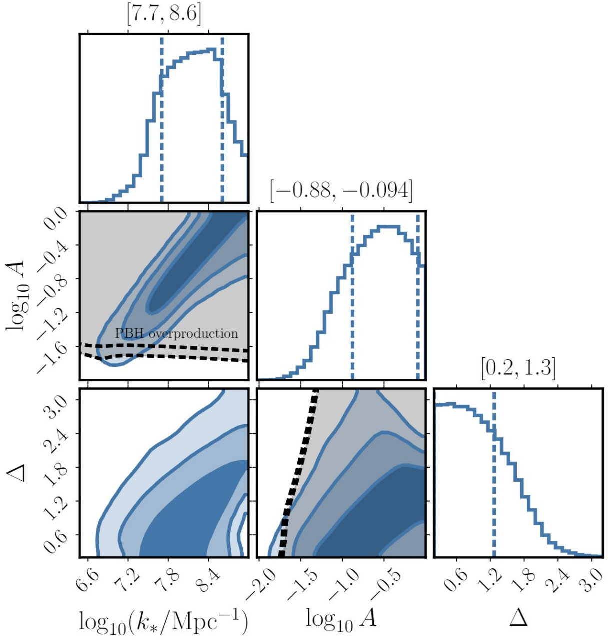
Domain Walls: emission of GWs due to DWs annihilation. See also arXiv:2306.17841 Y.Gouttenoire and E. Vitagliano

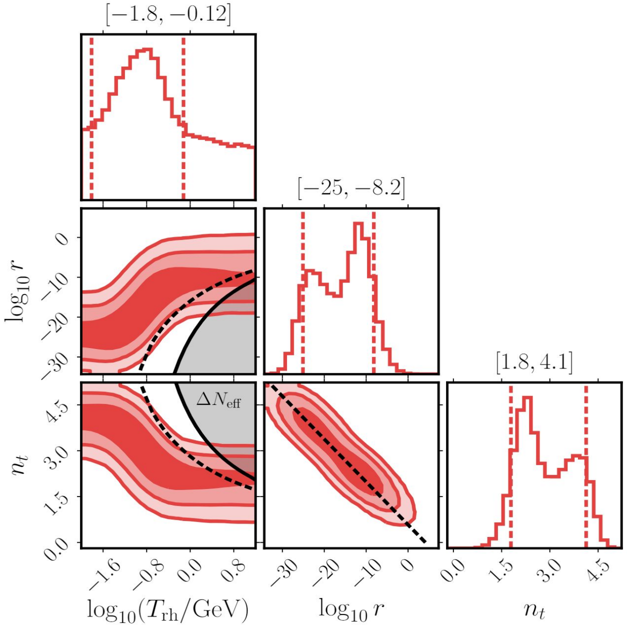


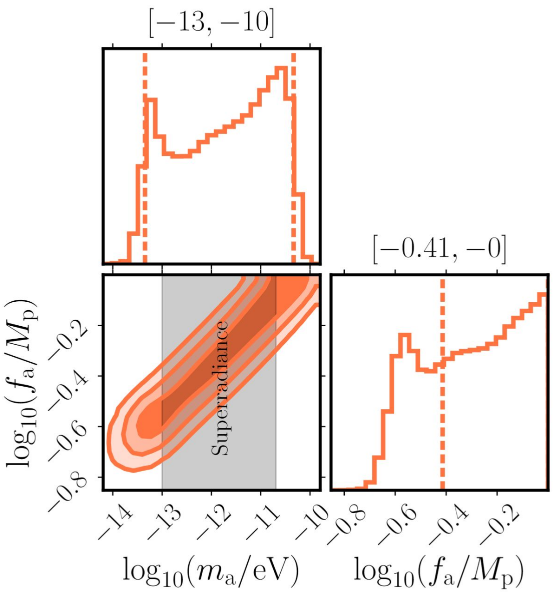
SIGWs: Large scalar cosmological perturbations. See this talk

FOGWs: Tensorial perturbations during Inflation. See also arXiv:2306.16912 S. Vagnozzi

Audible axions: Coupling to DP. Axion rolls down, tachionic instability for one of the DP helicities, causing vacuum fluctuations to grow. Anisotropic stress in energy-momentum tensor and then GWs.







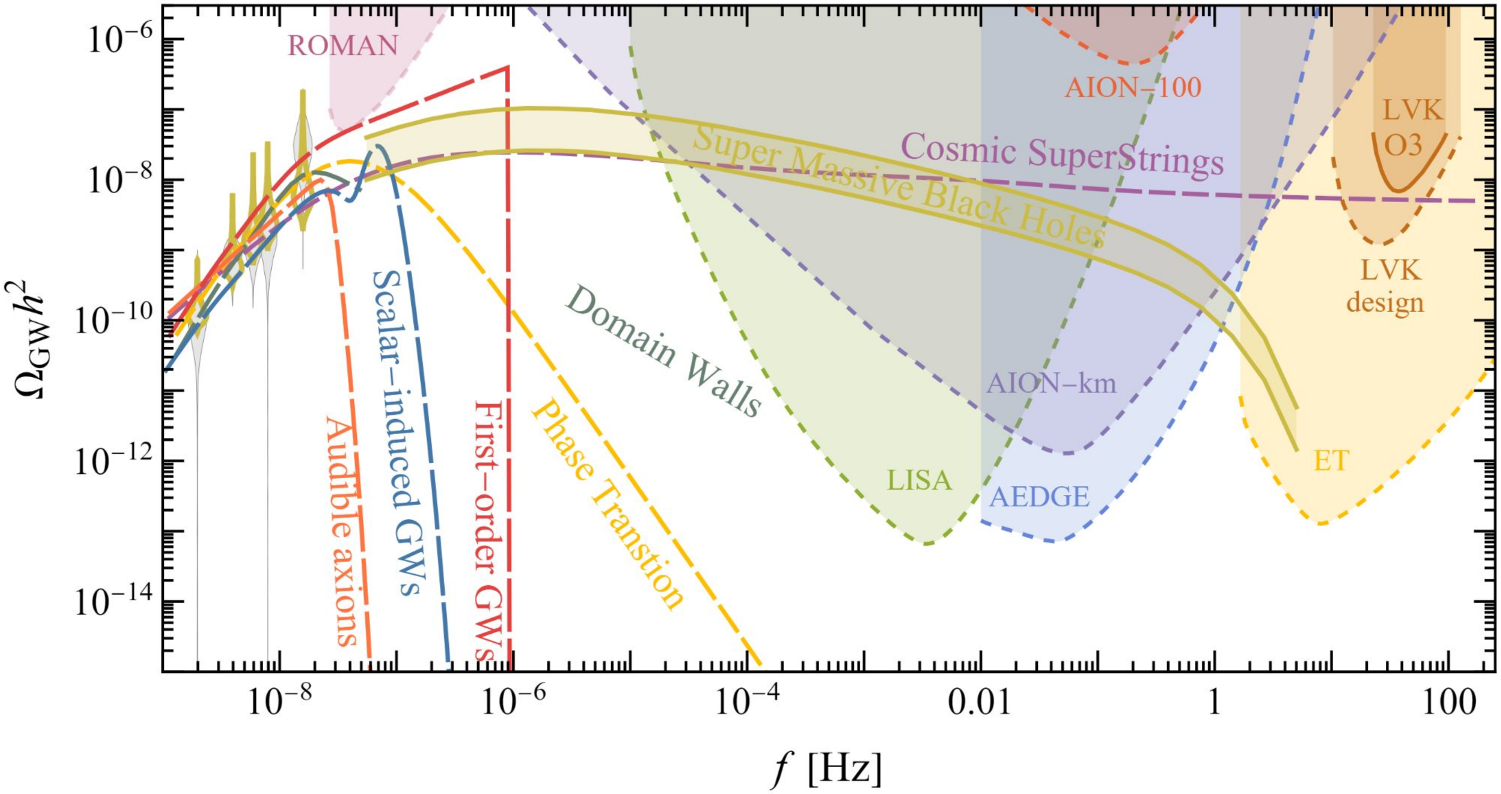
Results from Multi-Model Analysis (MMA)

Scenario	Best-fit parameters	ΔBIC	Signatures
GW-driven SMBH binaries	$p_{\rm BH} = 0.07$	6.0	FAPS, LISA, mid-f, ~~LVK, ET~~
GW + environment-driven SMBH binaries	$p_{\rm BH} = 0.84$ $\alpha = 2.0$ $f_{\rm ref} = 34$ nHz	Baseline (BIC = 53.9)	FAPS, LISA, mid-f, ~~LVK, ET~~
Cosmic (super)strings (CS)	$G\mu = 2 \times 10^{-12}$ $p = 6.3 \times 10^{-3}$	-1.2 (4.6)	~~FAPS~~, LISA, mid-f, LVK, ET
Phase transition (PT)	$T_* = 0.34$ GeV $\beta/H = 6.0$	-4.9 (2.9)	~~FAPS, LISA, mid-f, LVK, ET~~
Domain walls (DWs)	$T_{\rm ann} = 0.85$ GeV $\alpha_* = 0.11$	-5.7 (2.2)	~~FAPS~~, LISA?, ~~mid-f, LVK, ET~~
Scalar-induced GWs (SIGWs)	$k_* = 10^{7.7}$/Mpc $A = 0.06$ $\Delta = 0.21$	-2.1 (5.8)	~~FAPS, LISA, mid-f, LVK, ET~~
First-order GWs (FOGWs)	$\log_{10} r = -14$ $n_{\rm t} = 2.6$ $\log_{10}(T_{\rm rh}/{\rm GeV}) = -0.67$	-2.0 (6.0)	~~FAPS, LISA, mid-f, LVK, ET~~
"Audible" axions	$m_a = 3.1 \times 10^{-11}$ eV $f_a = 0.87\, M_{\rm P}$	-4.2 (3.7)	~~FAPS, LISA, mid-f, LVK, ET~~

FAPS ≡ fluctuations, anisotropies, polarization, sources, mid-f ≡ mid-frequency experiment, e.g., AION [308], AEDGE [310], LVK ≡ LIGO/Virgo/KAGRA [161–163], ET ≡ Einstein Telescope [312] (or Cosmic Explorer [313]), ~~signature~~ ≡ not detectable

TABLE I. *The parameters of the different models are defined in the text. For each model, we tabulate their best-fit values, and the Bayesian information criterion* $BIC \equiv -2\ell + k \ln 14$, *where* k *denotes the number of parameters, relative to that for the purely SMBH model with environmental effects that we take as the baseline. The quantity in the parentheses in the third column shows the* ΔBIC *for the best-fit combined SMBH+cosmological scenario. The last column summarizes the prospective signatures.*

Other experiments?



Conclusions

- *Fundamental to take into account both kind of NGs in the computation for the abundance.*

- *The tension between PTA and the PBH explanation can be alleviated for models where large negative NGs suppress the PBH abundance.*

- *GWs are an amazing tool to study the Universe.*
  A Myriad of GWs experiment and data are coming.

Backup Slides

Abundance of PBHs: The role of curvature perturbation ζ (or *R*).

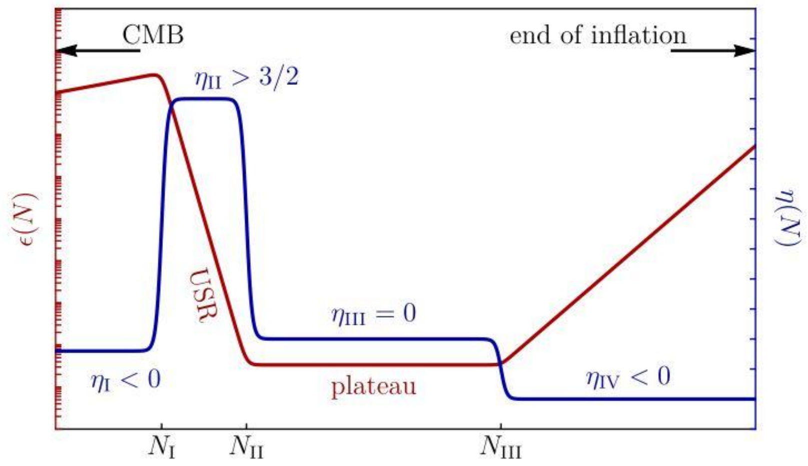
For the moment:

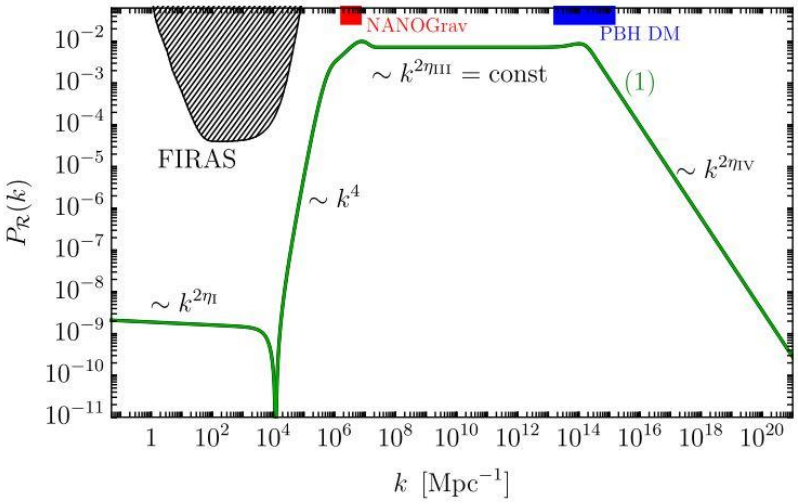
- ζ is gaussian
- $\delta(\vec{x},t) = -\frac{4}{9a^2H^2}\nabla^2\zeta(\vec{x})$ $\quad P_\delta(k,t) = \frac{16}{81}\frac{k^4}{a^4H^4}P_\zeta(k)$

In order to get 100 % of DM $P_\zeta \simeq$10^-2

USR $\quad \epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv -\frac{\ddot{H}}{2H\dot{H}} = \epsilon - \frac{1}{2}\frac{d\log\epsilon}{dN}$

G.Franciolini A.Urbano– arXiv:2207.10056

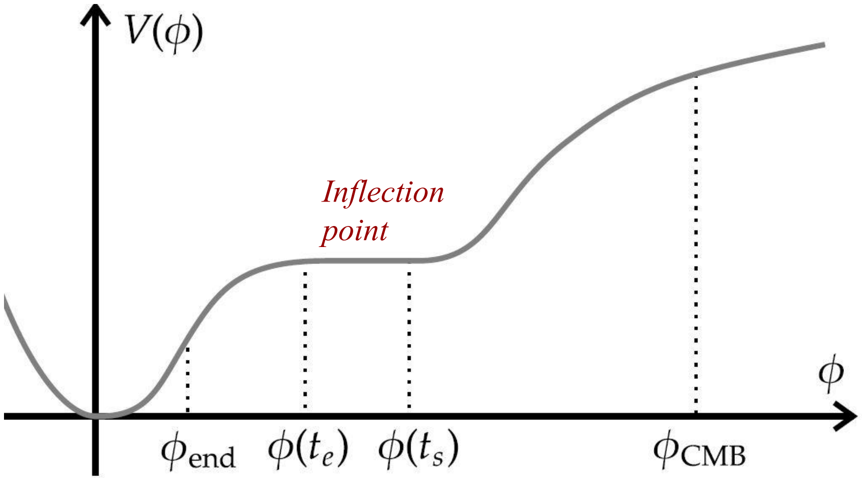




USR models

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm pl}^2 R - (\partial_\mu\phi)^2 - 2V(\phi)\right]$$

$$\phi(\mathbf{x},t) = \phi(t) + \delta\phi(\mathbf{x},t),$$
$$ds^2 = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$



We choose comoving gauge condition

$$\delta\phi(\mathbf{x},t) = 0,\ \ \gamma_{ij}(\mathbf{x},t) = a^2(t)[1 + 2\zeta(\mathbf{x},t)]\delta_{ij},$$

$$S^{(2)} = M_{\rm pl}^2\int dt\ d^3x\ a^3\epsilon\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right]$$

Famous Mukhanov-Sasaki equation

USR models

SR

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau)$$

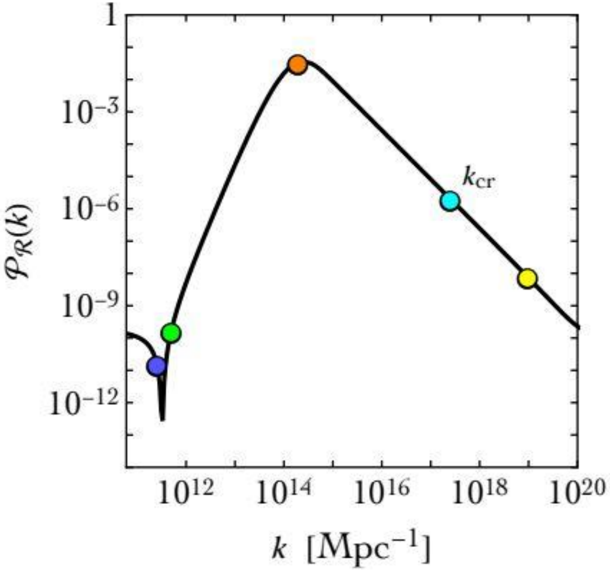
$$P_{\zeta(\rm SR)}(k) = \left(\frac{H^2}{8\pi^2 M_{\rm pl}^2\epsilon_{\rm SR}}\right)_\star$$

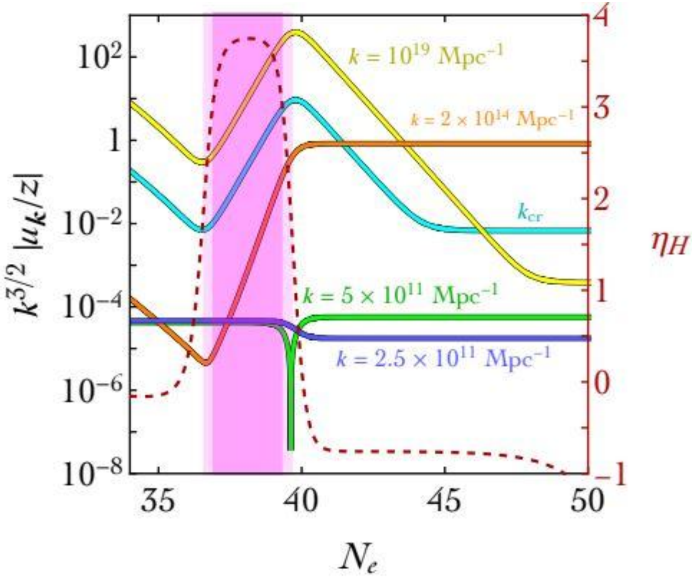
USR

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{1}{k^{3/2}}\mathcal{F}_k(\tau)$$

$$P_{\zeta(\rm PBH)} \approx P_{\zeta(\rm SR)}(k_s)\left(\frac{k_e}{k_s}\right)^6$$

G.Ballesteros,
J. Rey,
M.Taoso,
A.Urbano.
ArXiv:
2001.08220





USR models

SR

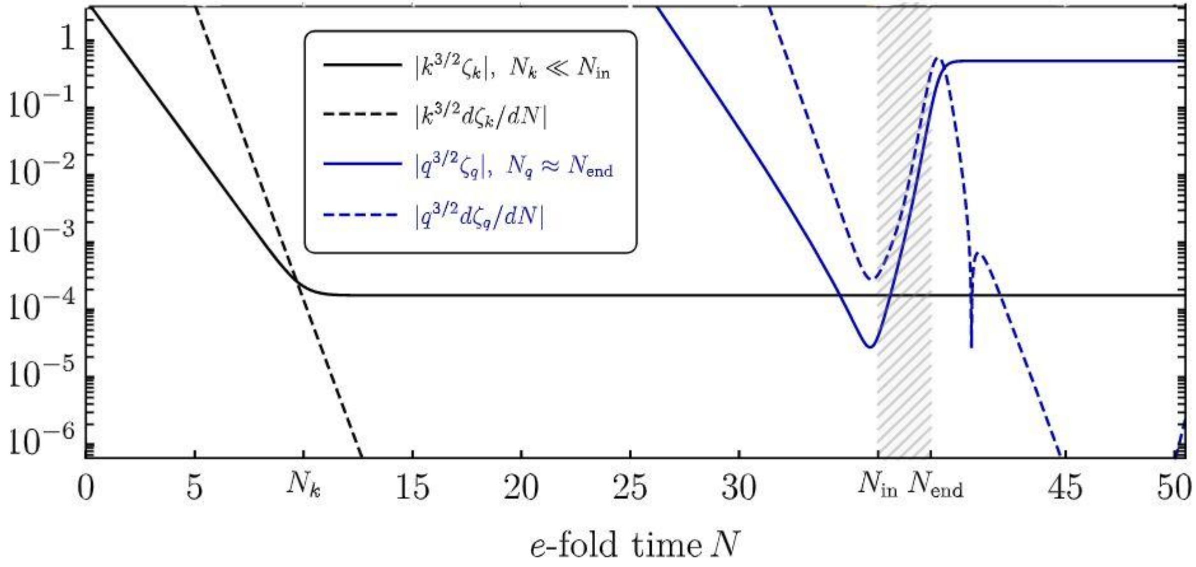
$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{e^{-ik\tau}}{k^{3/2}}(1+ik\tau)$$

$$P_{\zeta({\rm SR})}(k) = \left(\frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_{\rm SR}}\right)_\star$$

USR

$$\zeta_k(\tau) = \left(\frac{iH}{2M_{\rm pl}\sqrt{\epsilon_{\rm SR}}}\right)_\star \frac{1}{k^{3/2}}\mathcal{F}_k(\tau)$$

$$P_{\zeta({\rm PBH})} \approx P_{\zeta({\rm SR})}(k_s)\left(\frac{k_e}{k_s}\right)^6$$



Threshold statistics on the Compaction: Mathematical formulation

By integrating δ over the radial coordinate *r* we get the compaction function *C*

$$\mathcal{C}(r) = -2\Phi\, r\, \zeta'(r) \left[1 + \frac{r}{2}\zeta'(r)\right] = \mathcal{C}_1(r) - \frac{1}{4\Phi}\mathcal{C}_1(r)^2\,, \qquad \mathcal{C}_1(r) := -2\Phi\, r\, \zeta'(r)$$

In the presence of NG C_1 takes the form

$$\mathcal{C}_1(r) = -2\Phi\, r\, \zeta_{\rm G}'(r)\,\frac{dF}{d\zeta_{\rm G}} = \mathcal{C}_{\rm G}(r)\,\frac{dF}{d\zeta_{\rm G}}\,, \qquad \text{with} \quad \mathcal{C}_{\rm G}(r) := -2\Phi\, r\, \zeta_{\rm G}'(r)$$

From the two-dimensional joint PDF of ζ_G and C_G, called P_G

Later on confirmed also by A.Gow et al arXiv:2211.08348

NG PBH mass fraction-distribution adopting threshold statistics on the compaction function

$$\beta_{\rm NG} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C}-\mathcal{C}_{\rm th})^{\gamma}\, {\rm P}_{\rm G}(\mathcal{C}_{\rm G},\zeta_{\rm G})\, d\mathcal{C}_{\rm G}\, d\zeta_{\rm G}\,,$$

$$\mathcal{D} = \{\mathcal{C}_{\rm G},\, \zeta_{\rm G} \in \mathbb{R} \ :\ \mathcal{C}(\mathcal{C}_{\rm G},\zeta_{\rm G}) > \mathcal{C}_{\rm th} \ \wedge\ \mathcal{C}_1(\mathcal{C}_{\rm G},\zeta_{\rm G}) < 2\Phi\}\,,$$

$$f_{\rm PBH}(M_{\rm PBH}) = \frac{1}{\Omega_{\rm DM}} \int_{M_{\rm H}^{\rm min}(M_{\rm PBH})} d\log M_{\rm H} \left(\frac{M_{\rm eq}}{M_{\rm H}}\right)^{1/2} \left[1 - \frac{\mathcal{C}_{\rm th}}{\Phi} - \frac{1}{\Phi}\left(\frac{M_{\rm PBH}}{\mathcal{K} M_H}\right)^{1/\gamma}\right]^{-1/2} \frac{\mathcal{K}}{\gamma}\left(\frac{M_{\rm PBH}}{\mathcal{K} M_{\rm H}}\right)^{\frac{1+\gamma}{\gamma}}$$

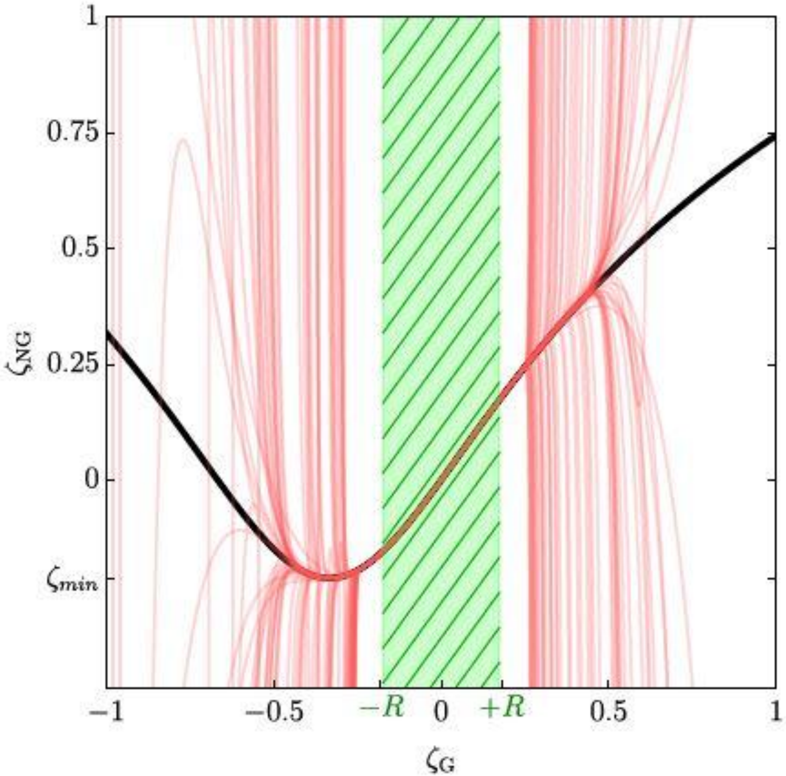
$$\times \int d\zeta_{\rm G} P_{\rm G}(\mathcal{C}_{\rm G}(M_{\rm PBH},\zeta_{\rm G}),\zeta_{\rm G}|M_{\rm H}) \left(\frac{dF}{d\zeta_{\rm G}}\right)^{-1}.$$

Failure of perturbative approach

$$\sum_{n=1}^{\infty} c_n(r_{\rm dec})\zeta_{\rm G}^n = \log\left[X(r_{\rm dec},\zeta_{\rm G})\right]$$

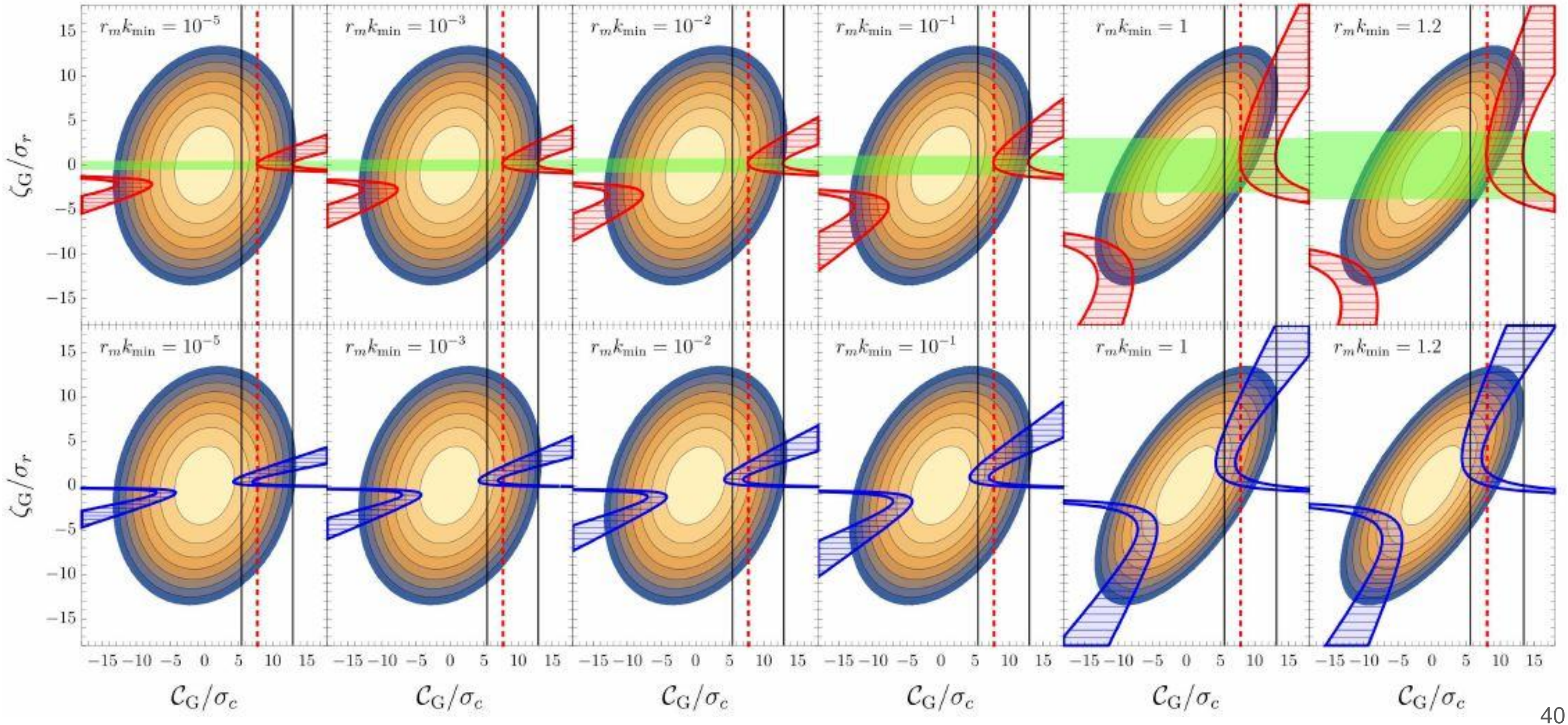
$$c_n(1) = \frac{(-1)^{n+1}}{n(2/3)^{n-1}}\,,$$

$$R = \lim_{n\to\infty}\left|\frac{c_n(1)}{c_{n+1}(1)}\right| = \frac{2}{3}\,,$$

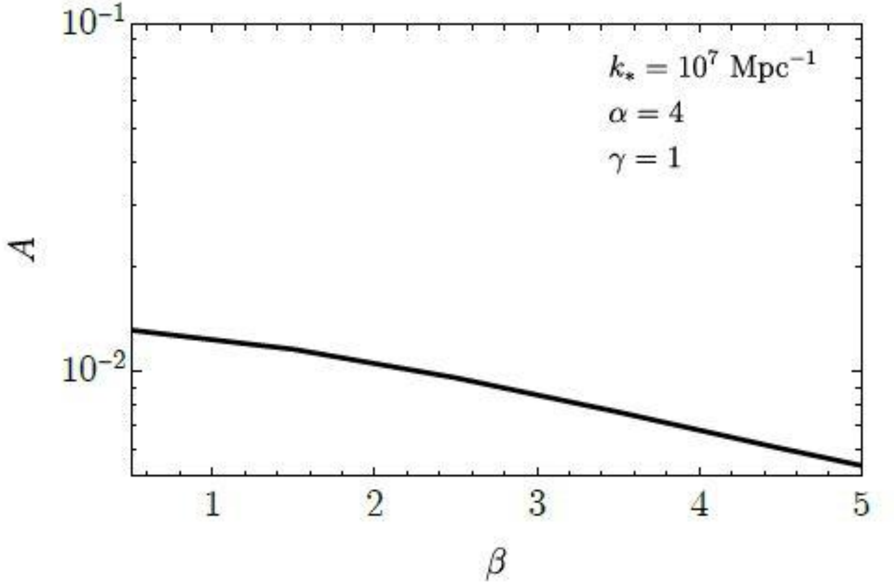


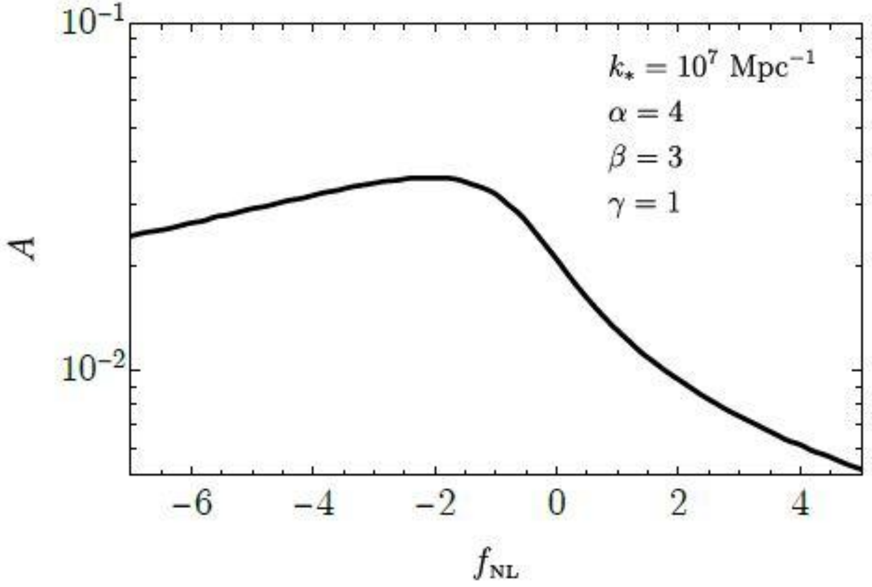
$$\frac{4(1-\sqrt{1-3\mathcal{C}_{\rm th}/2})}{3} < \mathcal{C}_{\rm G}\frac{dF}{d\zeta_{\rm G}} < \frac{4}{3}$$

Breaking of scale invariance



NG generic features

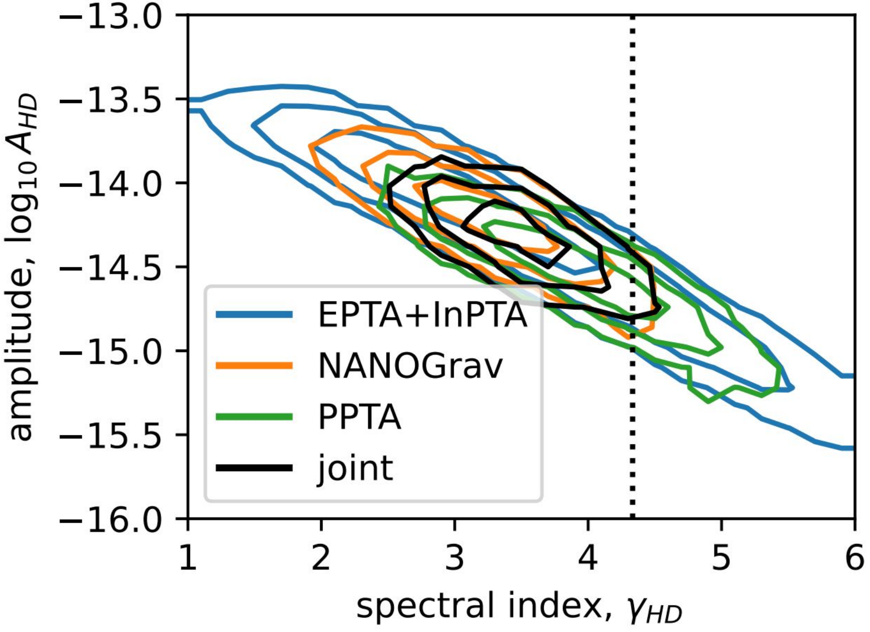




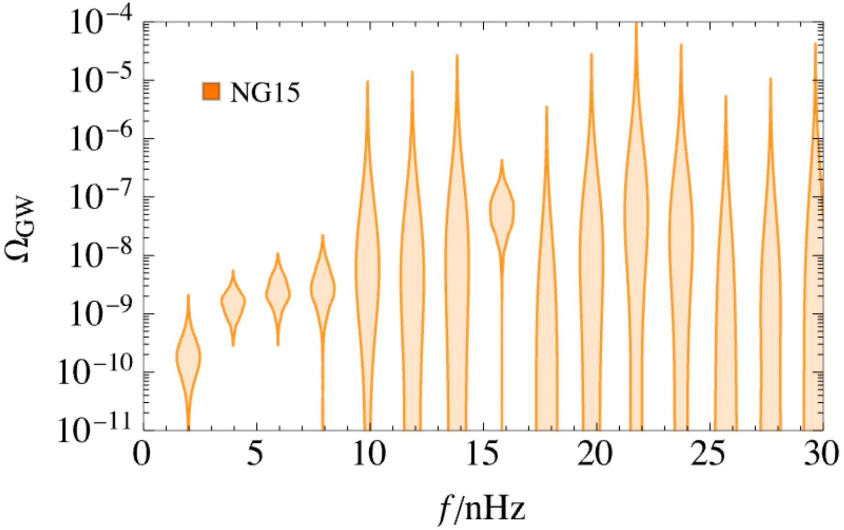
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693



NANOGrav – arXiv:2306.16213
arXiv:2306.16219



NG generic features

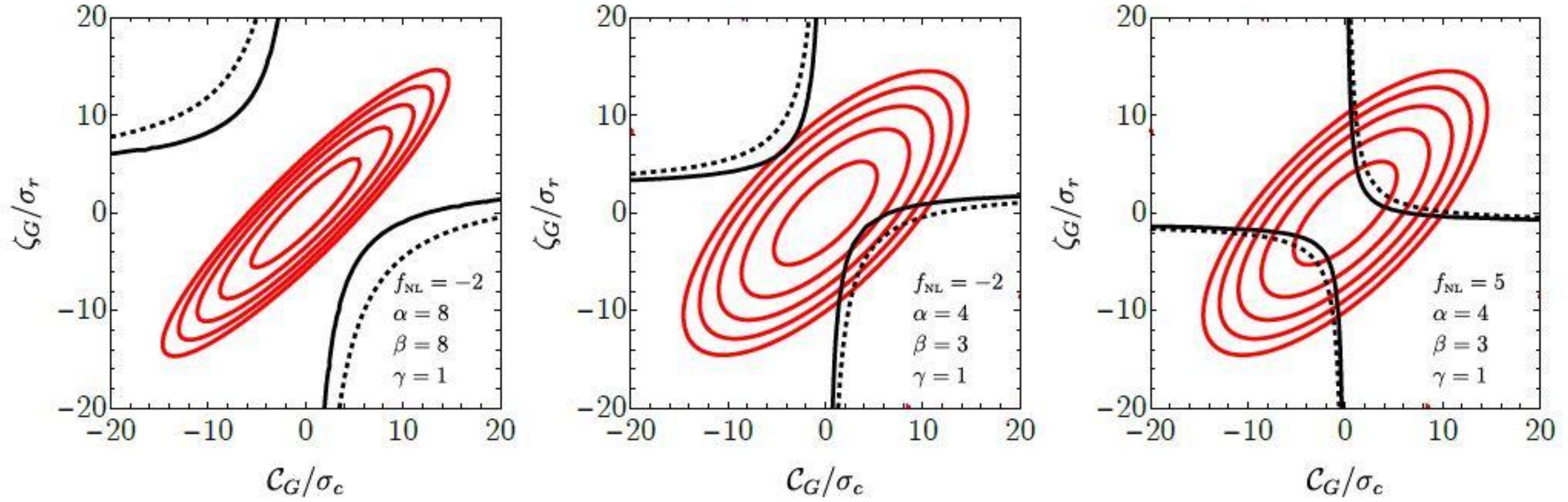


FIG. S4. Two dimensional PDF as a function of $(\mathcal{C}_G, \zeta_G)$ compared to the over-threshold condition $\mathcal{C} > \mathcal{C}_{th}$. In all panels, we considered the BPL power spectrum with an amplitude $A = 0.05$. The red lines indicates the contour lines corresponding to $\log_{10}(P_G) = -45, -35, -25, -15, -5$. The collapse of type-I PBHs take place between the black solid and dashed lines (see more details in Ref. [195]). *Left panel:* Example of a very narrow power spectrum with $\alpha = \beta = 8$. The abundance is suppressed in the presence of negative f_{NL} by the strong correlation between $\mathcal{C}_G$ and ζ_G obtained for narrow spectra. *Center panel:* Example of negative non-Gaussianity and representative BPL spectrum. The PBH formation is sourced by regions of small ζ_G and positive $\mathcal{C}_G$ or both negative $\mathcal{C}_G$ and ζ_G. *Right panel:* Example with positive f_{NL}, showing the region producing PBHs populates the correlated quadrants of the plot, at odds with that is found in the other panels.

Small improvements left

- Compute the threshold including NGs

  A. Escriva, Y. Tada, S. Yokoyama, and C.Yoo.– arXiv:2202.01028

- Variation of speed of sound due to QCD

  K.T.Abe, Y. Tada, I.Ueda.– arXiv:2010.06193

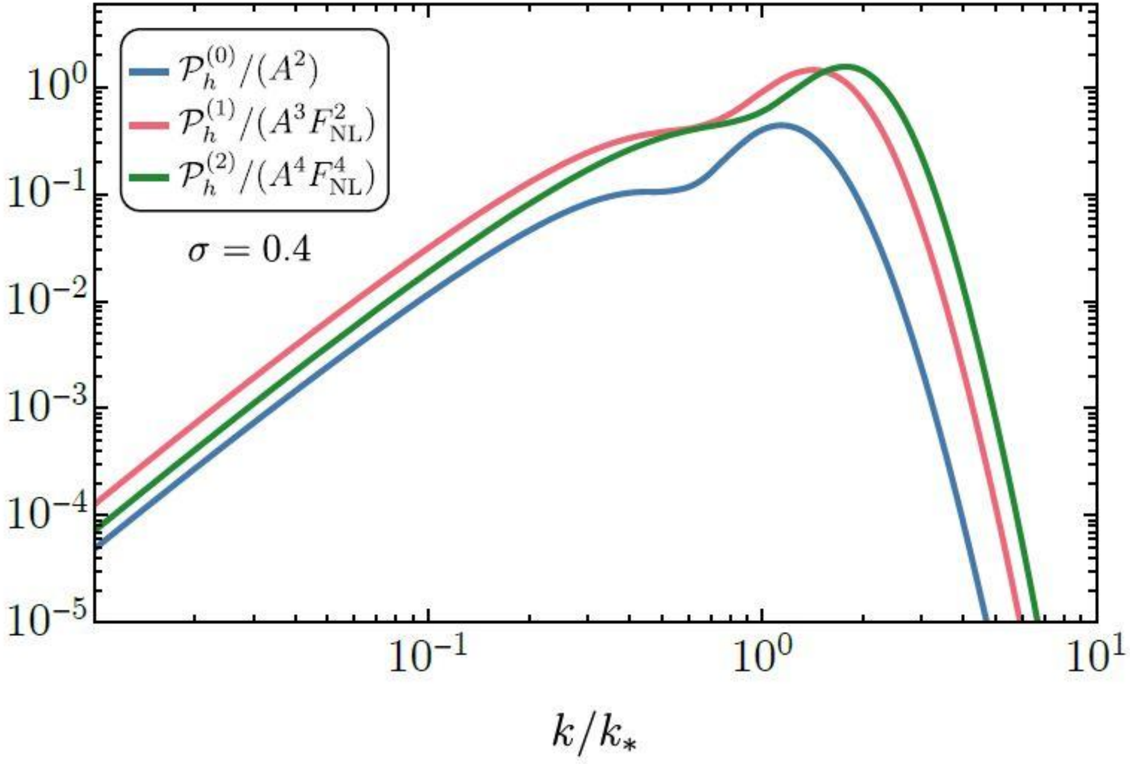
- NGs directly in GWs $\Omega_{\rm GW}^{\rm NLO}/\Omega_{\rm GW} \propto A(3f_{\rm NL}/5)^2$

  R. Cai, S. Pi, and M. Sasaki– arXiv:1810.11000
  K. T. Abe, R. Inui, Y. Tada, and S. Yokoyama-arXiv:2209.13891

NGs directly in GWs

arXiv:2308.08546
J. Ellis, M. Fairbarn, A.J.I. et al

PTA prefers IR tail, so HO corrections do not affect significantly the results showed before.



NGs directly in GWs

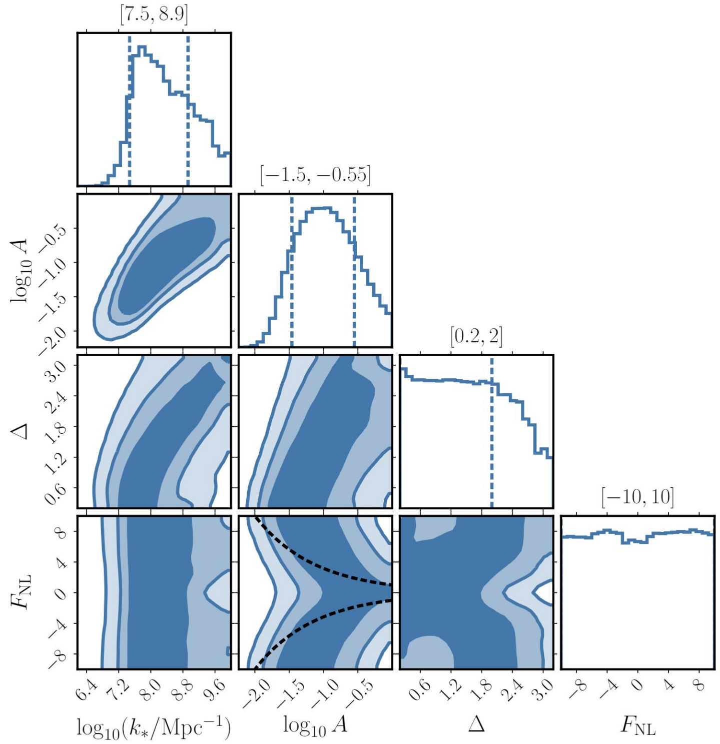
arXiv:2308.08546 (PRD)
J. Ellis et al

HO corrections do not affect significantly the results showed before.

$$\Omega_{GW}^{NLO}/\Omega_{GW} \propto A(3f_{NL}/5)^2$$

R. Cai, S. Pi, and M. Sasaki–arXiv:1810.11000
K. T. Abe, R. Inui, Y. Tada, and S. Yokoyama-arXiv:2209.13891

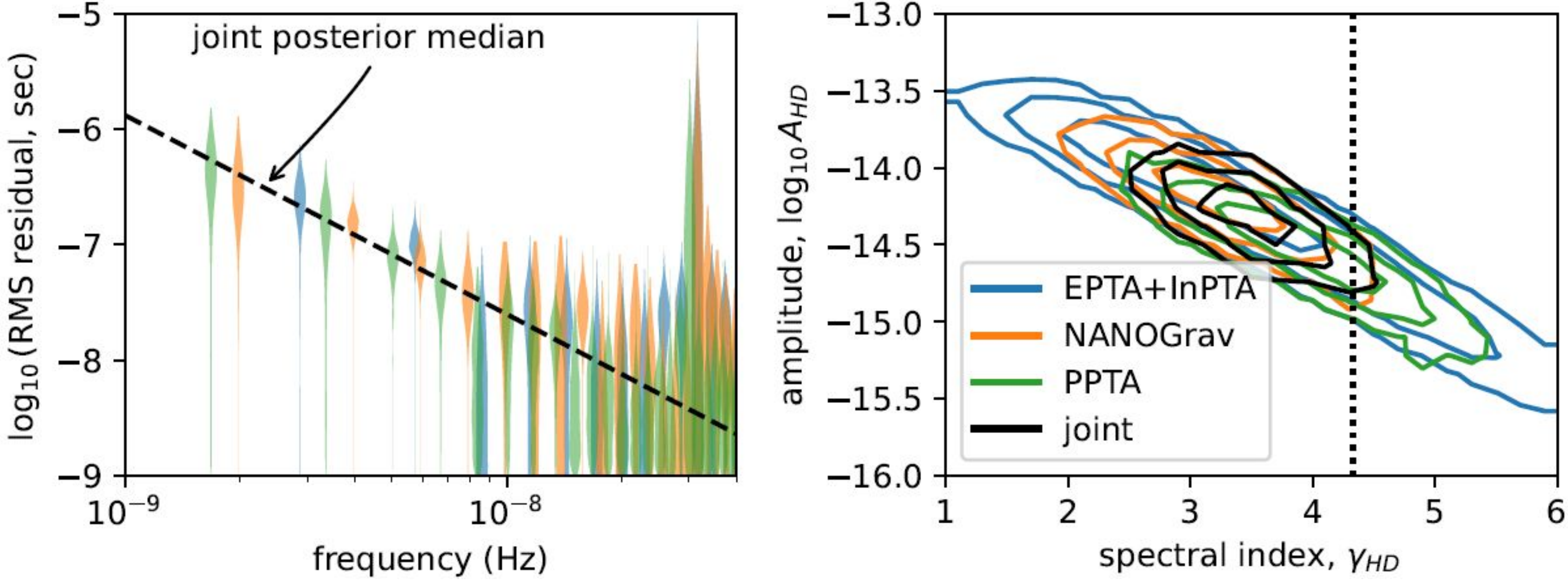
We cannot constrain the presence of NGs at PTA scales.
Large values of FNL are possible, provided the PS amplitude is sufficiently small.



PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

IPTA – arXiv:2309.00693



if the evolution is not only via GW emission

$$\dot{E} = -\dot{E}_{\rm GW} - \dot{E}_{\rm env}$$

Extra mechanism to lose energy with a different time scale,

$$t_{\rm GW} \equiv E/\dot{E}_{\rm GW} = 4\tau\,, \qquad t_{\rm env} \equiv E/\dot{E}_{\rm env}\,,$$

$$\frac{{\rm d}E_{\rm GW}}{{\rm d}\ln f_r} = \frac{{\rm d}E_{\rm GW}}{{\rm d}t}\frac{{\rm d}t}{{\rm d}\ln f_r} = \frac{1}{3}\frac{(\pi f_r)^{\frac{2}{3}}\,\mathcal{M}^{\frac{5}{3}}}{1 + t_{\rm GW}/t_{\rm env}}$$

which affects the relation between time and frequency.

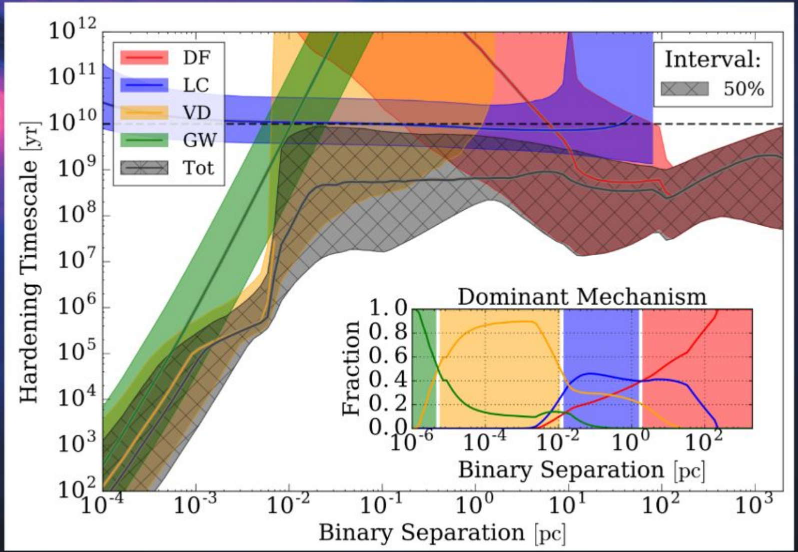
Phenomenological approach

Courtesy of J.Urrutia

We opted for a **generic form** of energy loss, that **respects the main results** in the literature but can **accommodate many models**

$$\frac{t_{\rm env}}{t_{\rm GW}} = \left(\frac{f_r}{f_{\rm GW}}\right)^{\alpha}, \quad f_{\rm GW} = f_{\rm ref}\left(\frac{\mathcal{M}}{10^9 M_\odot}\right)^{-\beta}$$

$\beta \in (0.2, 0.8)$



https://arxiv.org/pdf/1606.01900.pdf