Gravitational Waves in Einstein-Cartan Theory

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OUTLINE

1. EINSTEIN-CARTAN THEORY: A BRIEF INTRODUCTION

2. BLANCHET-DAMOUR APPROACH IN EINSTEIN-CARTAN THEORY

3. GRAVITATIONAL-WAVE GENERATION PROBLEM

4. A FIRST APPLICATION TO BINARY NEUTRON STAR AND BLACK HOLE SYSTEMS

5. CONCLUSIONS

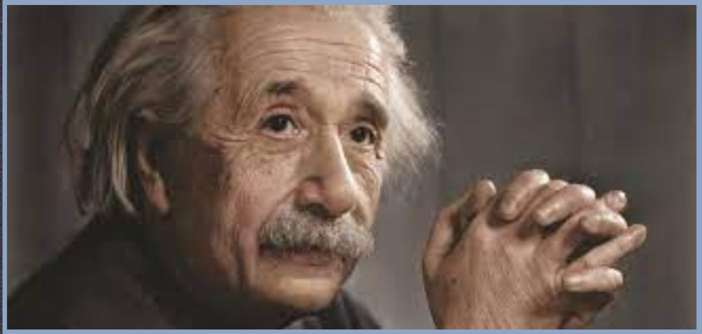
EINSTEIN-CARTAN THEORY (1)

- Einstein-Cartan (EC) theory has been formulated to extend the concepts of general relativity (GR) to the microphysical realm.

- Quantum intrinsic spin carried by elementary particles is described geometrically by means of the torsion tensor.

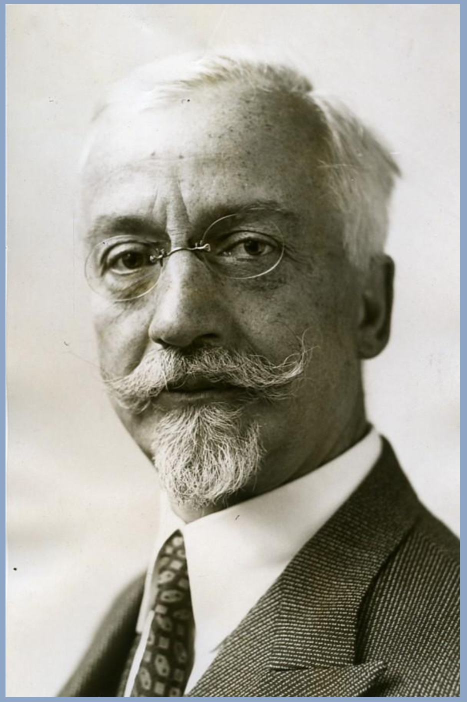
Mass-energy ⟷ Curvature

Energy-momentum tensor



Spin ⟷ Torsion

Spin density tensor



EINSTEIN-CARTAN THEORY (2)

- Riemann-Cartan spacetime U_4 endowed with the metric tensor $g_{\alpha\beta}$ and the most general nonsymmetric metric-compatible (i.e., $\nabla_\mu g_{\alpha\beta} = 0$) affine connection $\Gamma^\lambda_{\mu\nu}$:

$$\Gamma^\lambda_{[\mu\nu]} \equiv \frac{1}{2}(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}) \equiv S_{\mu\nu}{}^\lambda,$$

Torsion tensor

$$\Gamma^\lambda_{\mu\nu} = \hat{\Gamma}^\lambda_{\mu\nu} + (S^\lambda{}_{\mu\nu} + S_{\mu\nu}{}^\lambda - S_\nu{}^\lambda{}_\mu)$$

Affine connection

$$\hat{\Gamma}^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$$

Christoffel symbols

$$\Gamma^\lambda_{\mu\nu} = \hat{\Gamma}^\lambda_{\mu\nu} - K_{\mu\nu}{}^\lambda$$

Contortion tensor

$$K_{\mu\nu}{}^\lambda = S_\nu{}^\lambda{}_\mu - S^\lambda{}_{\mu\nu} - S_{\mu\nu}{}^\lambda$$

EINSTEIN-CARTAN THEORY (3)

- **EC theory**, **GR theory**, **special relativity**:

General affine manifold A_4

↓ $\nabla_\mu g_{\alpha\beta} = 0$ — **Metric-compatible connection**

Riemann-Cartan manifold U_4

↓ $S_{\mu\nu}{}^{\alpha} = 0$ — **Torsionless connection**

Riemann manifold V_4

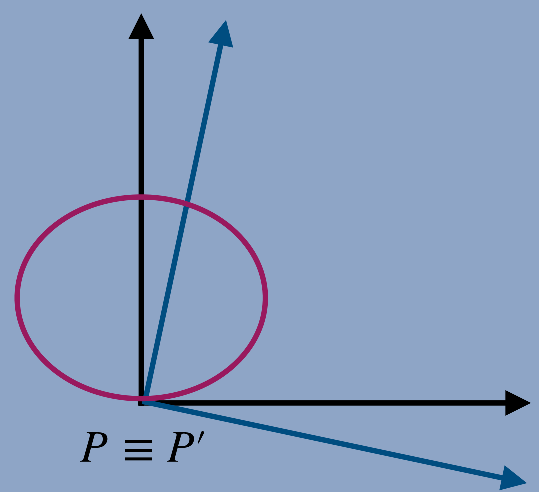
↓ $\hat{R}^{\alpha}{}_{\mu\nu\lambda} = 0$ — **Zero curvature**

Minkowski manifold M_4

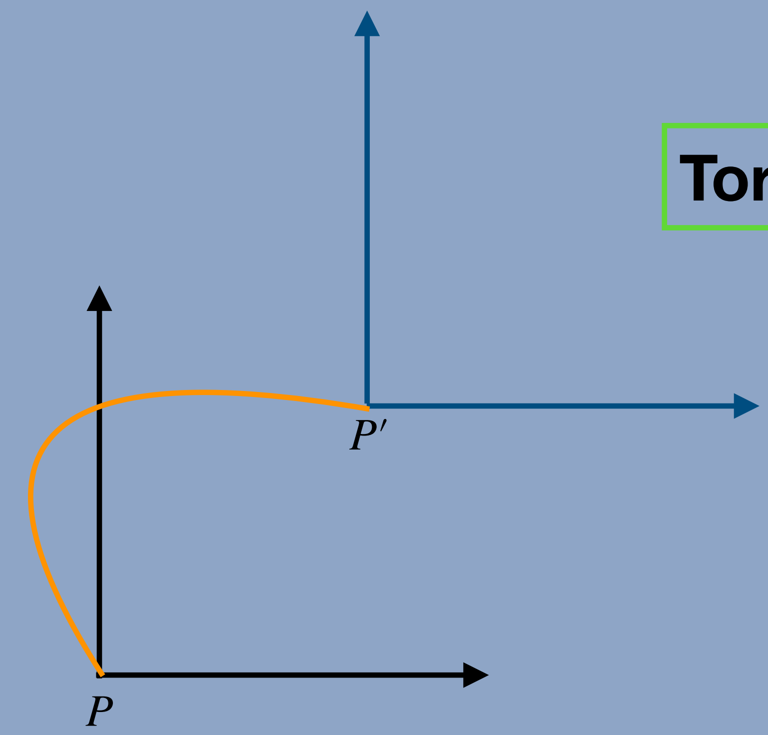
EINSTEIN-CARTAN THEORY (4)

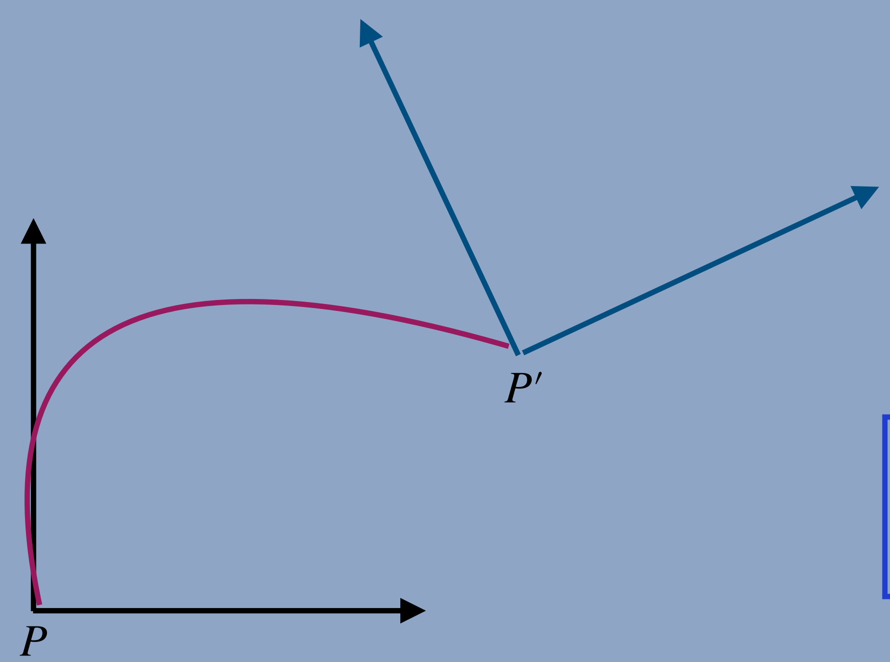
- **Curvature is related to the rotation of a vector; torsion is related to the translation of a vector.**

Curvature



Torsion





Curvature and torsion

EINSTEIN-CARTAN THEORY (5)

- **Riemann tensor in EC theory:**

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho \Gamma^{\mu}_{\sigma\nu} - \partial_\sigma \Gamma^{\mu}_{\rho\nu} + \Gamma^{\mu}_{\rho\alpha}\Gamma^{\alpha}_{\sigma\nu} - \Gamma^{\mu}_{\sigma\alpha}\Gamma^{\alpha}_{\rho\nu}$$

with $\Gamma^{\lambda}_{\mu\nu} = \hat{\Gamma}^{\lambda}_{\mu\nu} - K_{\mu\nu}{}^{\lambda}$

Riemann tensor of GR

$\hat{\nabla}_\mu \equiv$ **Covariant derivative wrt the Christoffel symbols** $\hat{\Gamma}^{\mu}{}_{\alpha\beta}$

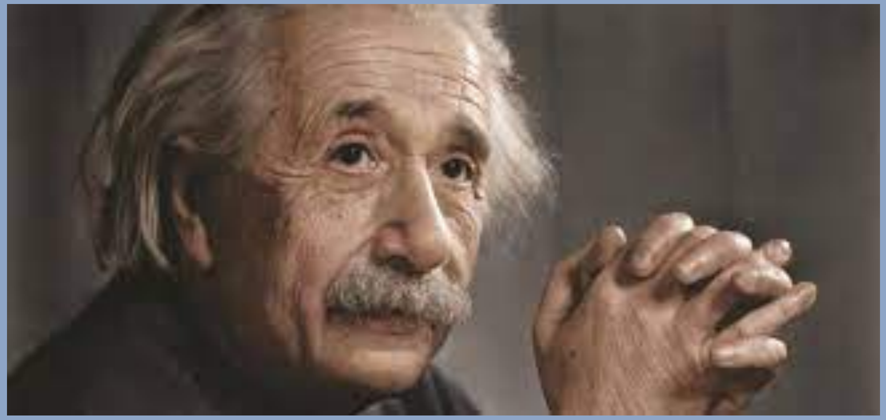
$$R_{\mu\nu\rho\sigma} = \hat{R}_{\mu\nu\rho\sigma} + \hat{\nabla}_\sigma K_{\rho\nu\mu} - \hat{\nabla}_\rho K_{\sigma\nu\mu} + K_{\rho\alpha\mu}K_{\sigma\nu}{}^{\alpha} - K_{\sigma\alpha\mu}K_{\rho\nu}{}^{\alpha},$$

$K_{\mu\nu\alpha} \equiv$ **Contortion tensor**

EINSTEIN-CARTAN THEORY (6)

- **Einstein tensor** $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ **is not symmetric and satisfies**

$$G_{[\mu\nu]} = R_{[\mu\nu]} = \nabla_\alpha ({S_{\mu\nu}}^{\alpha} + 2\delta^{\alpha}_{[\mu} {S_{\nu]\beta}}^{\beta}) + 2{S_{\alpha\rho}}^{\rho} ({S_{\mu\nu}}^{\alpha} + 2\delta^{\alpha}_{[\mu} {S_{\nu]\beta}}^{\beta})$$



- **Contracted Bianchi identity**

$$\nabla_\alpha {G_\mu}^{\alpha} = -2{S_{\alpha\rho}}^{\rho} {G_\mu}^{\alpha} - 2{S_{\alpha\mu}}^{\nu} {G_\nu}^{\alpha} + ({S_{\alpha\beta}}^{\nu} + 2\delta^{\nu}_{[\alpha} {S_{\beta]\rho}}^{\rho}) {R_{\mu\nu}}^{\alpha\beta}$$

$\nabla_\alpha \equiv$ **Covariant derivative wrt the connection coefficients** $\Gamma^{\mu}_{\alpha\beta}$

EINSTEIN-CARTAN FIELD EQUATIONS (1)

- **Total action of EC theory**

Matter Lagrangian

$$\mathscr{L}_{\mathrm{m}} = \mathscr{L}_{\mathrm{m}}(\psi, \partial\psi, \eta) \longrightarrow \mathscr{L}_{\mathrm{m}} = \mathscr{L}_{\mathrm{m}}(\psi, \partial\psi, g, \partial g, S)$$

Minimal coupling

$\eta_{\mu\nu} \longrightarrow g_{\mu\nu}$; $\partial_\mu \longrightarrow \nabla_\mu$

Gravitational Lagrangian

$$\mathscr{L}_{\mathrm{g}}(g, \partial g, S, \partial S) = \frac{c^4}{16\pi G} R$$

Total action of EC theory

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\mathscr{L}_{\mathrm{m}}(\psi, \partial\psi, g, \partial g, S) + \frac{c^4}{16\pi G} R \right]$$

EINSTEIN-CARTAN FIELD EQUATIONS (2)

- **EC field equations**

$$G^{\mu\nu} = \frac{8\pi G}{c^4} \mathbb{T}^{\mu\nu}$$

$$S_{\alpha\beta}{}^{\nu} + 2\delta^{\nu}_{[\alpha} S_{\beta]\rho}{}^{\rho} = \frac{8\pi G}{c^4} \tau_{\alpha\beta}{}^{\nu}$$

where

$$\mathbb{T}_{\beta}{}^{\alpha} = \frac{1}{\sqrt{-g}} \left[\delta_{\beta}{}^{\alpha} \mathcal{L}_{\mathrm{m}} - \frac{\partial \mathcal{L}_{\mathrm{m}}}{\partial(\partial_{\alpha}\Psi)} \nabla_{\beta}\Psi \right]$$

Canonical energy-momentum tensor

$$\tau_{\gamma}{}^{\beta\alpha} = \frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}_{\mathrm{m}}}{\delta K_{\alpha\beta}{}^{\gamma}},$$

Canonical spin angular momentum tensor

EINSTEIN-CARTAN FIELD EQUATIONS (3)

- **Generalized conservation laws**

$$\left(\nabla_\nu + 2S_{\nu\alpha}{}^{\alpha}\right)\mathbb{T}_\mu{}^\nu = 2\mathbb{T}_\lambda{}^\nu S_{\mu\nu}{}^\lambda - \tau_{\nu\rho}{}^\sigma R_{\mu\sigma}{}^{\nu\rho},$$

Conservation of energy-momentum

$$\nabla_\alpha G_\mu{}^\alpha = -2S_{\alpha\rho}{}^\rho G_\mu{}^\alpha - 2S_{\alpha\mu}{}^\nu G_\nu{}^\alpha + (S_{\alpha\beta}{}^\nu + 2\delta^\nu_{[\alpha}S_{\beta]\rho}{}^\rho)R_{\mu\nu}{}^{\alpha\beta}$$

$$\left(\nabla_\lambda + 2S_{\lambda\alpha}{}^{\alpha}\right)\tau_{\mu\nu}{}^\lambda = \mathbb{T}_{[\mu\nu]},$$

Conservation of angular momentum

$$G_{[\mu\nu]} = \nabla_\alpha(S_{\mu\nu}{}^\alpha + 2\delta^\alpha_{[\mu}S_{\nu]\beta}{}^\beta) + 2S_{\alpha\rho}{}^\rho(S_{\mu\nu}{}^\alpha + 2\delta^\alpha_{[\mu}S_{\nu]\beta}{}^\beta)$$

EINSTEIN-CARTAN FIELD EQUATIONS (4)

- **EC field equations**

$$\hat{G}^{\alpha\beta} = \frac{\chi}{2}\Theta^{\alpha\beta},$$

$\hat{G}^{\alpha\beta} \equiv$ **Einstein tensor constructed with the Christoffel symbols** $\hat{\Gamma}^{\mu}{}_{\alpha\beta}$

$(\chi = 16\pi G/c^4)$

where

$$\Theta^{\alpha\beta} = T^{\alpha\beta} + \frac{\chi}{2}\mathcal{S}^{\alpha\beta},$$

combined energy-momentum tensor

$$T^{\alpha\beta} = \frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathscr{L}_m\right)}{\delta g_{\alpha\beta}}$$

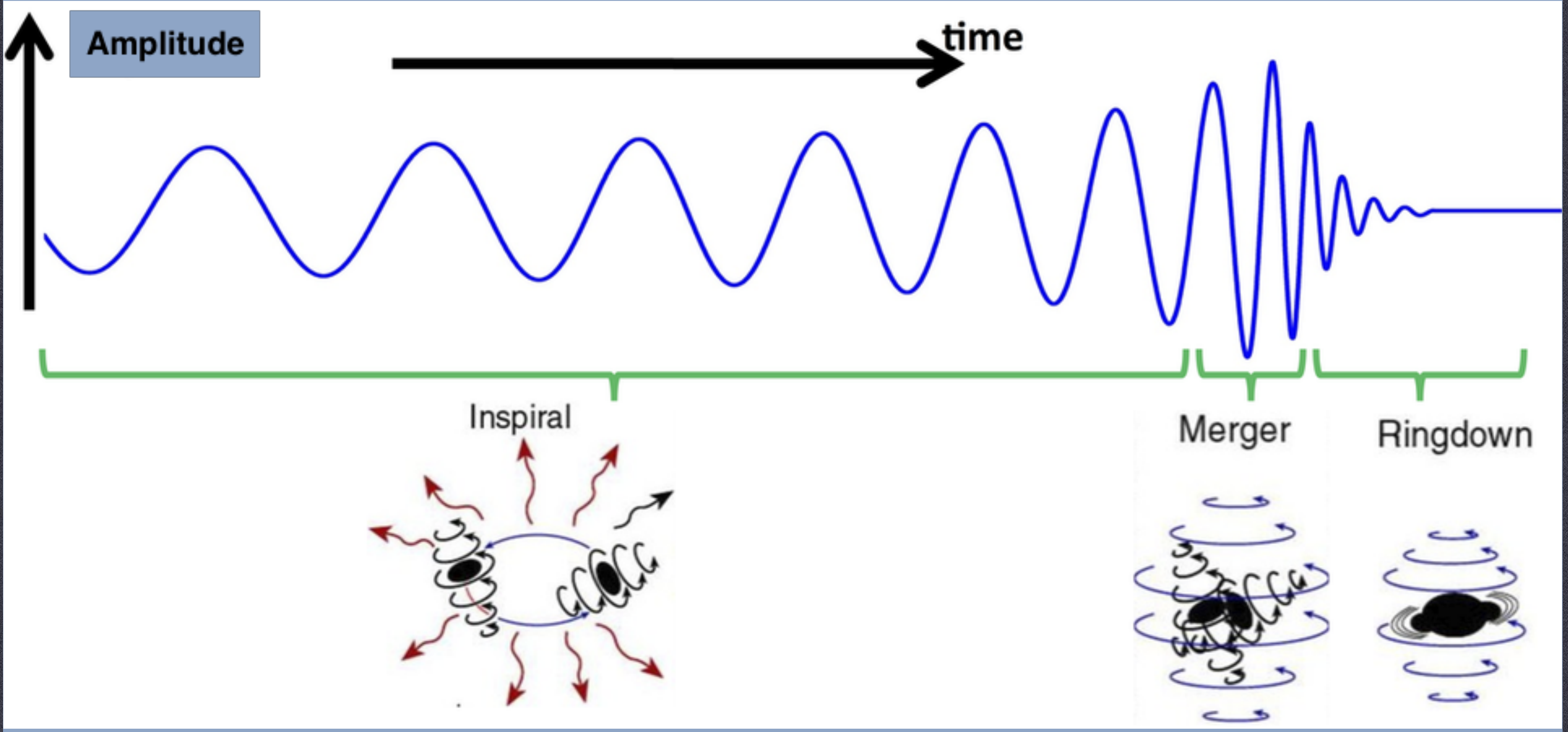
metric energy-momentum tensor

$$\mathcal{S}^{\alpha\beta} \equiv -4\tau^{\alpha\gamma}{}_{[\delta}\tau^{\beta\delta}{}_{\gamma]} - 2\tau^{\alpha\gamma\delta}\tau^{\beta}{}_{\gamma\delta} + \tau^{\gamma\delta\alpha}\tau_{\gamma\delta}{}^{\beta} + \frac{1}{2}g^{\alpha\beta}\left(4\tau_{\mu}{}^{\gamma}{}_{[\delta}\tau^{\mu\delta}{}_{\gamma]} + \tau^{\mu\gamma\delta}\tau_{\mu\gamma\delta}\right),$$

Contribution due to spin ($\tau^{\alpha\beta}{}_{\gamma} \equiv$ canonical spin angular momentum tensor)

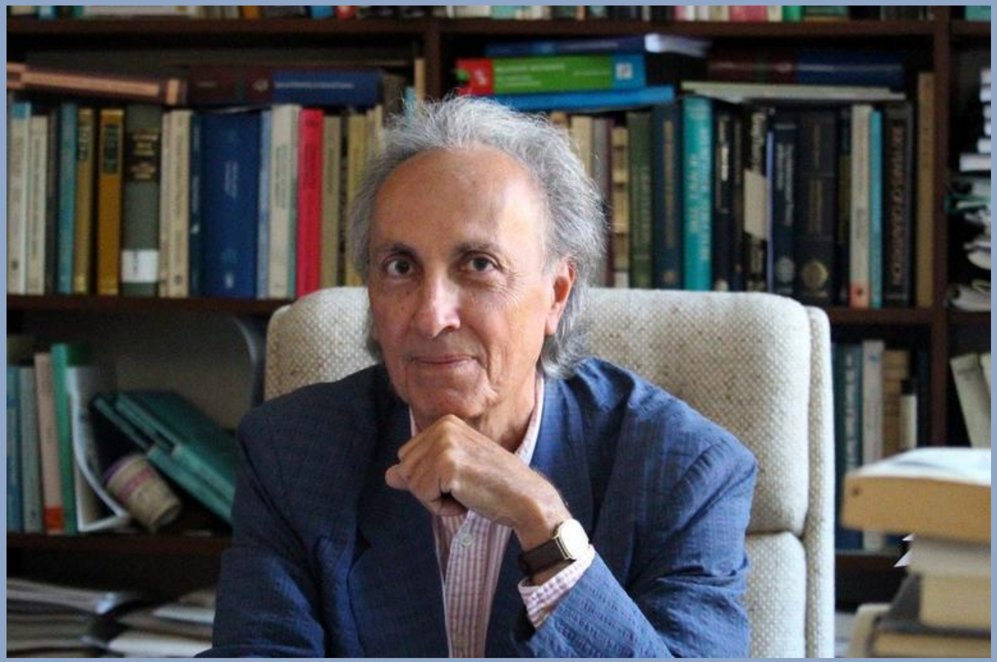
GRAVITATIONAL WAVES IN GR

- **In high-energy astrophysics, the main sources of gravitational waves (GWs) are compact binary systems: black holes (BHs) and neutron stars (NSs).**



BLANCHET-DAMOUR APPROACH IN EC THEORY (1)

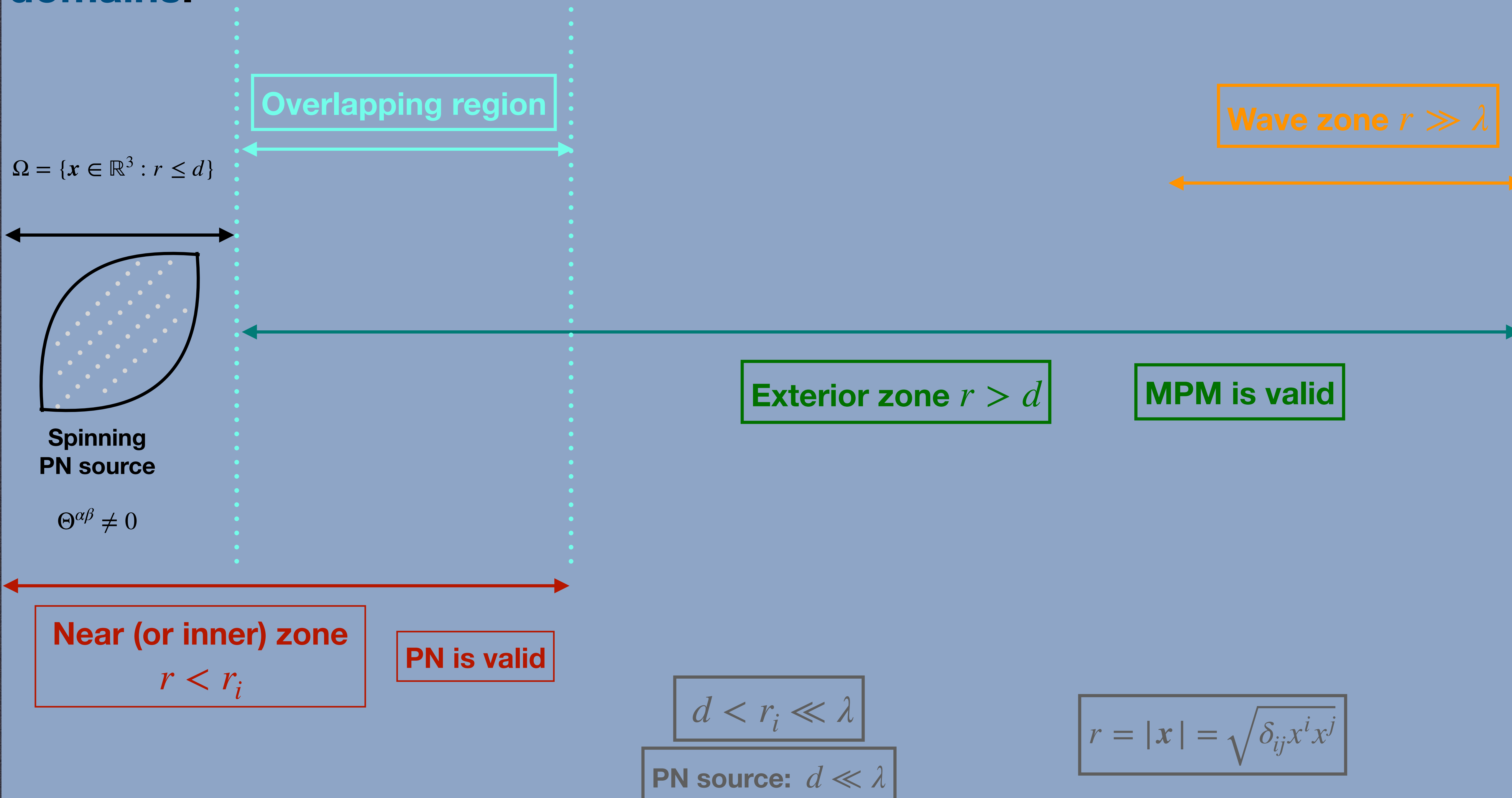
- **Spinning, weakly self-gravitating, weakly stressed, and slowly moving sources (i.e., spinning PN sources).**
- **Motion and radiation of binary systems in their early inspiralling stage.**



- **GW generation problem: relating the asymptotic gravitational-wave form generated by some isolated spinning PN source and which we observe via a detector (located in the wave zone of the source), to the material content of the source, i.e., its tensor $\Theta^{\alpha\beta}$, using some suitable approximation methods.**

BLANCHET-DAMOUR APPROACH IN EC THEORY (2)

Let us introduce a set of harmonic coordinates $x^\mu = (ct, \boldsymbol{x})$. The spatial part $\mathbb{R}^3$ of the spacetime manifold U_4 is decomposed in the following domains:



BLANCHET-DAMOUR APPROACH IN EC THEORY (3)

- **Post-Newtonian (PN) approximation scheme: valid under the assumptions of weak gravitational field inside the source and slow internal motions; an expansion in $v/c \ll 1$ is employed.**

$$F(t - r/c) = F(t) - \frac{r}{c}\dot{F}(t) + \frac{r^2}{2c^2}\ddot{F}(t) + \dots$$

$$\omega/c = 2\pi/\lambda$$

- **Post-Minkowskian (PM) approximation scheme: valid over all the spacetime; it operates by means of an expansion in the Newton gravitational constant G:**

$$\sqrt{-g}\,g^{\alpha\beta} = \eta^{\alpha\beta} + G\mathfrak{h}_1^{\alpha\beta} + G^2\mathfrak{h}_2^{\alpha\beta} + \dots$$

$$\mathfrak{h}^{\alpha\beta} = \sum_{n=1}^{\infty} G^n \mathfrak{h}_n^{\alpha\beta}$$

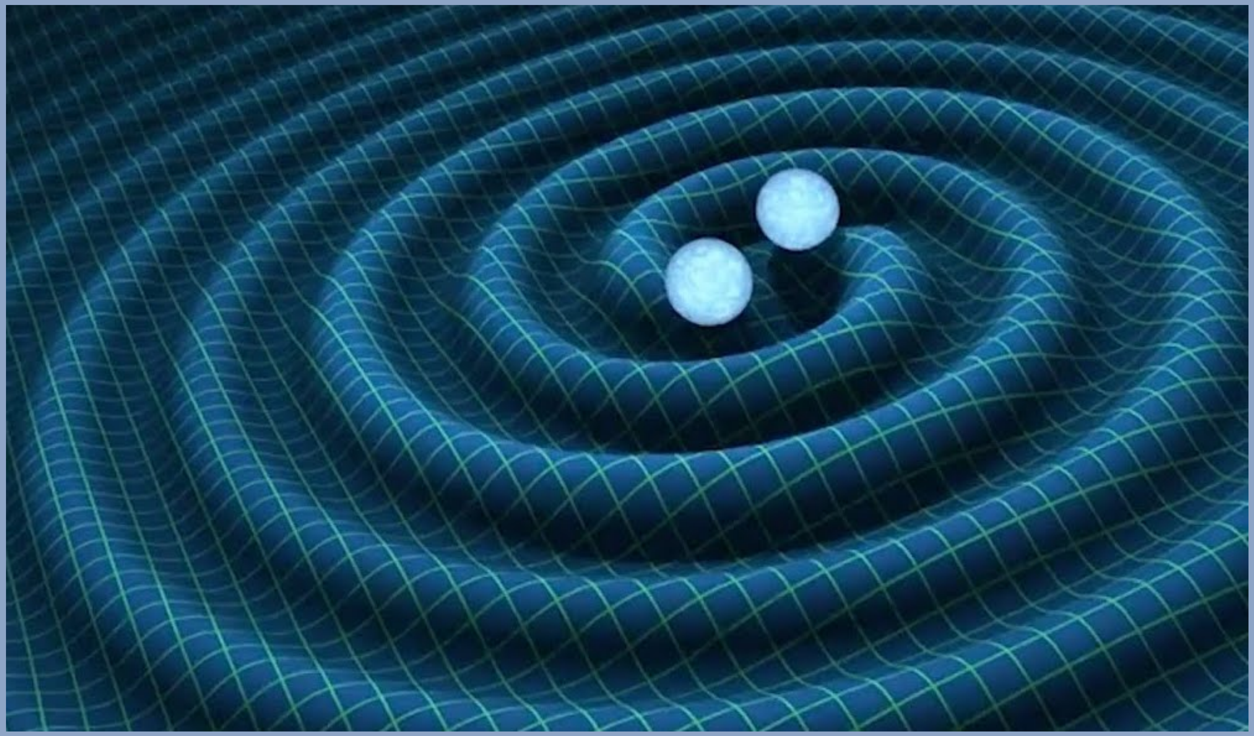
$\sqrt{-g}\,g^{\alpha\beta}$: **gothic metric**

basic variable: $\mathfrak{h}^{\alpha\beta} = \sqrt{-g}\,g^{\alpha\beta} - \eta^{\alpha\beta}$

$g = \det(g_{\mu\nu})$

BLANCHET-DAMOUR APPROACH IN EC THEORY (4)

- **Multipole expansion: method used to describe the properties of the source as seen from its exterior ($r > d$); the spacetime metric is parametrized by symmetric and trace-free (STF) multipole moments.**



- **Multipolar-post-Minkowskian (MPM) method: it can be employed in the exterior weak-field region of the source to solve vacuum EC field equations and combines the PM algorithm and the multipole expansion.**

BLANCHET-DAMOUR APPROACH IN EC THEORY (5)

- **Blanchet-Damour formalism is based on two approximation schemes: MPM and PN methods. It allows to solve approximately the GW generation problem and employs a four-stage program:**

1. In the exterior domain, vacuum EC field equations are perturbatively solved by means of the MPM algorithm and the resulting solution is parametrized by STF source multipole moments I_L, J_L:

recall that
$\mathfrak{h}^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\beta} - \eta^{\alpha\beta}$

$$\mathfrak{h}^{\alpha\beta}_{\rm ext} = \sum_{n=1}^{\infty} G^n \mathfrak{h}^{\alpha\beta}_n$$

$$\mathfrak{h}^{\alpha\beta}_{\rm ext} = \mathfrak{h}^{\alpha\beta}_{\rm ext}(I_L, J_L)$$

I_L**: mass-type STF source multipole moment of order *l***

J_L**: current-type STF source multipole moment of order *l***

Multi-index notation, where *L* denotes the multi-index $i_1 i_2 \ldots i_l$ made of *l* spatial indices. Hence $I_L = I_{i_1 i_2 \ldots i_l}$

BLANCHET-DAMOUR APPROACH IN EC THEORY (6)

2. In the wave zone, a set of radiative coordinates $X^\mu = (cT, \boldsymbol{X})$ is invoked where the metric coefficients $\mathscr{H}^{\mu\nu}$ admit the radiative form

$$\mathscr{H}^{\mu\nu}(X) = \sum_{n=1}^{\infty} \frac{1}{\mathcal{R}^n} \mathscr{H}_n^{\mu\nu}(T - \mathcal{R}/c, \boldsymbol{\mathcal{N}})$$

$$\mathcal{R} = |\boldsymbol{X}| \equiv \left(\delta_{ij} X^i X^j\right)^{1/2} \to \infty$$

radiative distance from the source

$\mathcal{U} \equiv T - \mathcal{R}/c$ retarded time

$\boldsymbol{\mathcal{N}} \equiv \boldsymbol{X}/\mathcal{R}$ direction of propagation of GW

$$\mathscr{H}_{ij}^{\mathrm{TT}}(X^\mu) = \frac{4G}{c^2 \mathcal{R}} \mathscr{P}_{ijkl}(\boldsymbol{\mathcal{N}}) \sum_{l=2}^{\infty} \frac{1}{l! c^l} \left\{ \mathcal{N}_{L-2} U_{klL-2}(\mathcal{U}) - \frac{2l}{(l+1)c} \mathcal{N}_{aL-2} \epsilon_{ab(k} V_{l)bL-2}(\mathcal{U}) \right\},$$

Transverse-traceless (TT) projection of the leading term

U_L: mass-type STF radiative multipole moment of order *l*

V_L: current-type STF radiative multipole moment of order *l*

Physical Observables

BLANCHET-DAMOUR APPROACH IN EC THEORY (7)

Transverse-traceless (TT) projection operator onto the plane orthogonal to $\mathcal{N}$

$$\mathscr{P}_{ijkl}(\boldsymbol{\mathcal{N}}) \equiv \mathscr{P}_{ik}\mathscr{P}_{jl} - \frac{1}{2}\mathscr{P}_{ij}\mathscr{P}_{kl},$$
$$\mathscr{P}_{ij}(\boldsymbol{\mathcal{N}}) \equiv \delta_{ij} - \mathcal{N}_i\mathcal{N}_j.$$

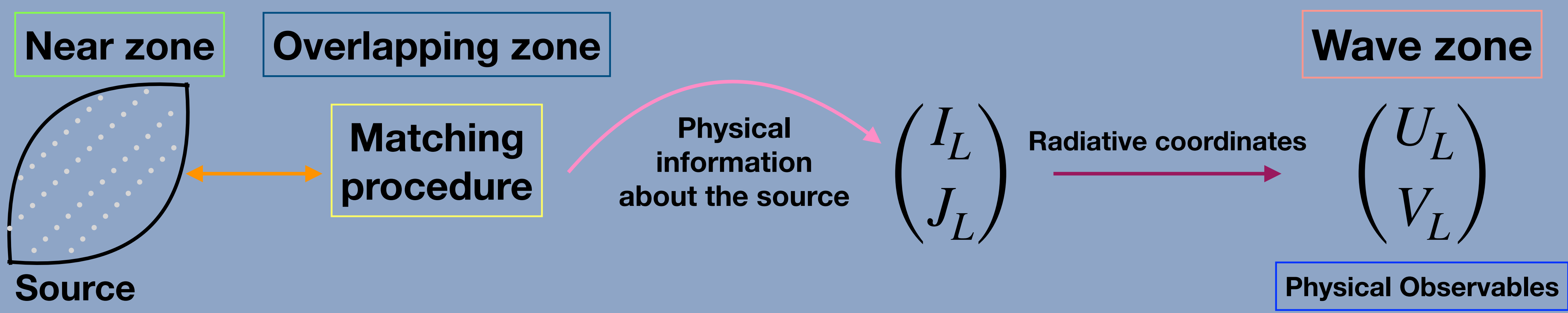
TT gauge

$\mathscr{H}^{\rm TT}_{0\mu} = 0$ $\quad$ $(\mathscr{H}^{\rm TT})^i{}_i = 0$ $\quad$ $\partial^j \mathscr{H}^{\rm TT}_{ij} = 0$

3. In the near zone of the source, the EC field equations are solved through the PN iteration: the inner metric $\mathfrak{h}^{\alpha\beta}_{\rm in}$ is obtained.

4. The matching procedure is exploited in the overlapping region, yielding the explicit expressions of both the source multipole moments I_L, J_L and the radiative moments U_L, V_L in terms of the combined energy-momentum tensor $\Theta^{\alpha\beta}$.

BLANCHET-DAMOUR APPROACH IN EC THEORY (8)



- **The matching procedure allows to "fill" the otherwise "empty" expressions of both the source and the radiative multipole moments with physical information about the source.**

- I_L, J_L **are given as well-defined (compact-support) integral expressions involving the source variables; in particular, they are given as integrals extending over the (compact-support) combined stress-energy tensor** $\Theta^{\alpha\beta}$ **of the material source.**

- **The radiative multipole moments** U_L, V_L **are obtained in the form of some (non-linear) functionals of the source moments** I_L, J_L**.**

GW GENERATION PROBLEM IN EC THEORY (1)

- **External metric at 1PN order in harmonic coordinates $x^\mu = (ct, \boldsymbol{x})$:**

$$\mathfrak{h}^{\alpha\beta}_{\text{ext}} = \mathfrak{h}^{\alpha\beta}_{\text{ext}}(I_L, J_L)$$

$$\mathfrak{h}^{\alpha\beta}_{\text{ext}} = \sqrt{-g}\, g^{\alpha\beta}_{\text{ext}} - \eta^{\alpha\beta}$$

$$g^{\text{ext}}_{00} = -\mathrm{e}^{-2V^{\text{ext}}/c^2} + \mathrm{O}(c^{-6}),$$
$$g^{\text{ext}}_{0i} = -\frac{4}{c^3} V^{\text{ext}}_i + \mathrm{O}(c^{-5}),$$
$$g^{\text{ext}}_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2} V^{\text{ext}}\right) + \mathrm{O}(c^{-4}),$$

The qualifier nPN refers to a correction of the order c^{-2n}

External potentials

$$V^{\text{ext}} = G \sum_{l=0}^{+\infty} \frac{(-1)^l}{l!} \partial_L \left(\frac{I_L(u)}{r}\right),$$
$$V^{\text{ext}}_i = -G \sum_{l=1}^{+\infty} \frac{(-1)^l}{l!} \left\{ \partial_{L-1} \left(\frac{\dot{I}_{iL-1}(u)}{r}\right) + \frac{l}{l+1} \epsilon_{iab} \partial_{aL-1} \left(\frac{J_{bL-1}(u)}{r}\right) \right\}.$$

$$r = |\boldsymbol{x}| = (x^i x_i)^{1/2}$$

$$u \equiv t - r/c$$

GW GENERATION PROBLEM IN EC THEORY (2)

- **Upon introducing in the wave zone the radiative coordinates $X^\mu = (cT, X)$, the analysis of the asymptotic expansion of the external metric yields**

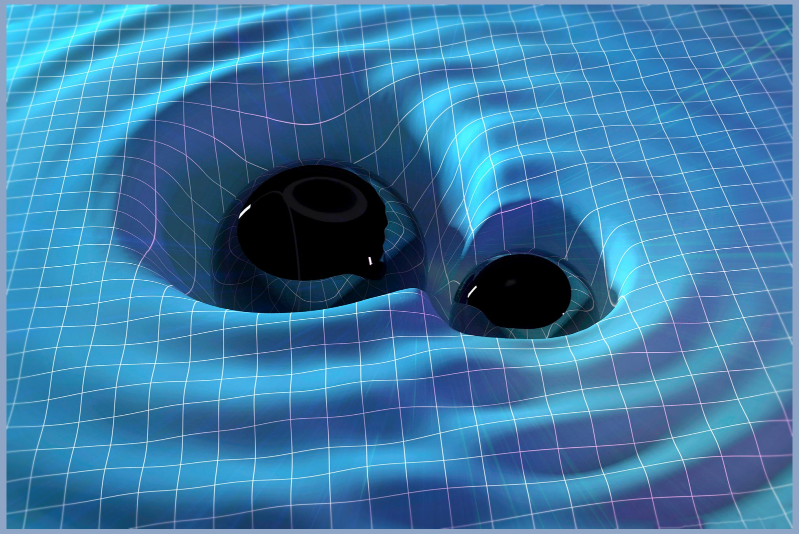
$(u = t - r/c)$

$(l \geq 2)$

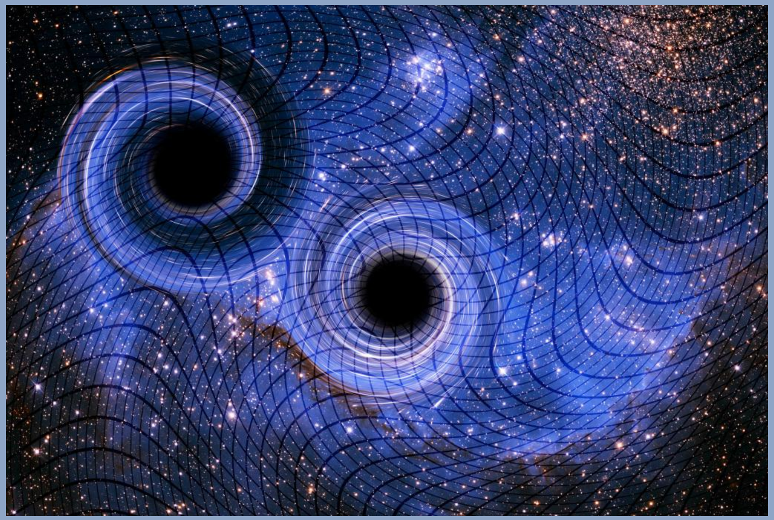
$$U_L(u) = \overset{(l)}{I}_L(u) + \mathrm{O}(c^{-3}),$$
$$V_L(u) = \overset{(l)}{J}_L(u) + \mathrm{O}(c^{-2}).$$

Relation between harmonic coordinates $x^\mu = (ct, \boldsymbol{x})$
and radiative coordinates $X^\mu = (cT, \boldsymbol{X})$:

$$\mathcal{U} = u + \mathrm{O}(c^{-3})$$



$$\mathcal{U} = T - |\boldsymbol{X}|/c$$



GW GENERATION PROBLEM IN EC THEORY (3)

- **Inner metric** at 1PN order in harmonic coordinates $x^\mu = (ct, \boldsymbol{x})$:

$$g_{00}^{\text{in}} = -\mathrm{e}^{-2V^{\text{in}}/c^2} + \mathrm{O}(c^{-6}),$$
$$g_{0i}^{\text{in}} = -\frac{4}{c^3} V_i^{\text{in}} + \mathrm{O}(c^{-5}),$$
$$g_{ij}^{\text{in}} = \delta_{ij}\left(1 + \frac{2}{c^2} V^{\text{in}}\right) + \mathrm{O}(c^{-4}),$$

Inner potentials

$$V^{\text{in}}(t, \boldsymbol{x}) = G \int \frac{\mathrm{d}^3 y}{|\boldsymbol{x} - \boldsymbol{y}|} \sigma\left(t - |\boldsymbol{x} - \boldsymbol{y}|/c, \boldsymbol{y}\right),$$
$$V_i^{\text{in}}(t, \boldsymbol{x}) = G \int \frac{\mathrm{d}^3 y}{|\boldsymbol{x} - \boldsymbol{y}|} \sigma_i\left(t - |\boldsymbol{x} - \boldsymbol{y}|/c, \boldsymbol{y}\right),$$

$$\sigma \equiv \frac{\Theta^{00} + \Theta^{kk}}{c^2} = \frac{T^{00} + T^{kk}}{c^2} + (8\pi G)\frac{\mathcal{S}^{00} + \mathcal{S}^{kk}}{c^6}$$

$$\sigma_i \equiv \frac{\Theta^{0i}}{c} = \frac{T^{0i}}{c} + (8\pi G)\frac{\mathcal{S}^{0i}}{c^5}$$

GW GENERATION PROBLEM IN EC THEORY (4)

- **Matching procedure in the overlapping domain: the internal field and the external metric should be isometric.**

$(u = t - r/c)$

$y_{\langle L\rangle} = \check{y}_L$ **STF projection of** y_L

$$I_L(u) = \int \mathrm{d}^3\boldsymbol{y}\, y_{\langle L\rangle}\sigma(\boldsymbol{y},u) + \frac{1}{2(2l+3)}\frac{1}{c^2}\frac{\mathrm{d}^2}{\mathrm{d}u^2}\int \mathrm{d}^3\boldsymbol{y}\, y_{\langle L\rangle}\,\boldsymbol{y}^2\sigma(\boldsymbol{y},u) - \frac{4(2l+1)}{(l+1)(2l+3)}\frac{1}{c^2}\frac{\mathrm{d}}{\mathrm{d}u}\int \mathrm{d}^3\boldsymbol{y}\, y_{\langle iL\rangle}\sigma_i(\boldsymbol{y},u) + \mathrm{O}(c^{-4}), \qquad (l \geqslant 0),$$

$$J_L(u) = \int \mathrm{d}^3\boldsymbol{y}\, \epsilon_{ab\langle i_l}\check{y}_{L-1\rangle a}\sigma_b(\boldsymbol{y},u) + \mathrm{O}(c^{-2}), \qquad (l \geqslant 1),$$

$$\sigma \equiv \frac{\Theta^{00}+\Theta^{kk}}{c^2} = \frac{T^{00}+T^{kk}}{c^2} + (8\pi G)\frac{\mathcal{S}^{00}+\mathcal{S}^{kk}}{c^6}$$

$$\sigma_i \equiv \frac{\Theta^{0i}}{c} = \frac{T^{0i}}{c} + (8\pi G)\frac{\mathcal{S}^{0i}}{c^5}$$

$$U_L(u) = \overset{(l)}{I}_L(u) + \mathrm{O}(c^{-3}),$$
$$V_L(u) = \overset{(l)}{J}_L(u) + \mathrm{O}(c^{-2}).$$

$(l \geq 2)$

Solution of GW generation problem in EC theory at 1PN level

GW GENERATION PROBLEM IN EC THEORY (5)

- **1PN-accurate asymptotic gravitational radiation amplitude (or waveform)**

$$\mathscr{H}_{ij}^{\mathrm{TT}}(X^\mu) = \frac{2G}{c^4 \mathcal{R}} \mathscr{P}_{ijkl}(\mathcal{N}) \left\{ U_{kl}(\mathcal{U}) + \frac{1}{c}\left[\frac{1}{3}\mathcal{N}_a U_{kla}(\mathcal{U}) + \frac{4}{3}\epsilon_{ab(k} V_{l)a}(\mathcal{U})\mathcal{N}_b\right] + \frac{1}{c^2}\left[\frac{1}{12}\mathcal{N}_{ab} U_{klab}(\mathcal{U}) + \frac{1}{2}\epsilon_{ab(k} V_{l)ac}(\mathcal{U})\mathcal{N}_{bc}\right] + \mathrm{O}(c^{-3}) \right\}.$$

- **Total radiated power (or luminosity or flux) of the source at 1PN order**

$$\mathcal{F}(\mathcal{U}) = \frac{G}{c^5}\left\{ \frac{1}{5} \overset{(1)}{U}_{ij}(\mathcal{U}) \overset{(1)}{U}_{ij}(\mathcal{U}) + \frac{1}{c^2}\left[\frac{1}{189} \overset{(1)}{U}_{ijk}(\mathcal{U}) \overset{(1)}{U}_{ijk}(\mathcal{U}) + \frac{16}{45} \overset{(1)}{V}_{ij}(\mathcal{U}) \overset{(1)}{V}_{ij}(\mathcal{U}) \right] + \mathrm{O}(c^{-4}) \right\}.$$

APPLICATION TO A BINARY NS SYSTEM (1)

- ***N* weakly self-gravitating, slowly moving, widely separated spinning bodies.**

$$U_{ij} = \frac{\mathrm{d}^2}{\mathrm{d}t^2} \sum_{A=1}^{N} m_A \left\{ r_A^{\langle i} r_A^{j\rangle} \left[1 + \frac{1}{c^2} \left(\frac{3}{2} v_A^2 - \sum_{B\neq A} \frac{G m_B}{|\boldsymbol{r}_A - \boldsymbol{r}_B|} \right) \right] + \frac{1}{14c^2} \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left(r_A^2 r_A^{\langle i} r_A^{j\rangle} \right) - \frac{20}{21c^2} \frac{\mathrm{d}}{\mathrm{d}t} \left(v_A^k r_A^{\langle i} r_A^j r_A^{k\rangle} \right) \right\}$$

$$+ \frac{\mathrm{d}^2}{\mathrm{d}t^2} \sum_{A=1}^{N} \left\{ \frac{4}{c^2} \left[(\boldsymbol{v}_A \times \boldsymbol{s}_A)^i r_A^j + (\boldsymbol{v}_A \times \boldsymbol{s}_A)^j r_A^i - \frac{2}{3} \delta^{ij} (\boldsymbol{v}_A \times \boldsymbol{s}_A) \cdot \boldsymbol{r}_A \right] - \frac{4}{3c^2} \frac{\mathrm{d}}{\mathrm{d}t} \left[(\boldsymbol{r}_A \times \boldsymbol{s}_A)^i r_A^j + (\boldsymbol{r}_A \times \boldsymbol{s}_A)^j r_A^i \right] \right\} + \mathrm{O}\left(\mathrm{c}^{-3}\right),$$

mass quadrupole moment

$$U_{ijk} = \frac{\mathrm{d}^3}{\mathrm{d}t^3} \sum_A m_A r_A^{\langle i} r_A^j r_A^{k\rangle} + \mathrm{O}\left(\mathrm{c}^{-2}\right),$$

mass octupole moment

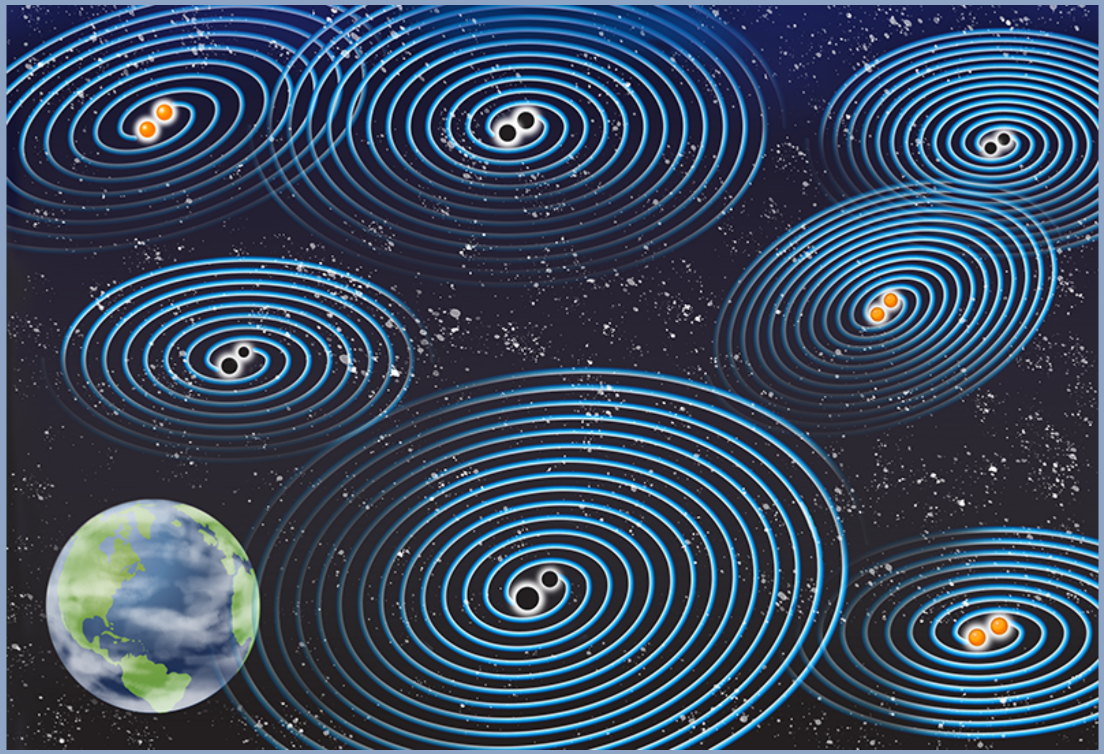
$$V_{ij} = \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left\{ \sum_A m_A \epsilon^{kl\langle i} r_A^{j\rangle} r_A^k v_A^l + \frac{1}{2} \sum_A \left[3 \left(s_A^i r_A^j + s_A^j r_A^i \right) - 2\delta^{ij} \boldsymbol{s}_A \cdot \boldsymbol{r}_A \right] \right\} + \mathrm{O}\left(\mathrm{c}^{-2}\right),$$

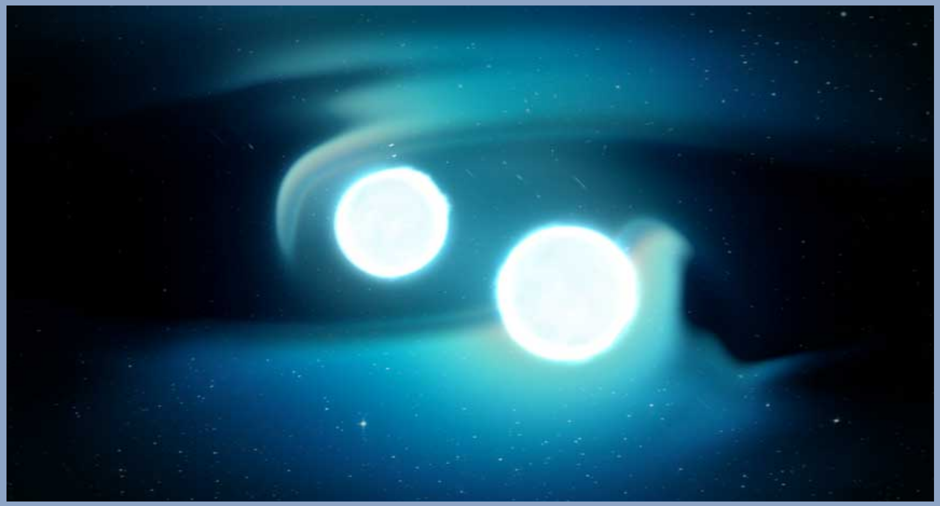
current quadrupole moment

APPLICATION TO A BINARY NS SYSTEM (2)

$$U_{ijkl} = \frac{\mathrm{d}^4}{\mathrm{d}t^4} \sum_{A=1}^{N} m_A r_A^{\langle i} r_A^j r_A^k r_A^{l\rangle} + \mathrm{O}\left(\mathrm{c}^{-2}\right),$$

mass 2^4-pole moment





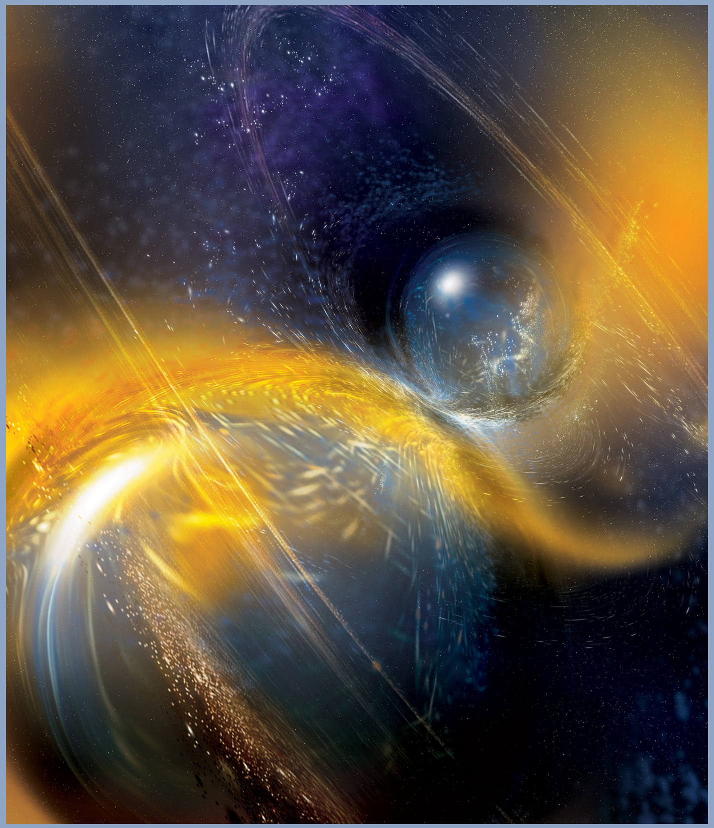
$$V_{ijk} = \frac{\mathrm{d}^3}{\mathrm{d}t^3} \sum_{A=1}^{N} \left[m_A r_A^{\langle i} r_A^j \epsilon^{k\rangle lp} r_A^l v_A^p + 2\left(r_A^n s_A^q\, \delta_n^{\langle i} r_A^j \delta_q^{k\rangle} - \boldsymbol{r}_A \cdot \boldsymbol{s}_A\, \delta_n^{\langle i} r_A^j \delta_n^{k\rangle} + s_A^q\, r_A^{\langle i} r_A^j \delta_q^{k\rangle} \right) \right] + \mathrm{O}\left(\mathrm{c}^{-2}\right).$$

current octupole moment

APPLICATION TO A BINARY NS SYSTEM (3)

- **Let us consider a binary NS system**

$$
\begin{aligned}
m_1 &= 1.60 M_\odot \\
m_2 &= 1.17 M_\odot \\
|\boldsymbol{s}_1| &= 1.21 \times 10^{57} \hbar \\
|\boldsymbol{s}_2| &= 4.73 \times 10^{56} \hbar \\
R_{\text{av}} &= 4.69 \times 10^{8}\,\text{m}
\end{aligned}
$$



$$
\mathcal{E}_{\mathcal{F}}(t) \equiv \left| \frac{\mathcal{F}_{\text{EC}}(t)}{\mathcal{F}_{\text{GR}}(t)} \right|,
$$

$$
\mathcal{E}_{\mathscr{H}}(t) \equiv |\mathscr{H}_{11}^{\text{GR}}(t)| - |\mathscr{H}_{11}^{\text{EC}}(t)|.
$$

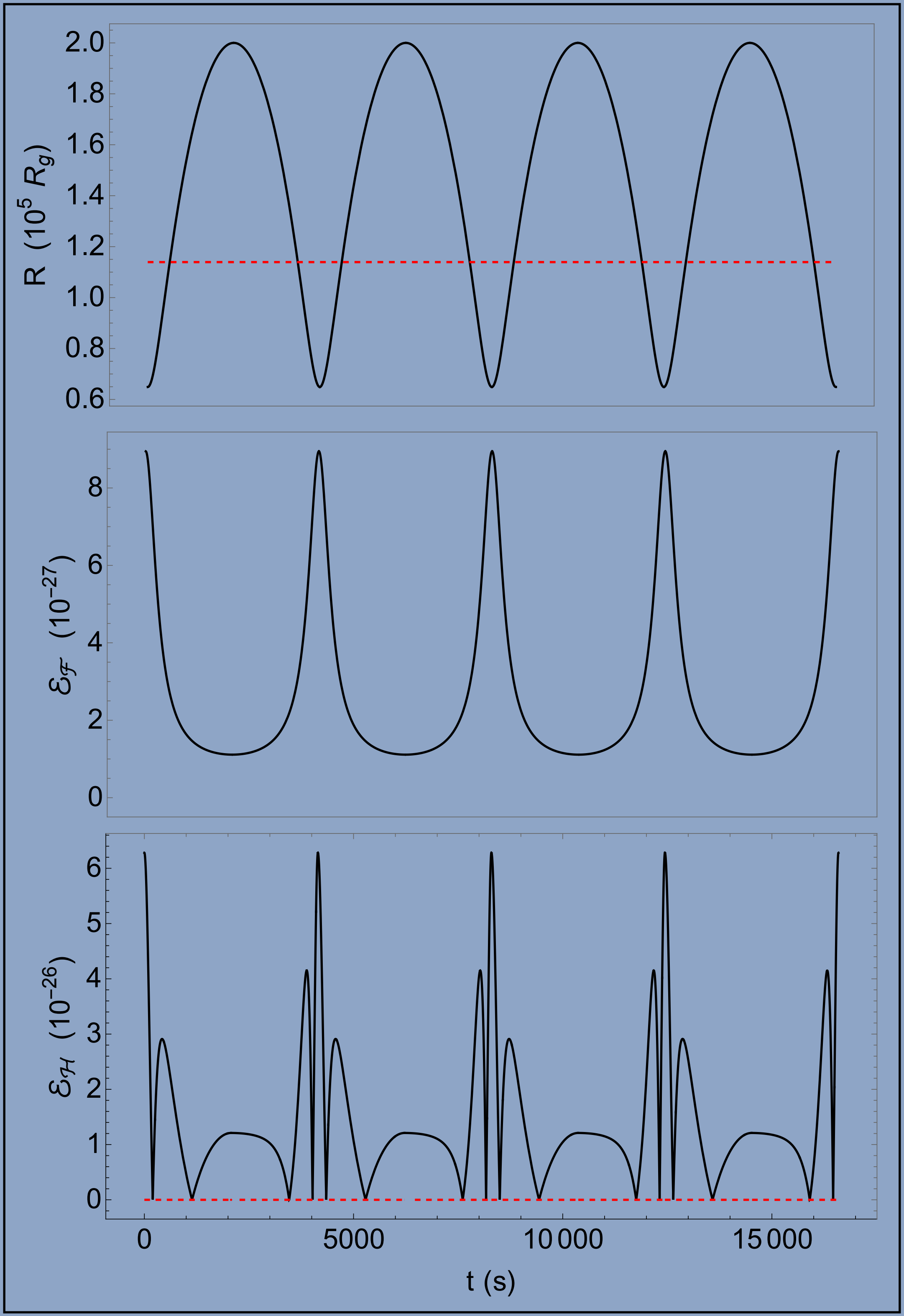
EC contribution to GR flux

EC contribution to GR waveform

APPLICATION TO A BINARY NS SYSTEM (4)

- **Plots**

The average EC contributions are smaller than GR ones by a factor 10^{-23}

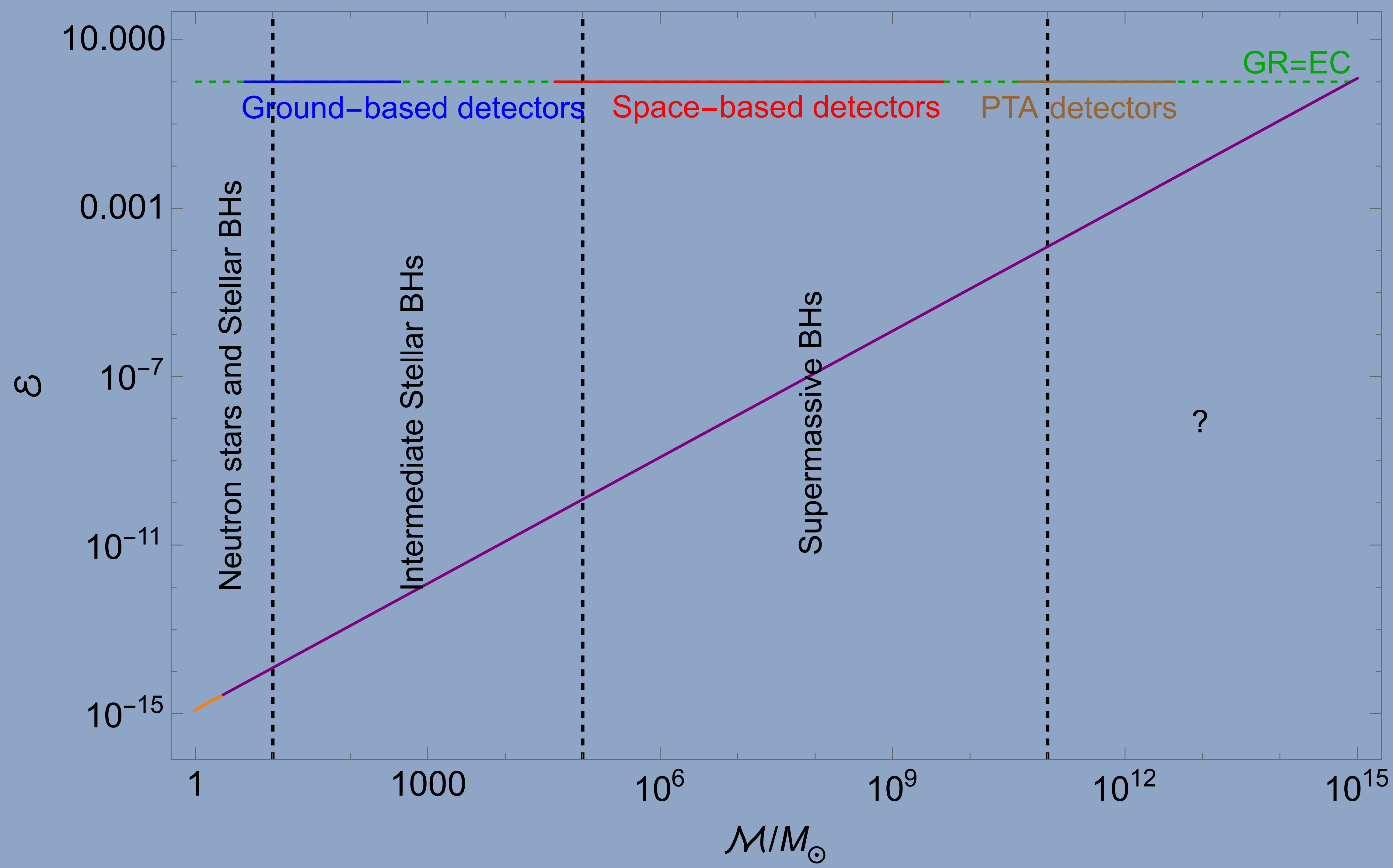


Function $R(t)$

Function $\mathcal{E}_{\mathcal{F}}(t)$

Function $\mathcal{E}_{\mathcal{H}}(t)$

APPLICATION TO BINARY BH SYSTEMS



GR and EC effects are comparable for masses of the order of $10^{14} M_\odot$, which corresponds to a GW frequency of the order of 10^{-12} Hz.

CONCLUSIONS (1)

- The research activity underlying this seminar aims at understanding possible quantum imprints in the propagation of GWs produced by spinning PN sources in EC theory, namely spinning, weakly self-gravitating, slowly moving, and weakly stressed sources.

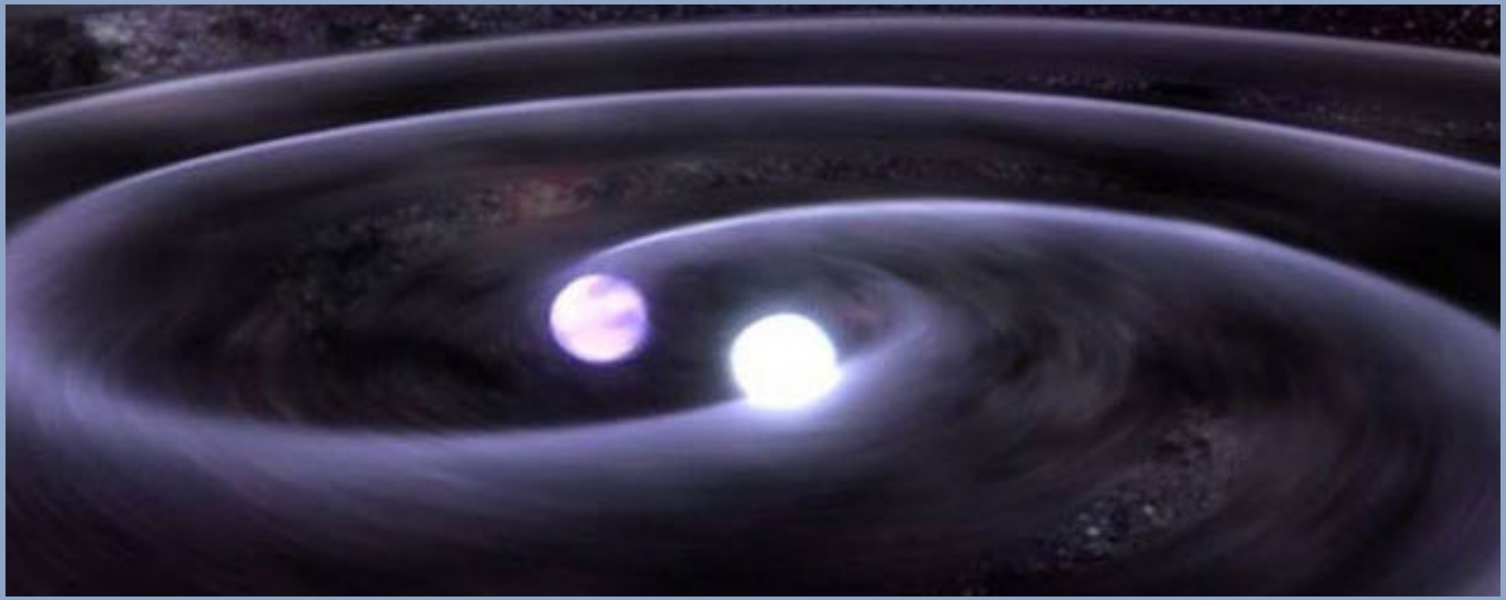
- We have solved the GW generation problem at 1PN level by extending the Blanchet-Damour approach to EC theory.

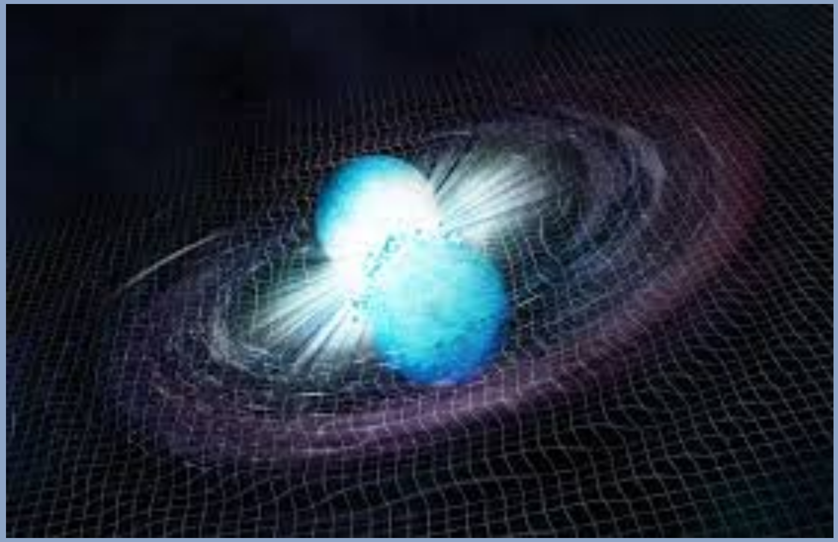
- We have evaluated the 1PN-accurate asymptotic gravitational waveform and the luminosity of the source.

CONCLUSIONS (2)

- **We have provided a concrete application by applying the Blanchet-Damour method to a binary NS system**

- **The case of binary BH systems has also been considered. We have seen that EC corrections imprinted in their gravitational-wave signal can be potentially detected by means of the pulsar timing array technique.**





- **Future work: analysis of the behavior of compact binaries in their later evolution phases (i.e., plunge, merger, ringdown)**

CONCLUSIONS (3)

- **Further details can be found in:**

- "*First post-Newtonian generation of gravitational waves in Einstein-Cartan theory*" (Emmanuele Battista and Vittorio De Falco), Phys. Rev. D 104, 084067 (2021)
- "*Gravitational waves at the first post-Newtonian order with the Weyssenhoff fluid in Einstein-Cartan theory*" (Emmanuele Battista and Vittorio De Falco), Eur. Phys. J. C 82, 628 (2022)
- "*First post-Newtonian N-body problem in Einstein-Cartan theory with the Weyssenhoff fluid: equations of motion*" (Emmanuele Battista and Vittorio De Falco), Eur. Phys. J. C 82, 782 (2022)
- "*First post-Newtonian N-body problem in Einstein-Cartan theory with the Weyssenhoff fluid: Lagrangian and first integrals*" (Emmanuele Battista, Vittorio De Falco, and Davide Usseglio), Eur. Phys. J. C 83, 112 (2023)
- *"Analytical results for binary dynamics at the first post-Newtonian order in Einstein-Cartan theory with the Weyssenhoff fluid"* (Vittorio De Falco and Emmanuele Battista), Phys. Rev. D 108, 064032 (2023)
- *"Radiative losses and radiation-reaction effects at the first post-Newtonian order in Einstein-Cartan theory"* (Vittorio De Falco, Emmanuele Battista, Davide Usseglio, and Salvatore Capozziello), to appear on Eur. Phys. J. C