Quantum Black Hole Physics from the Event Horizon

a life at the edge

Manuel Del Piano

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with S. Hohenegger and F. Sannino

SSM Scuola Superiore Meridionale

INFN

ℏQTC

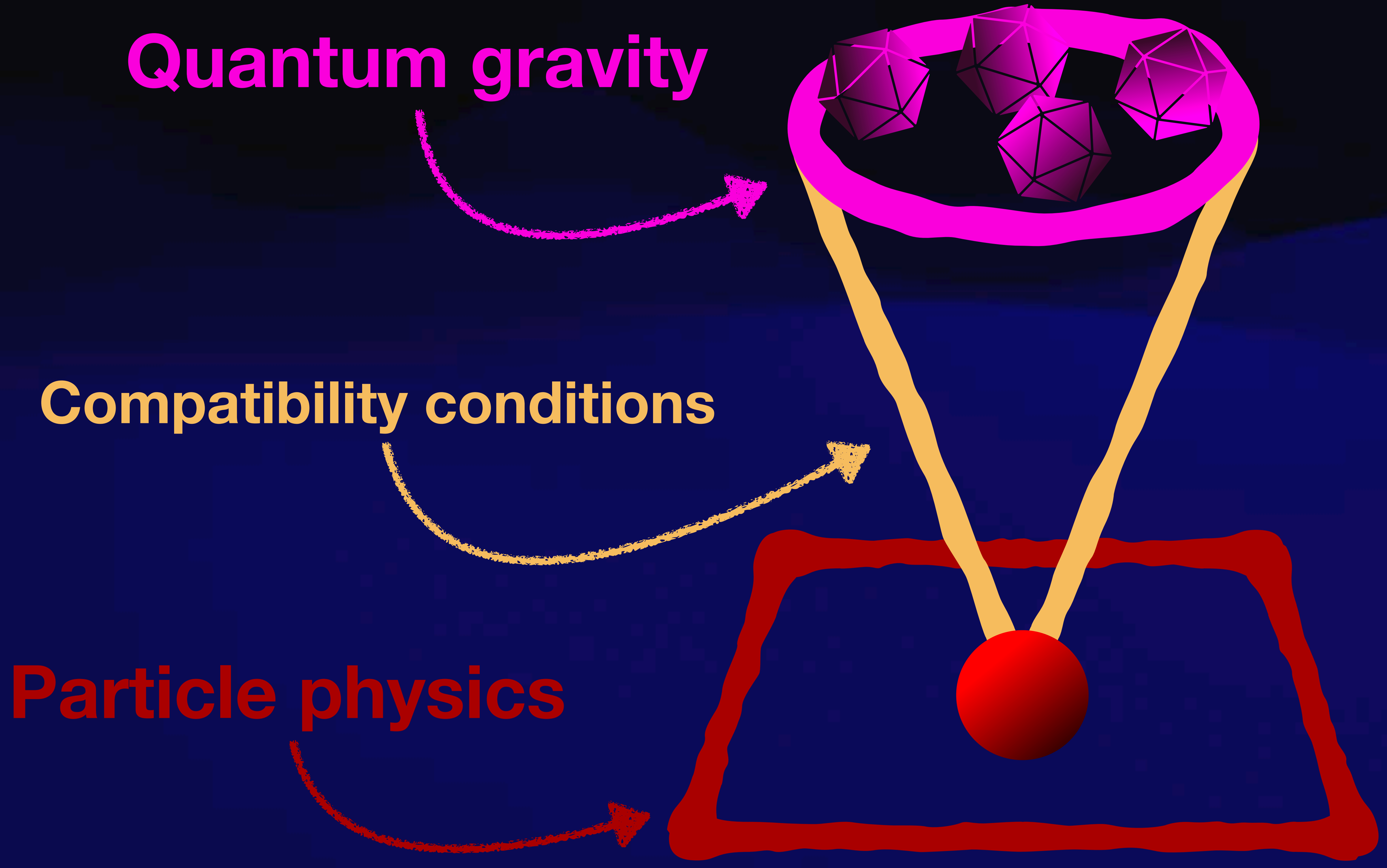
What we will delve into…

1. The problem of quantum gravity and classical BH
2. Set up the effective framework
3. Thermodynamics
4. Examples
5. Asymptotic expansions
6. Conclusions and outlooks

The problem of Quantum Gravity

Quantization of General Relativity is a rather difficult task

Numerous approaches → String theory, LQG, ASG, CDT, EFT…



Swampland program

Seeking for universal underlying structures

Astrophysical observations and advances

Classical Black Hole recap

Static and Spherically Symmetric (SSS) spacetime

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2$$

Schwarzschild: vacuum solution to **Einstein equations**

$$h(r) = f(r) = f_{\mathrm{S}}(r) = 1 - \frac{2G_{\mathrm{N}}M}{r}$$

Dimensionless quantities:

reference mass-scale M_{P}

$$\begin{cases} z := M_{\mathrm{P}}r \\ \chi := M/M_{\mathrm{P}} \end{cases}$$

$$1 - \frac{2\chi}{z}G_{\mathrm{N}}M_{\mathrm{P}}^2 = 1 - \frac{2\chi}{z}$$

SSS Event horizon = Killing horizon

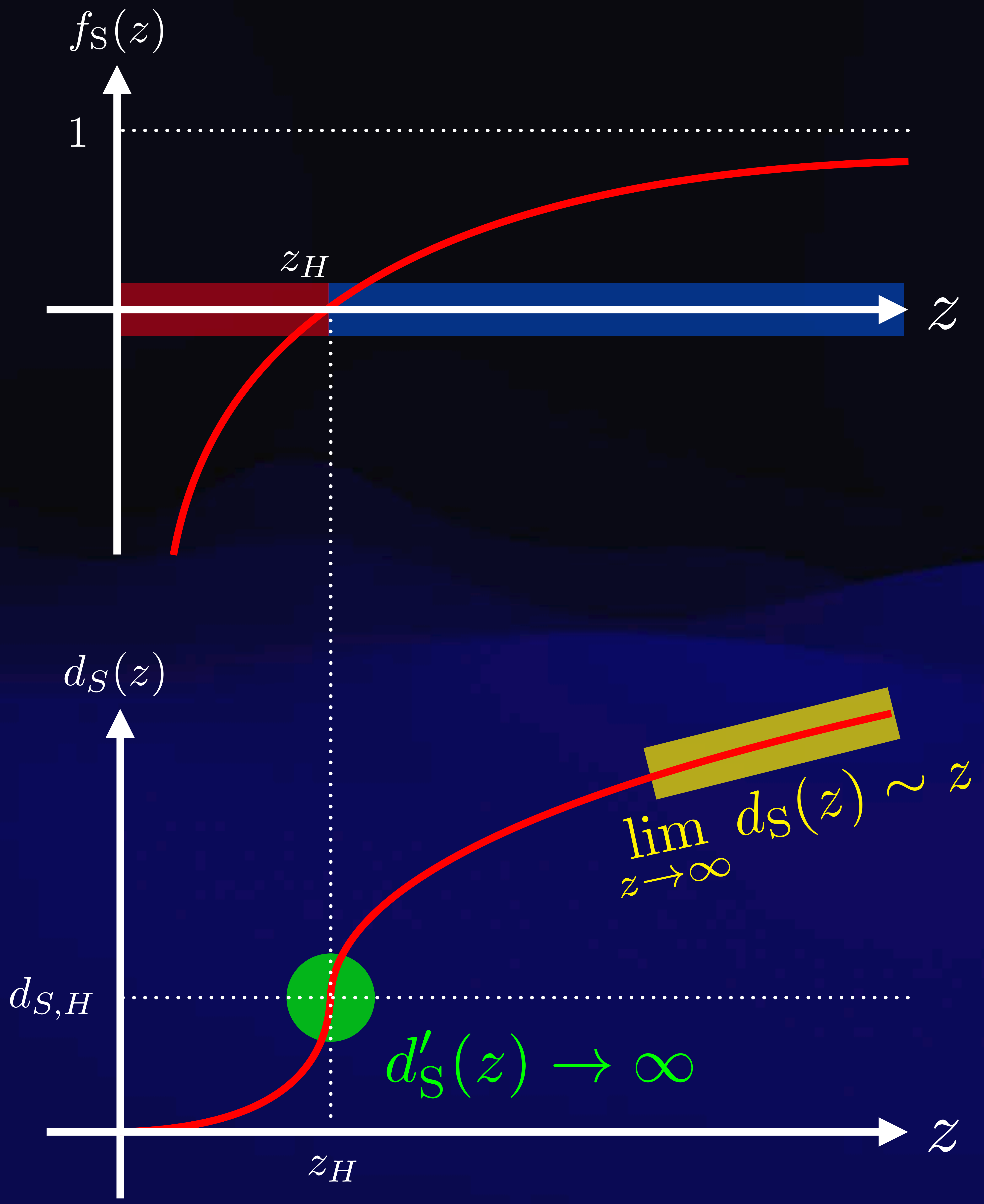
$$(K^t)^\mu = \delta^{0\mu} \implies (K^t)^\mu (K^t)_\mu|_{z=z_H} = 0$$

$$f_S(z_H) = 1 - \frac{2\chi}{z_H} = 0 \implies z_H = 2\chi$$

We want an invariant (*physical*) quantity

The radial proper distance

$$d_S(z) = M_P \int_0^{z/M_P} \mathrm{d}s = \int_0^z \frac{\mathrm{d}z'}{\sqrt{|f_S(z')|}}$$



Setting up the framework

1. Upgrade M_{P} to a physical scale that governs the transition quantum-classical

2. Dependence on the new scale in all new quantities

$$f_{\mathrm{S}}(z) \longrightarrow f(z, u, M_{\mathrm{P}}) = 1 - \frac{2\chi}{z} v^{(u, M_{\mathrm{P}})}(z)$$

spurious scale to compensate the physical scale

Physical quantities must be independent on u

f must preserve the same coordinate transformations of f_{S}

Deformation function can only depend on physical quantities: we chose proper distance

$$v^{(\mathcal{X})}(\mathcal{X}) = e^{\Phi(d(z))} \longrightarrow \Phi\left(\frac{1}{d(z)}\right)$$

$$\mathcal{X} \equiv d(z) := \int_0^z \frac{\mathrm{d}z'}{\sqrt{|f(z')|}}$$

Setting up the framework

3. Same procedure for the temporal component $$h(z) = 1 - \frac{2\chi}{z} e^{\Psi\left(\frac{1}{d(z)}\right)}$$

4. General *SSS* of a quantum deformed metric

$$M_{\mathrm{P}}^2 \mathrm{d}s^2 = -h(z) M_{\mathrm{P}}^2 \mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + z^2 \mathrm{d}\theta^2 + z^2 \sin^2\theta \, \mathrm{d}\varphi^2$$

quantum corrections are embedded into 2 independent functions of the physical distance

$$\Phi\left(\frac{1}{d(z)}\right) \quad \text{and} \quad \Psi\left(\frac{1}{d(z)}\right)$$

5. Asymptotic flatness recovered

$$\Phi(0) = 0 = \Psi(0)$$

Two issues

a. Implicit definition of the metric functions

$$f(z) = 1 - \frac{2\chi}{z} e^{\Phi(1/d(z))} \quad \text{with} \quad d(z) = \int_0^z \frac{\mathrm{d}z'}{\sqrt{|f(z')|}}$$

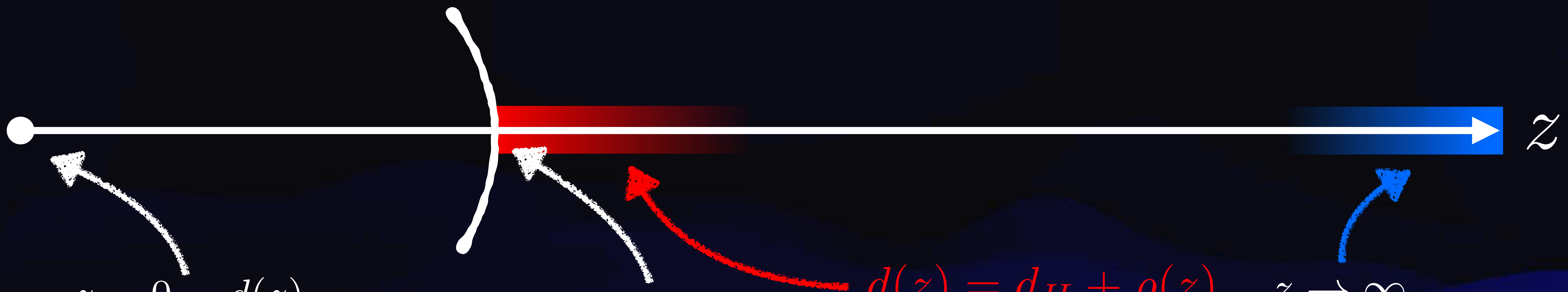
previously it was addressed with an *approximation* $d(z) \sim d_{\mathrm{S}}(z)$

b. Problems with the derivatives at the horizon

$$\left.\frac{\mathrm{d}f(z)}{\mathrm{d}z}\right|_{z=z_H} \supset \left.\frac{\mathrm{d}d(z)}{\mathrm{d}z}\right|_{z=z_H} = \frac{1}{\sqrt{|f(z_H)|}} \to \infty$$

Need to build a self-consistent approach and regularity conditions

Self-consistent approach



$z = 0 = d(z)$

Center of the BH

possible singularity

$z = z_H$

$d(z_H) = d_H$

Event horizon

$d(z) = d_H + \rho(z)$

Near horizon region

$z \to \infty$

$d(z) \to \infty$

Asymptotic region

flat space

Series expansion close to the **event horizon**

$$z(\rho) = z_H + \sum_{n=1}^{\infty} a_n \rho^n \qquad 2\chi e^{\Phi\left(\frac{1}{d_H+\rho}\right)} = \sum_{n=0}^{\infty} \xi_n \rho^n \quad \text{and} \quad 2\chi e^{\Psi\left(\frac{1}{d_H+\rho}\right)} = \sum_{n=0}^{\infty} \theta_n \rho^n$$

What is the minimal set of input parameters?

Self-consistent approach

Consistency of the distance framework

$$d(z) = \int_0^z \frac{\mathrm{d}z'}{\sqrt{|f(z')|}} \rightarrow \frac{\mathrm{d}\rho}{\mathrm{d}z} = \frac{1}{\sqrt{f(z)}}$$

$$\xi_0 = z_H(1 - a_1^2) \equiv z_H$$

$$\xi_1 = a_1(1 - a_1^2 - 4z_H a_2)$$

$$\xi_2 = a_2(1 - a_1^2) - [z_H(6a_1 a_3 + 4a_2^2) + 4a_1^2 a_2]$$

$$a_1 = 0$$

$$\xi_2 = a_2(1 - 4z_H a_2)$$

$$a_2 = \frac{1 + \sqrt{1 - 16 z_H \xi_2}}{2}$$

$$\xi_1 = 0 \quad \xi_2 \leq \frac{1}{16 z_H}$$

Iterative relation for $p \geq 3$

$$a_p = \frac{1}{1 - 4p\, z_H a_2}\left[\xi_p + z_H \sum_{n=3}^{p-1}(p-n+2) n a_n a_{p-n+2} + \sum_{n=2}^{p-2}\sum_{m=2}^{n}(n-m+2) m a_{p-n} a_m a_{n-m+2}\right]$$

Regularity conditions

Investigate the behavior of invariant quantities at the horizon

Ricci scalar $$R = \frac{fh^{(2)}}{h} + \frac{f(h^{(1)})^2}{2h^2} - \frac{(zf^{(1)} + 4f)h^{(1)}}{2zh} - \frac{2(zf^{(1)} + f - 1)}{z^2}$$

Consistency of power series expansion in ρ of h requires constraints

$$\theta_1 = 0 \quad \theta_2 \leq \frac{1 + \sqrt{1 - 16 z_H \xi_2}}{8 z_H}$$

Not enough to avoid singular terms in the Ricci scalar

$$R = \frac{(1 + 3\sqrt{1 - 16 z_H \xi_2})\theta_3 + 2\xi_3}{(1 + 3\sqrt{1 - 16 z_H \xi_2})(1 - 8 z_H \theta_2 + \sqrt{1 - 16 z_H \xi_2})} \rho^{-1} + \mathcal{O}(\rho^0)$$

$$\xi_3 = -\frac{1}{2}\left(1 + 3\sqrt{1 - 16 z_H \xi_2}\right)\theta_3$$

Summing up the constraints

Self consistency

$$\xi_1 = 0 \quad \xi_2 \leq \frac{1}{16 z_H}$$

Divergence-free Ricci scalar at the event horizon

$$\theta_1 = 0 \quad \theta_2 \leq \frac{1 + \sqrt{1 - 16 z_H \xi_2}}{8 z_H}$$

$$\xi_3 = -\frac{1}{2}\left(1 + 3\sqrt{1 - 16 z_H \xi_2}\right)\theta_3$$

Hawking temperature

We are now able to compute physical quantities defined at the event horizon

Time-like Killing vector is needed to define the surface gravity

$$\kappa^2 = -\frac{1}{2M_{\rm P}^2}\nabla_\mu (K^t)_\nu \nabla^\mu (K^t)^\nu \big|_{z=z_H}$$ dimensionless

Which is related to the Hawking temperature

$$T_{\rm H} := \frac{\kappa}{2\pi} = \frac{1}{4\pi}\sqrt{f_H^{(1)} h_H^{(1)}}$$

Achieved in a universal form expressing the power series expansions

$$T_{\rm H} = \frac{\sqrt{1 - 8z_H\theta_2 + \sqrt{1 - 16z_H\xi_2}}}{4\sqrt{2}\pi z_H}$$

Entropy

Coarse grained definition of entropy from the first law of thermodynamics

$$S = \int \frac{\mathrm{d}\chi}{T_{\mathrm{H}}(\chi)}$$

The integral is to be performed once the dependence of the parameters on the mass is given

$$z_H(\chi) \qquad d_H(\chi) \qquad \xi_2(\chi) \qquad \theta_2(\chi)$$

An example

Bonanno-Reuter Black Hole

framework: Asymptotically Safe Gravity

Idea: promote the Newton coupling to a running scale-dependent coupling

$$G_{\rm N} \to G(k) = \frac{G(k=0)}{1+\omega G(k=0)k^2}$$

scale identification $k(z) = \dfrac{\xi}{d_{\rm BR}(z)}$

The corresponding metric functions $f(z) = h(z) \equiv f_{BR}(z) = 1 - \dfrac{2\chi}{z}\dfrac{\cancel{e^{\Phi}}\;1}{1+\frac{\omega\xi^2}{d_{\rm BR}(z)^2}}$

Self-consistency would require $d_{\rm BR}(z) = \displaystyle\int_0^z \frac{{\rm d}z'}{\sqrt{|f_{\rm BR}(z')|}}$

approximation

$$d_{\rm BR}(z) = \left(\frac{z^3}{z+\frac{9}{2}\chi}\right)^{1/2}$$

regular at the origin, not on the horizon

$$\xi_1 = \theta_1 = \frac{4\chi\omega\xi^2 d_{{\rm BR},H}}{(\omega\xi^2 + d_{{\rm BR},H}^2)^2} \neq 0$$

✗

Two more examples

Hayward Black Hole

framework: effective regular BH

$$f(z) = h(z) \equiv f_{\text{Hay}}(z) = 1 - \frac{2\chi}{z}\frac{z^3}{z^3 + 2\gamma\chi}$$

$$\xi_1 = \theta_1 = \xi_3 = \theta_3 = 0 \qquad \xi_2 = \theta_2 = \frac{6\gamma\chi^3 z_H^3 (z_H^3 - 4\gamma\chi)}{(2\gamma\chi + z_H^3)^4}$$

Dymnikova Black Hole

framework: regular BH with effective EMT

$$f(z) = h(z) \equiv f_{\text{Dymn}}(z) = 1 - \frac{2\chi}{z}\left(1 - e^{-z^3/2\chi z_0^2}\right)$$

$$\xi_1 = \theta_1 = \xi_3 = \theta_3 = 0 \qquad \xi_2 = \theta_2 = \frac{3e^{-z_H^3/\chi z_0^2}}{4z_0^4}\left(2\chi z_0^2\left(e^{z_H^3/2\chi z_0^2} - 1\right) - 3z_H^3\right)$$

A minimal model

We provide a model abiding by the constraints

$$\Phi\left(\frac{1}{d_H+\rho}\right) \equiv \Psi\left(\frac{1}{d_H+\rho}\right) = -\frac{3\phi_2}{2d_H^2} + \frac{\rho^2(d_H+3\rho)\phi_2}{2d_H^2(d_H+\rho)^3}$$

such that $\Phi_H^{(1)} = 0\ , \quad \Phi_H^{(2)} = \phi_2 \quad \text{and} \quad \Phi_H^{(n)} = 0 \quad \text{for} \quad n \geq 3$

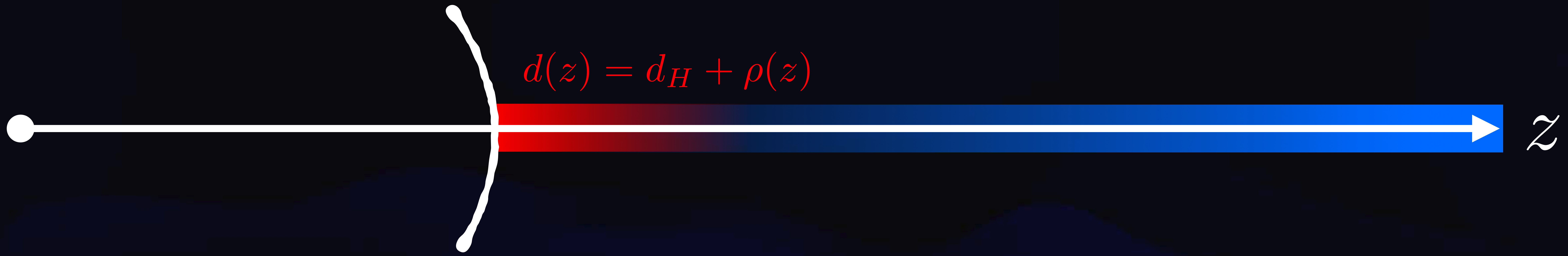
The event horizon and the Hawking temperature are easily computed

$$z_H = 2\chi e^{-3\phi_2/2d_H^2} \quad \text{and} \quad T_{\mathrm{H}} = \frac{1}{8\pi z_H}\left[1+\left(1-\frac{8z_H^2\phi_2}{d_H^4}\right)^{1/2}\right]$$

The entropy can be computed by extracting the leading contributions in the large mass limit

$$S = 4\pi\chi^2\left[1-\frac{3\pi^2-16}{3\pi^2\chi^2}\log(\chi^2)+\mathcal{O}(\chi^{-4})\right]$$

Asymptotic expansions



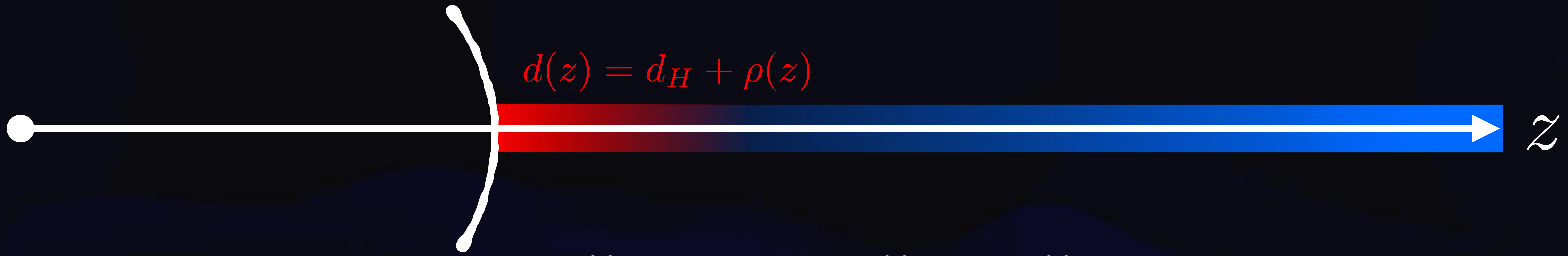
$$f(z) = 1 - \frac{2\chi}{z}\left(1 + \sum_{n=1}^{\infty} \frac{\omega_n}{d(z)^n}\right) \qquad h(z) = 1 - \frac{2\chi}{z}\left(1 + \sum_{n=1}^{\infty} \frac{\gamma_n}{d(z)^n}\right)$$

Convergence criteria $\limsup_{n\to\infty} |\omega_n|^{1/n} \leq d_H$ and $\limsup_{n\to\infty} |\gamma_n|^{1/n} \leq d_H$

Rescale the coefficients $\omega_n = \overline{\omega}_n d_H^n$ and $\gamma_n = \overline{\gamma}_n d_H^n$

Horizon constraint $\sum_{n=1}^{\infty} \overline{\omega}_n = \sum_{n=1}^{\infty} \overline{\gamma}_n = \frac{z_H}{2\chi} - 1$

Asymptotic expansions



Regularity of derivatives

$$\frac{1}{2\chi}\sum_{p=0}^{\infty}\xi_p\rho^p = 1 + \sum_{n=1}^{\infty}\overline{\omega}_n\Big(\sum_{k=0}^{\infty}\Big(-\frac{\rho}{d_H}\Big)^k\Big)^n$$

$$\xi_0 = 2\chi\Big(1+\sum_{n=1}^{\infty}\overline{\omega}_n\Big) \equiv z_H \qquad \xi_p = \frac{2\chi}{p!(-d_H)^p}\sum_{n=1}^{\infty}\overline{\omega}_n\frac{(n+p-1)!}{(n-1)!}$$

$$\xi_1 = 0 = \xi_3 \to \sum_{n=1}^{\infty} n\overline{\omega}_n = 0 = \sum_{n=1}^{\infty} n^2(n+3)\overline{\omega}_n \qquad \theta_1 = 0 = \theta_3 \to \sum_{n=1}^{\infty} n\overline{\gamma}_n = 0 = \sum_{n=1}^{\infty} n^2(n+3)\overline{\gamma}_n$$

$$\xi_2 \to \sum_{n=1}^{\infty} n^2\overline{\omega}_n \le \frac{d_H^2}{16 z_H\chi} \qquad \theta_2 \to \sum_{n=1}^{\infty} n^2\overline{\gamma}_n \le \frac{d_H^2}{8 z_H\chi}\left(1+\sqrt{1-8z_H\xi_2}\right)$$

Asymptotic expansions

Hawking temperature in the large mass limit $\lim_{\chi\to\infty}\sum_{n=1}^{\infty} n^r\overline{\omega}_n = 0 = \lim_{\chi\to\infty}\sum_{n=1}^{\infty} n^r\overline{\gamma}_n \quad \forall r \in \mathbb{N}$

$$T_{\mathrm{H}} = \frac{1}{8\pi\chi}\left[1 - \frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(4n^2(\overline{\omega}_n + \overline{\gamma}_n) + \pi^2\overline{\omega}_n\right) + \ldots\right]$$

Two types of leading corrections to the entropy $d_H \simeq \pi\chi + \mathfrak{o}(\chi)$

1. $(\omega_1, \gamma_1) \neq (0,0)$ $\quad S = 4\pi\chi^2\left(1 + \frac{8\gamma_1 + 2(4+\pi^2)\omega_1}{\pi^3\chi} + \frac{\alpha}{\chi^2}\log(\chi)\ldots\right)$

2. $(\omega_1, \gamma_1) = (0,0)$, $(\omega_2, \gamma_2) \neq (0,0)$ $\quad S = 4\pi\chi^2\left(1 + \frac{(16\gamma_2 + (16+\pi^2)\omega_2)^2}{\pi^4\chi^2}\log\left(\pi^4\chi^2 - 16\gamma_2 - (16+\pi^2)\omega_2\right) + \ldots\right)$

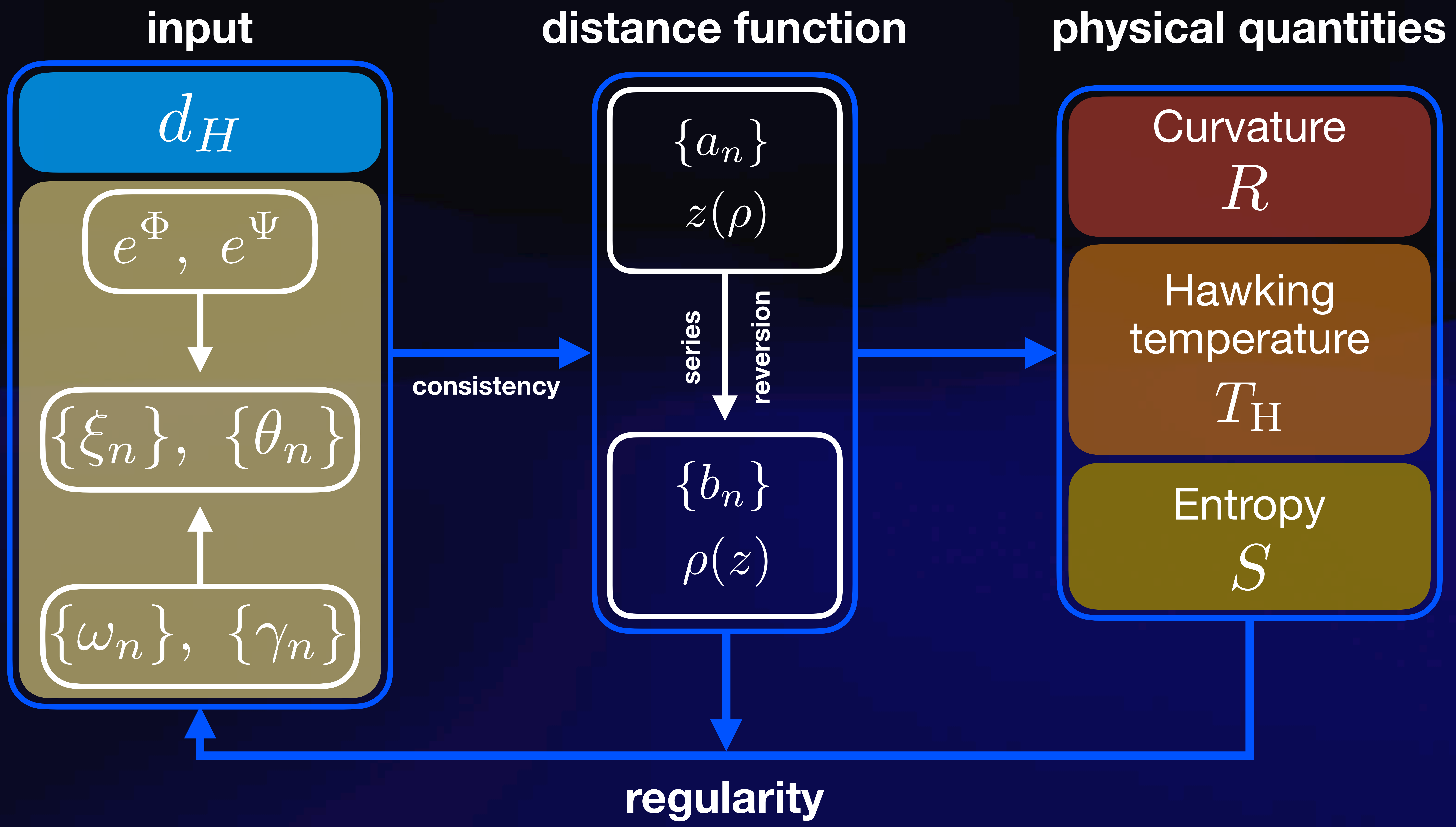
Further results

i. We can demand that the metric must be infinitely times differentiable

$$f_H^{(n)},\ h_H^{(n)} \ \rightarrow\ \theta_{2p+1} = 0 = \xi_{2p+1} \quad \text{for} \quad p \in \mathbb{N}_0$$

ii. Regularity constraints can be applied also at internal horizons

Conclusions



Conclusions

1. We introduced a **model-independent framework** to describe effective quantum black holes deformations with physical quantities

2. Although universal, we obtained **non-trivial conditions** on the admissible deformations

3. Determine **self-consistently** physical quantities that are defined at the event horizon, *i.e.* the universal form of the Hawking temperature

Outlooks

1. Extend the definition of *scheme* (*EMD*) to local invariants, *i.e.* Ricci and Kretschmann scalars

2. Generalize the deformation to other black hole solutions, *i.e.* Reissner–Nordström, Kerr, regular BHs, other dimensions, *AdS*, holography, *etc*…

3. Phenomenology, *i.e.* QNM and GWs, shadow, precession, astroparticles, *etc…*

Mange tak! 🇩🇰
Thank you! 🇬🇧
Grazie! 🇮🇹

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