THE THEORY OF LARKS

ħQTC Journal Club

Sofie Martins &
Mattia Damia Paciarini

WHAT WE WILL BE TALKING ABOUT

1. Large-N SU(N) and String theory
   1. The ‘t Hooft-limit
   2. The Veneziano-Limit
   3. Lark theory
2. Two theory predictions

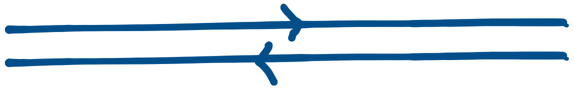
LARGE-N AND STRING THEORY

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

U(N) GAUGE THEORY

QUARKS

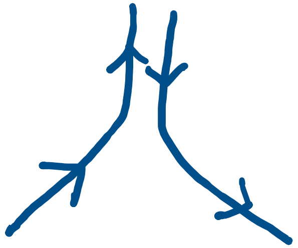
GAUGE FIELD



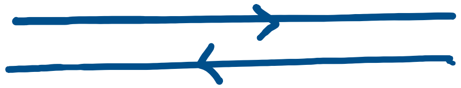
+ GHOSTS

U(N) GAUGE THEORY

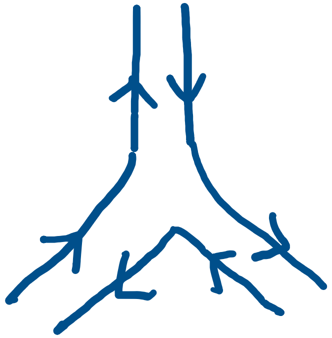
G. 't Hooft, Nucl. Phys. B 72 461 (1974)



$-g\gamma_\mu$

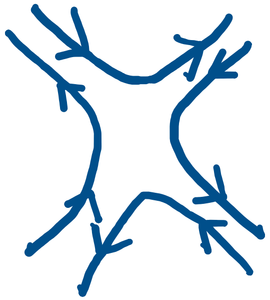


$-\dfrac{\delta_{\mu\nu}}{k^2-i\varepsilon}$



$ig\{\delta_{\alpha\nu}(k-q)_\mu + \delta_{\alpha\mu}(p-k)_\nu + \delta_{\mu\nu}(q-p)_\alpha\}$

$\dfrac{1}{m+i\gamma k-i\varepsilon}$

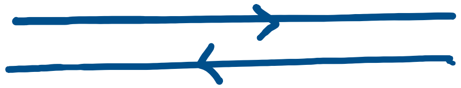


$g^2\{2\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\beta}\delta_{\mu\nu} - \delta_{\beta\mu}\delta_{\alpha\nu}\}$

+ GHOSTS

U(N) GAUGE THEORY

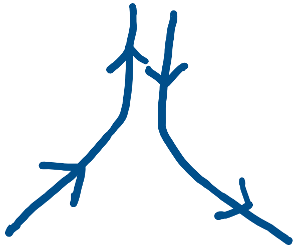
G. 't Hooft, Nucl. Phys. B 72, 461 (1974)



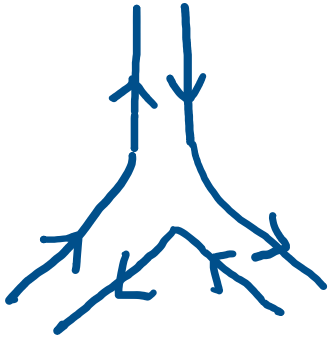
$\sim \mathcal{O}(1)$

$\sim \mathcal{O}(1)$

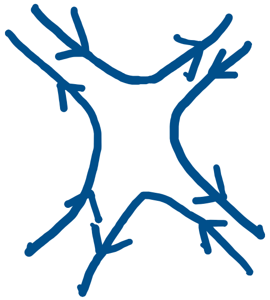
+ GHOSTS



$\sim \mathcal{O}(g)$



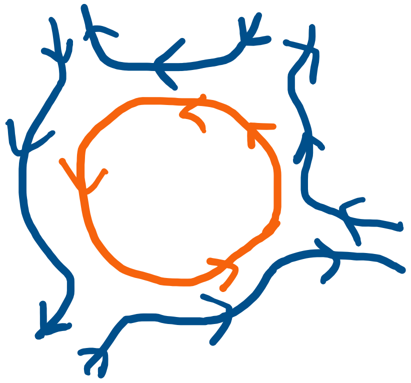
$\sim \mathcal{O}(g)$

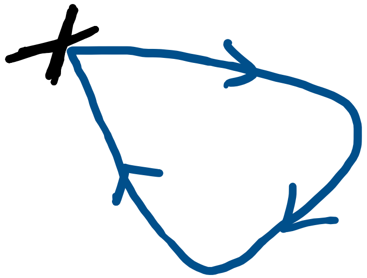


$\sim \mathcal{O}(g^2)$

U(N) GAUGE THEORY

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

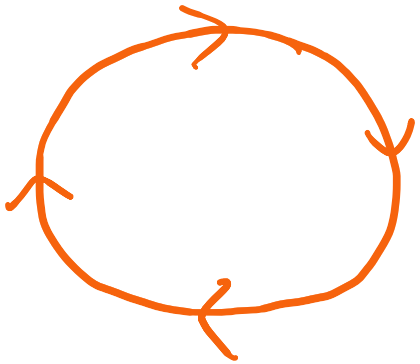




$$\sim \sum_i \delta_i^{\,i} = N$$

CLOSED INDEX LOOP

QUARK LOOP

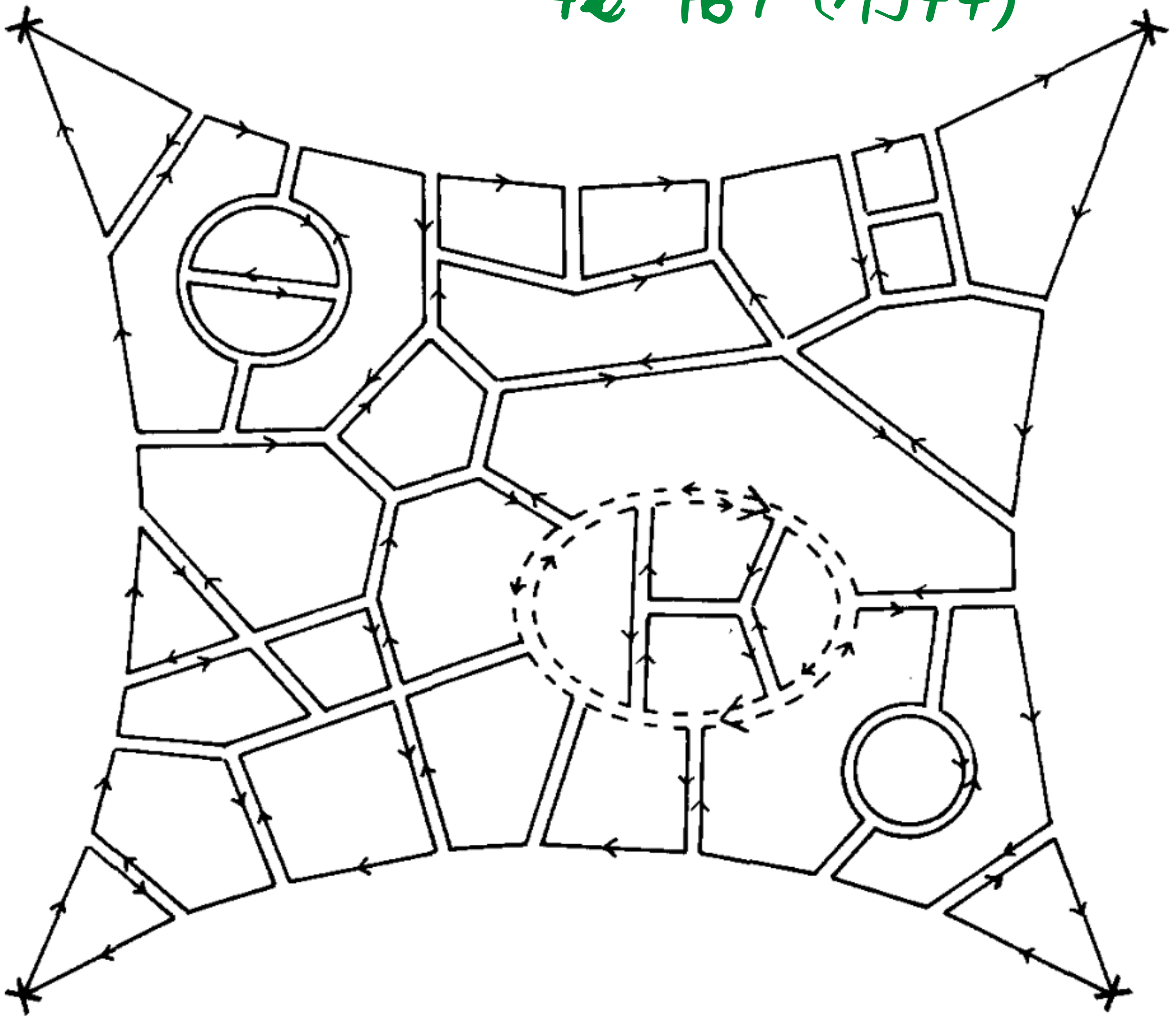


$$\sim 1$$

COLOR BLINDNESS

COMBINATORICS

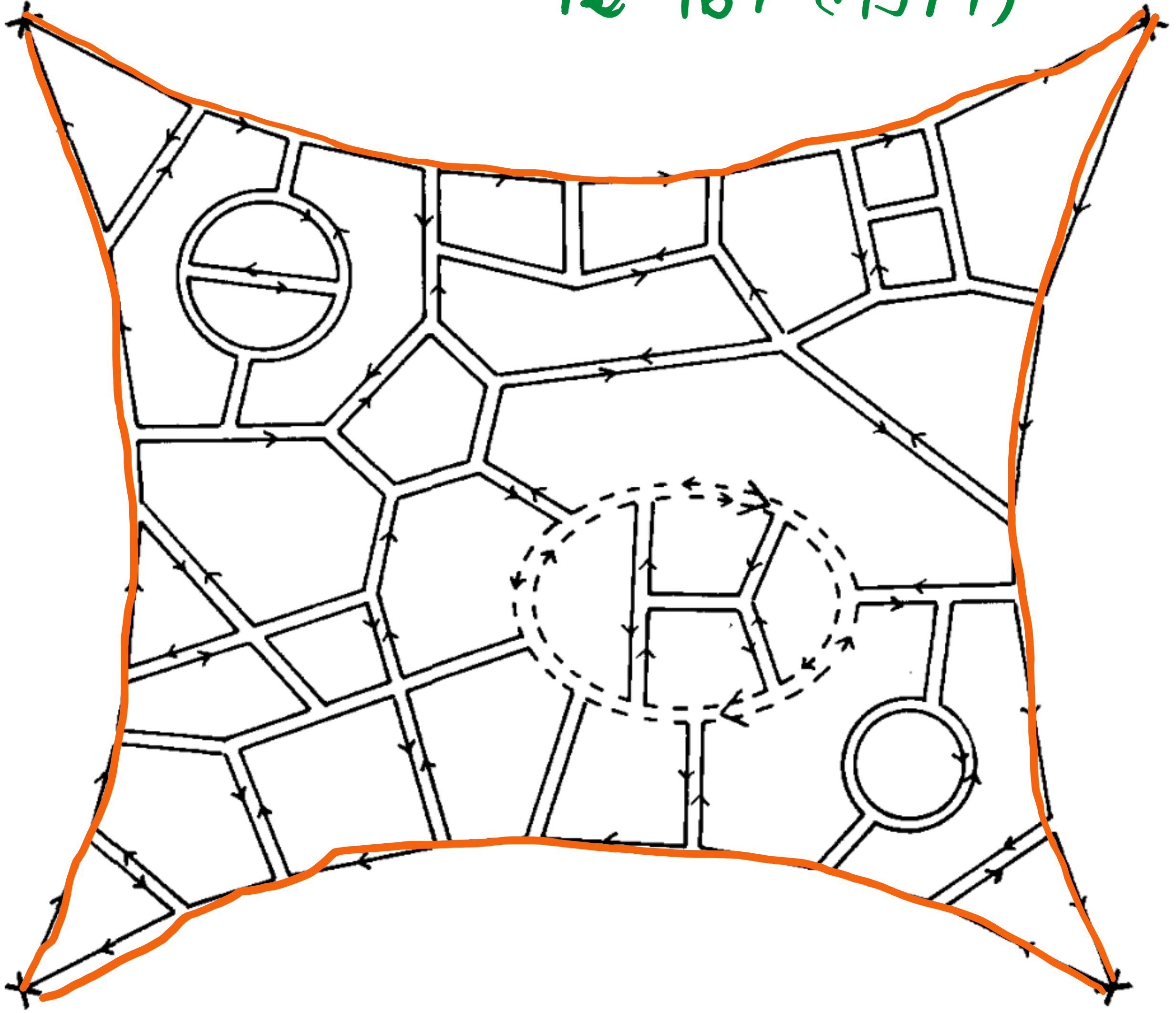
G. 't Hooft, Nucl. Phys. B 72, 461 (1974)



COMBINATORICS

OF QUARK LOOPS

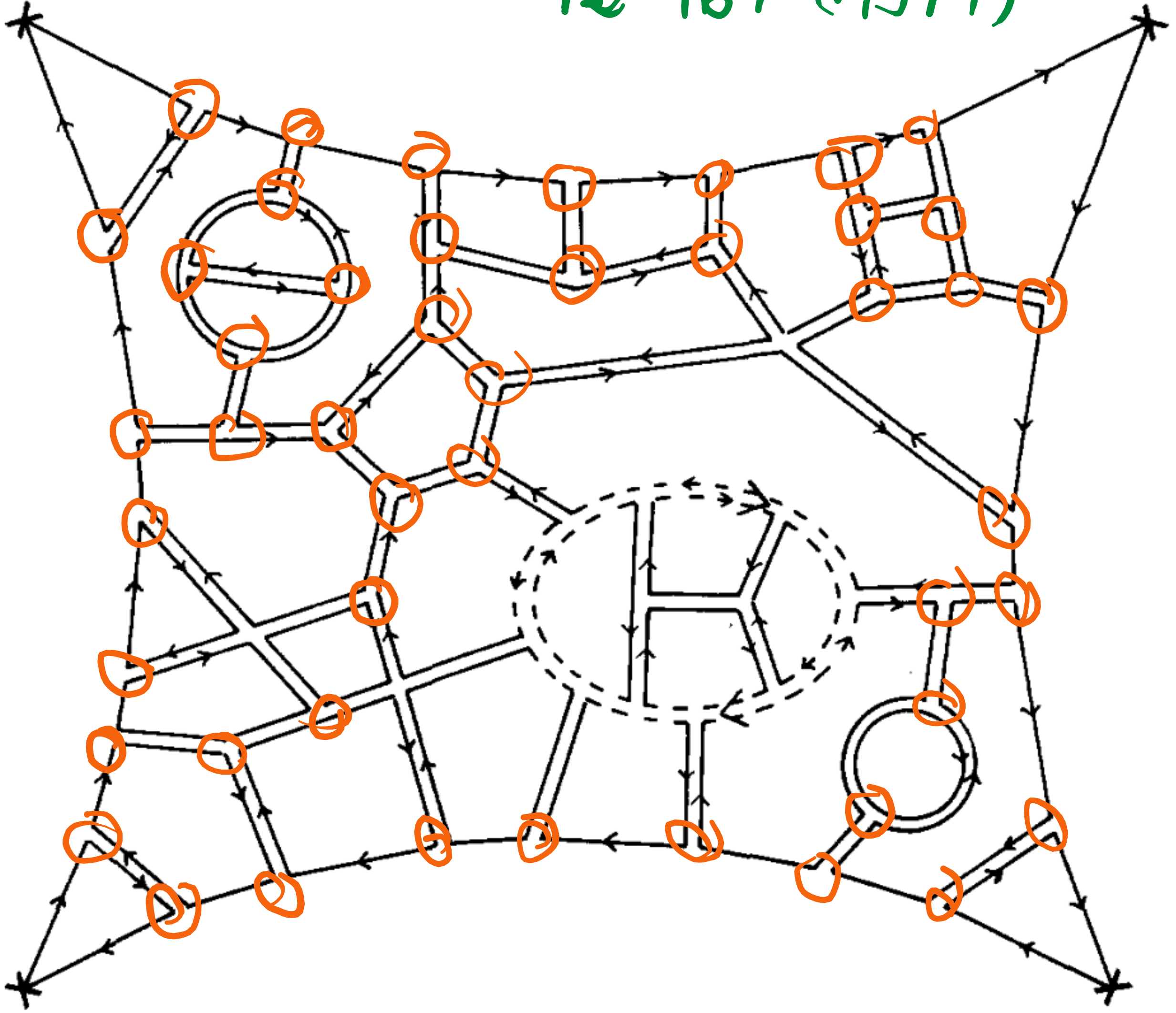
G. 't Hooft, Nucl. Phys. B 72 461 (1974)



COMBINATORICS

OF 3-PT VERTICES

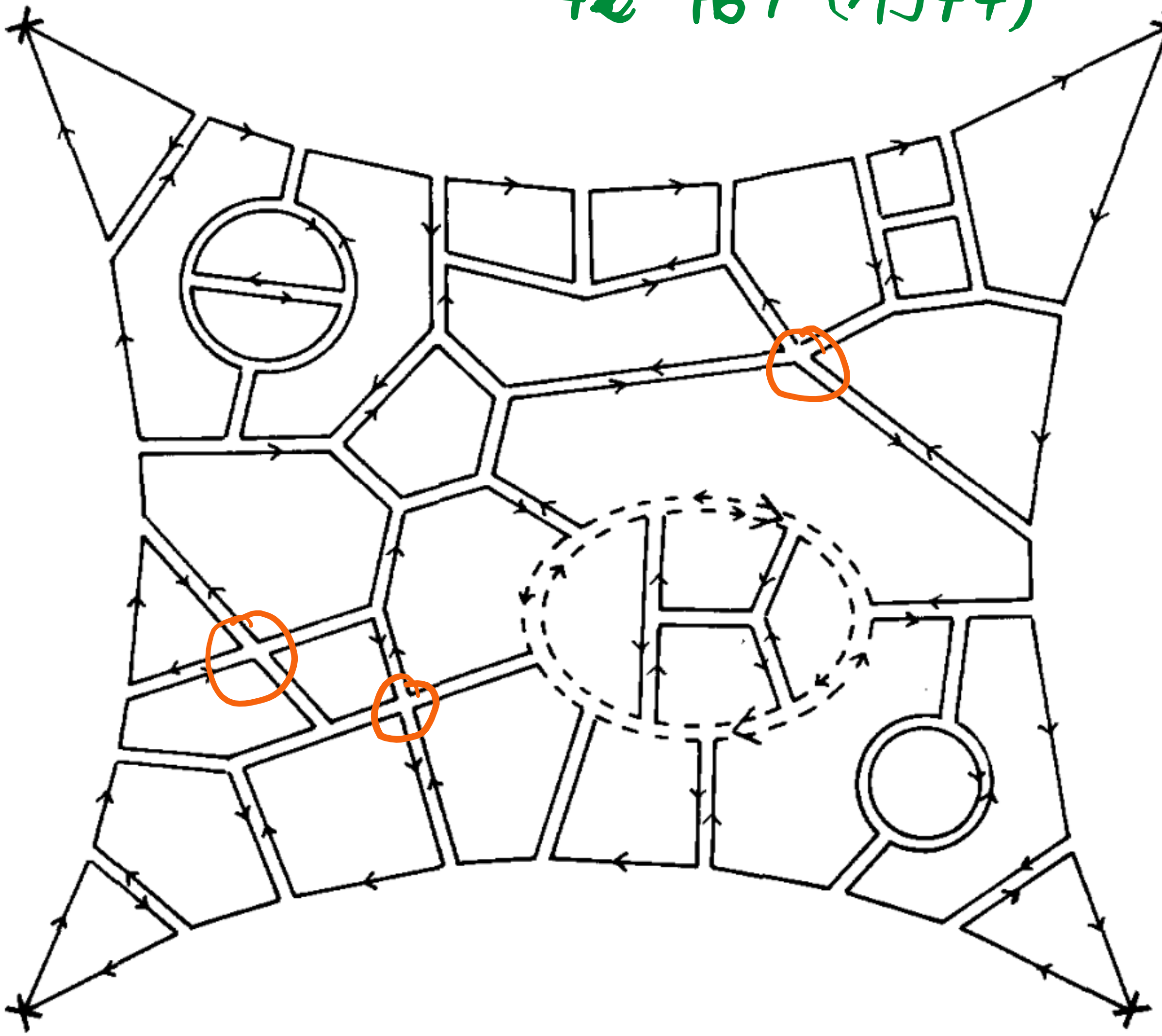
G. 't Hooft, Nucl. Phys. B 72 461 (1974)



COMBINATORICS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

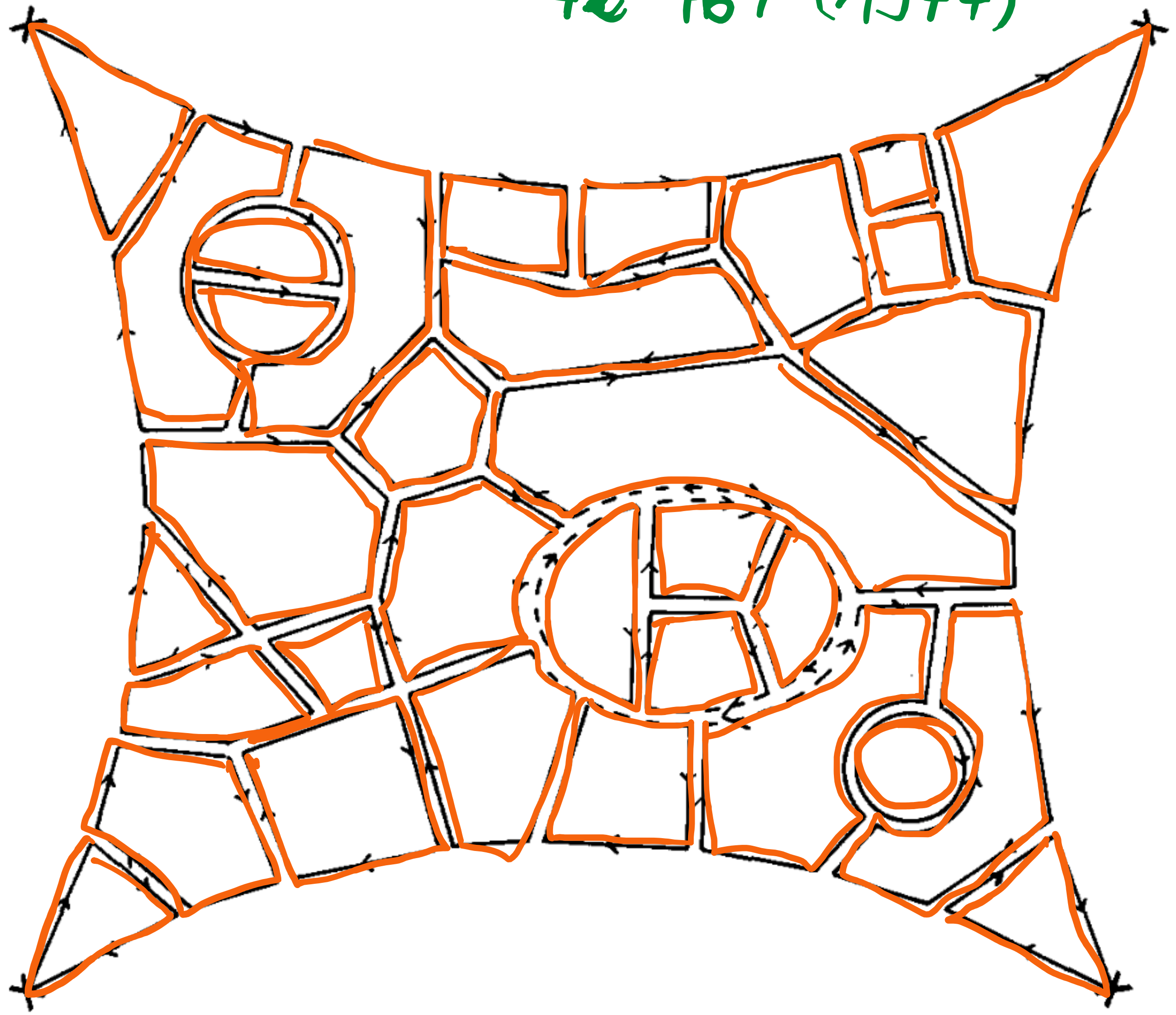
OF 4-PT VERTICES



COMBINATORICS

OF CLOSED INDEX LOOPS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)



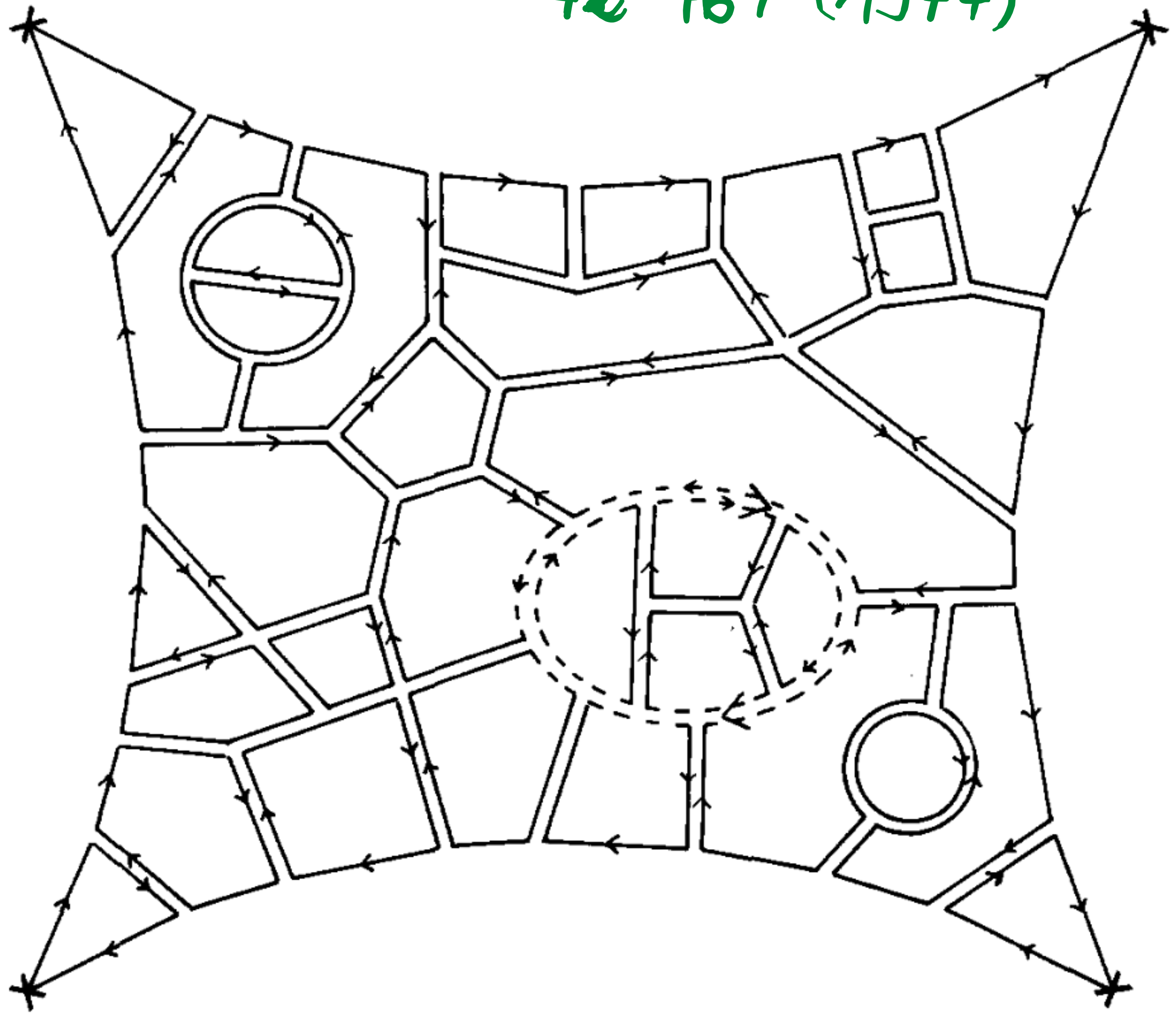
COMBINATORICS

SCALING OF THE DIAGRAM

$$g^{V_3}(g^2)^{V_4}\, N^{I}$$

3-pt 4-pt INDEX LOOPS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)



COMBINATORICS

Scaling of the diagram

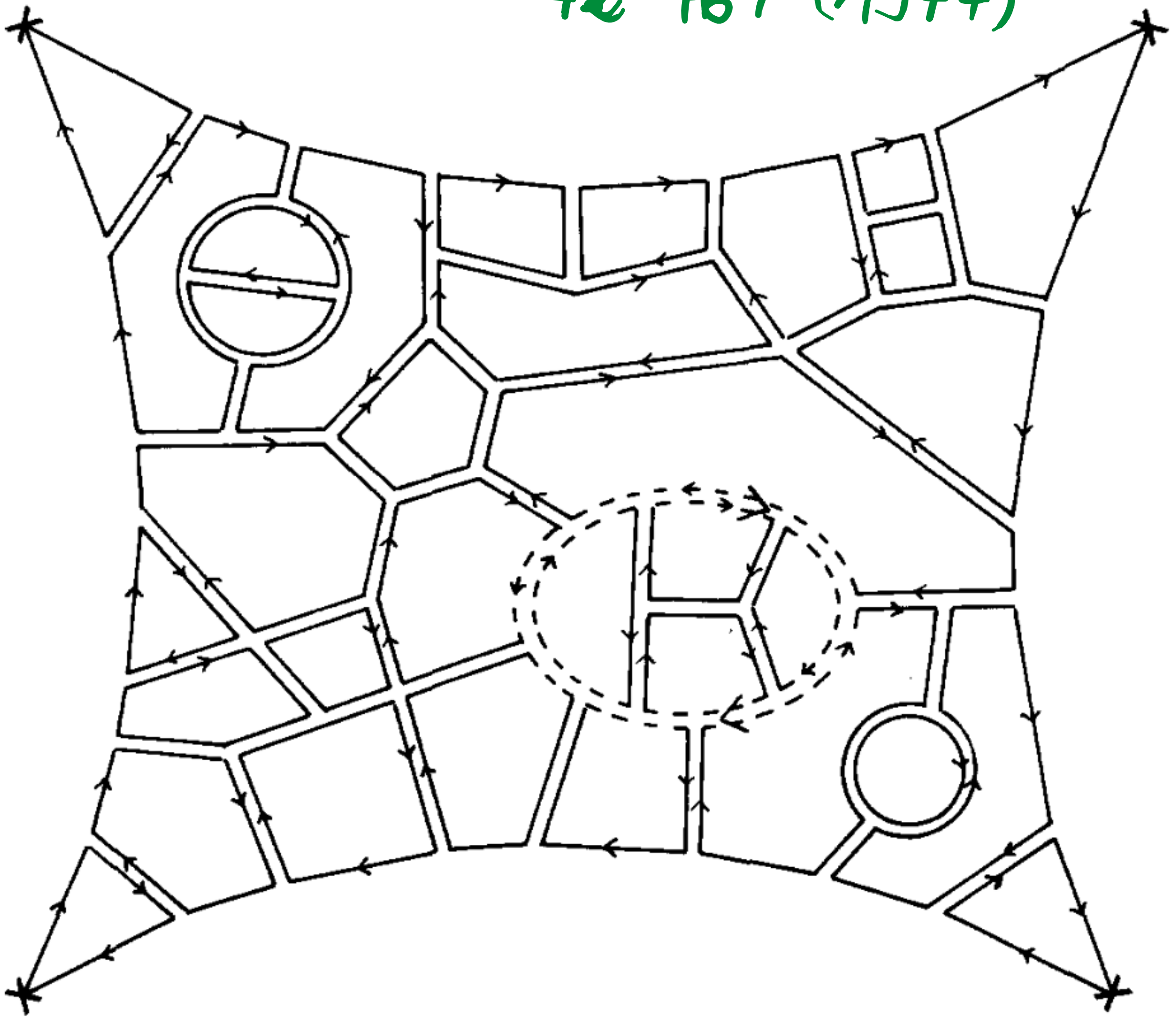
$$g^{V_3}(g^2)^{V_4}N^{I}$$

$L = I + Q \longrightarrow$ Quark loops

L ↓ Total # of loops

I ↓ Index loops

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

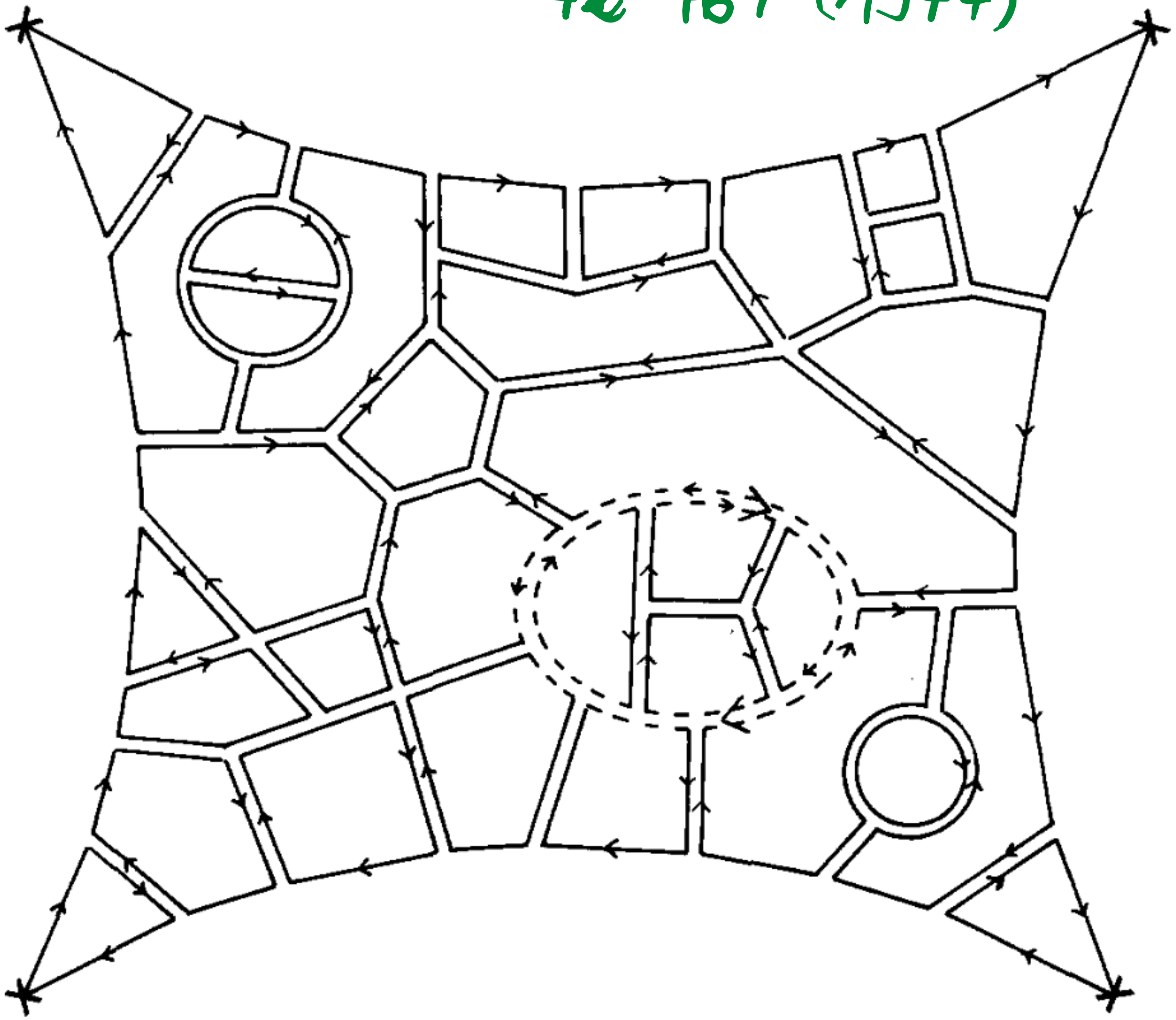


COMBINATORICS

SCALING OF THE DIAGRAM

$g^{V_3}(g^2)^{V_4} N^{L-Q}$

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

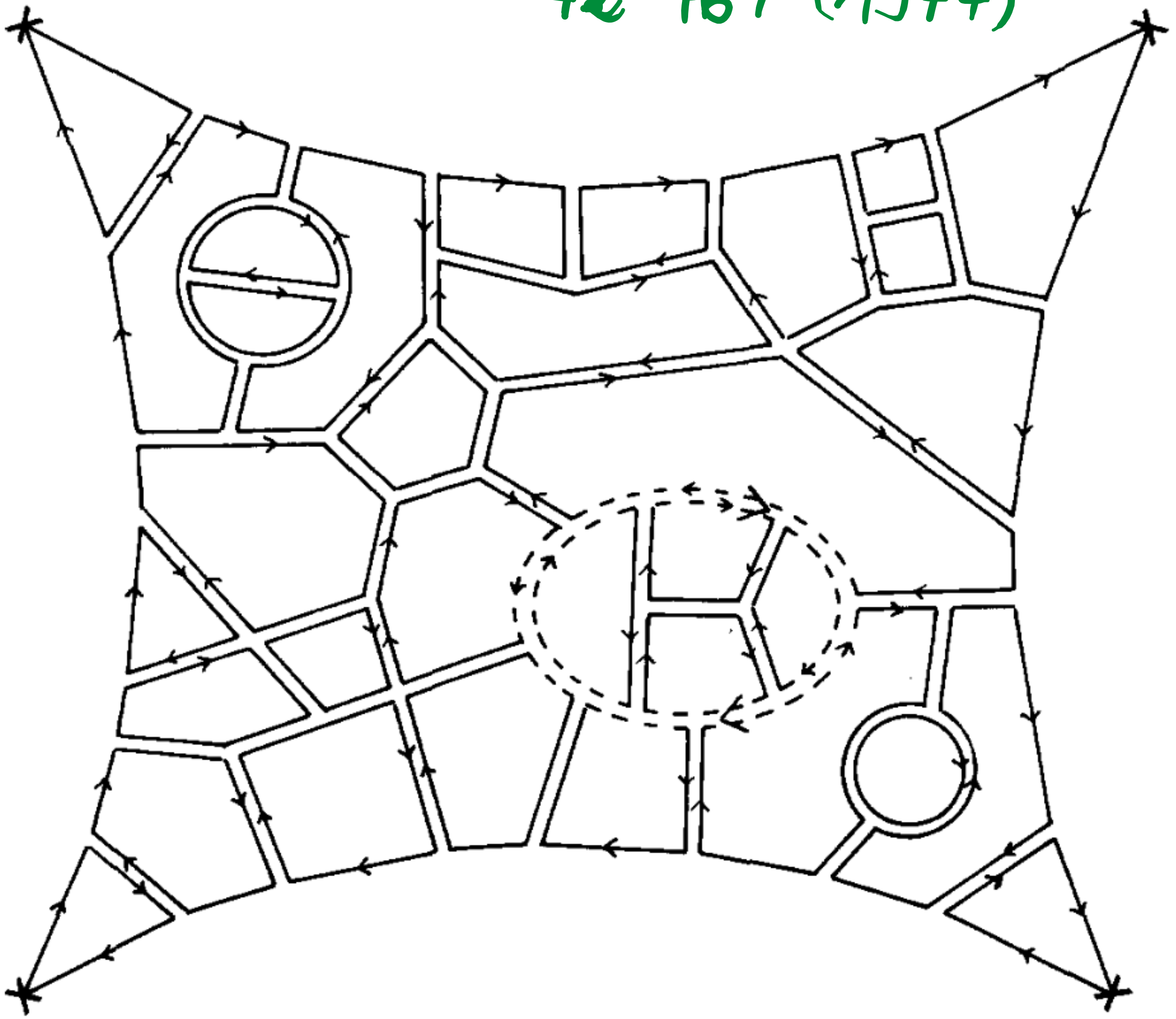


COMBINATORICS

SCALING OF THE DIAGRAM

$$g^{V_3+2V_4} N^{L-Q}$$

G. 't Hooft, Nucl. Phys. B 72 461 (1974)



COMBINATORICS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

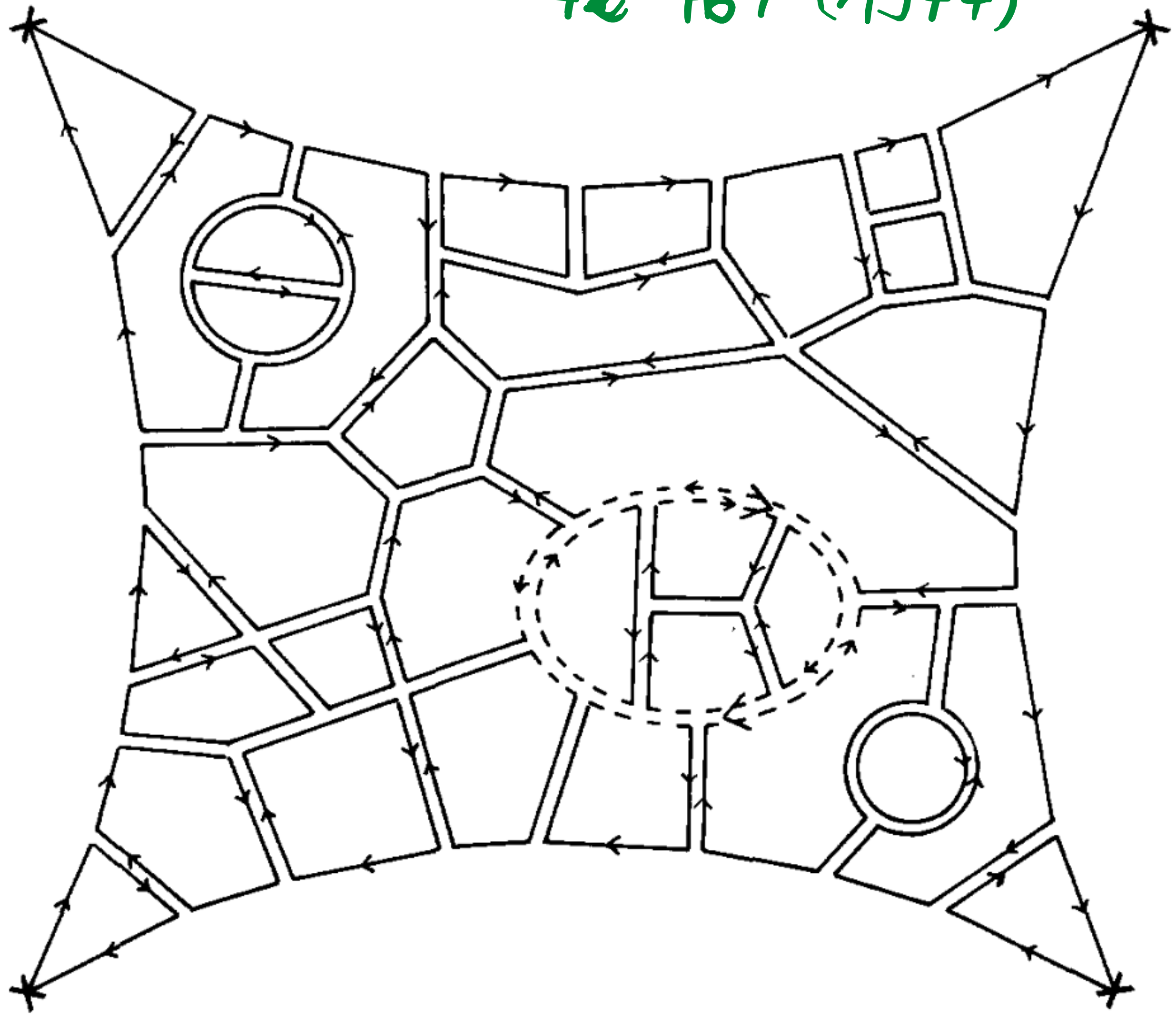
TOTAL # OF PROPAGATORS

$$2P = \sum_n n V_n = 3V_3 + 4V_4$$

TOTAL # OF VERTICES

$$V = \sum_n V_n = V_3 + V_4$$

$$\Rightarrow \quad V_3 + 2V_4 = 2P - 2V$$



COMBINATORICS

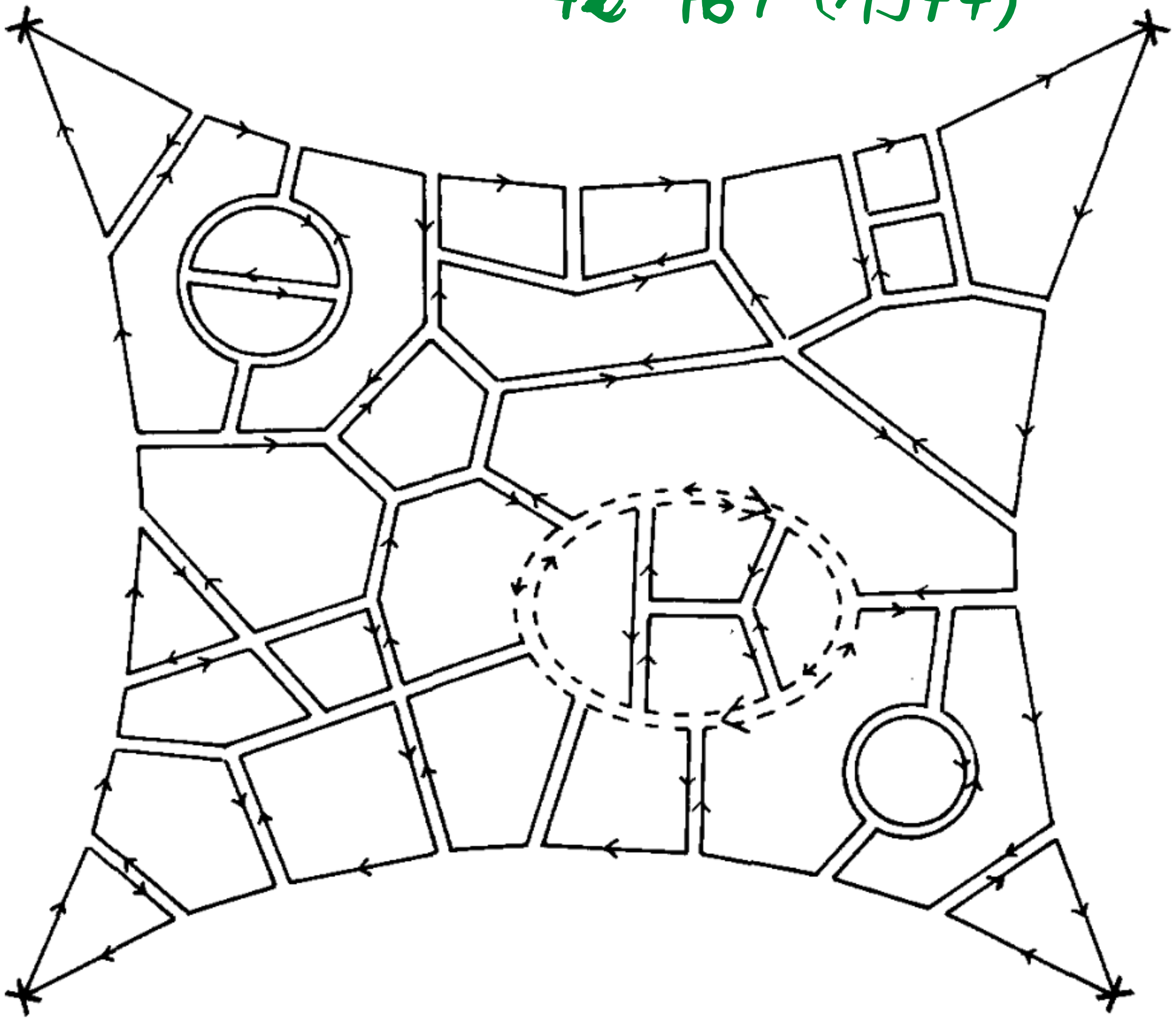
$V_3 + 2V_4 = 2P - 2V$

EULER'S THEOREM

$L - P + V = 2 - 2H$

H = HOLES IN THE SURFACE

G. 't Hooft, Nucl. Phys. B 72 461 (1974)



COMBINATORICS

$$V_3 + 2V_4 = 2P - 2V$$

EULER'S THEOREM

$$L - P + V = 2 - 2H$$

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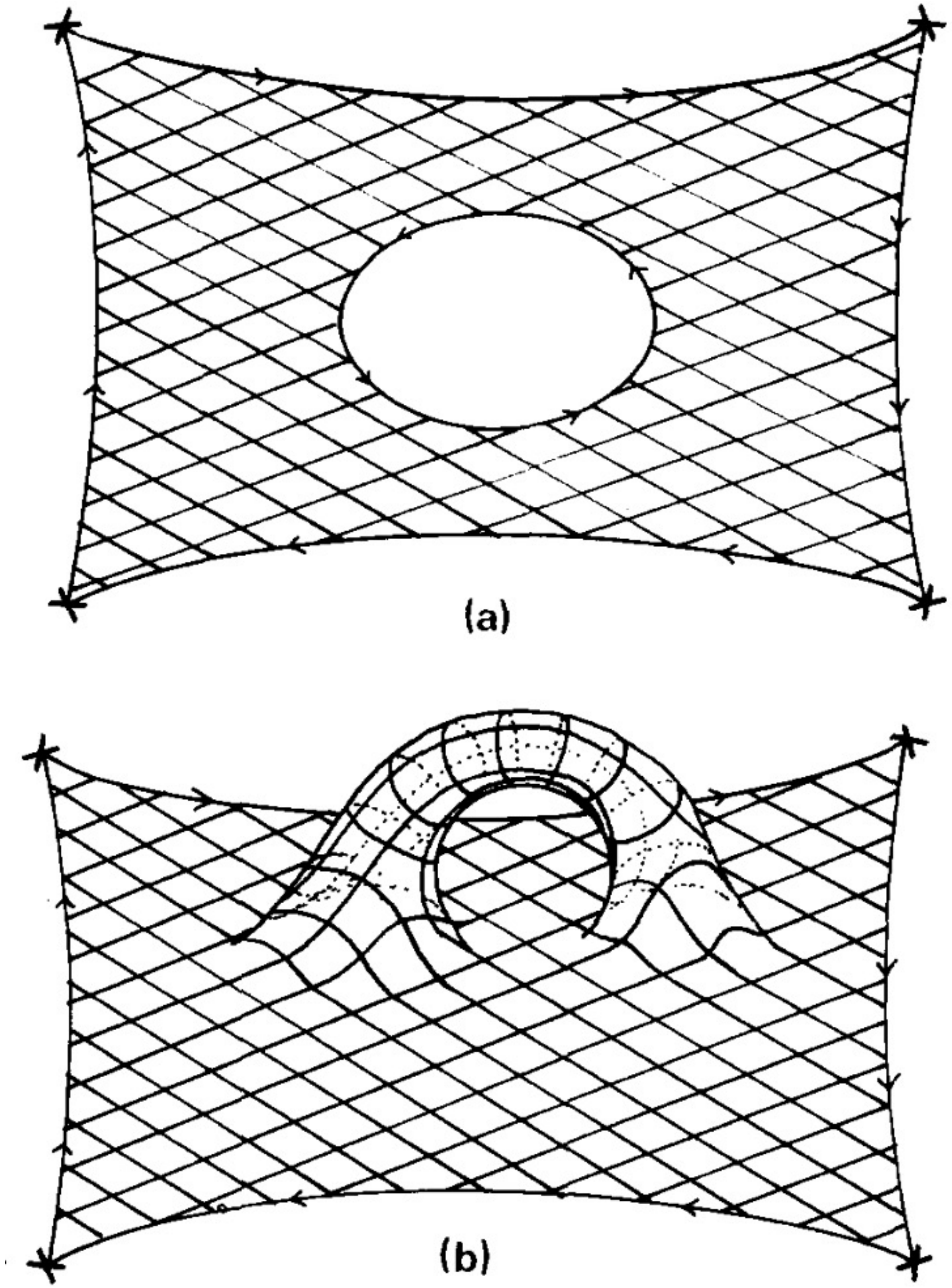


(a)

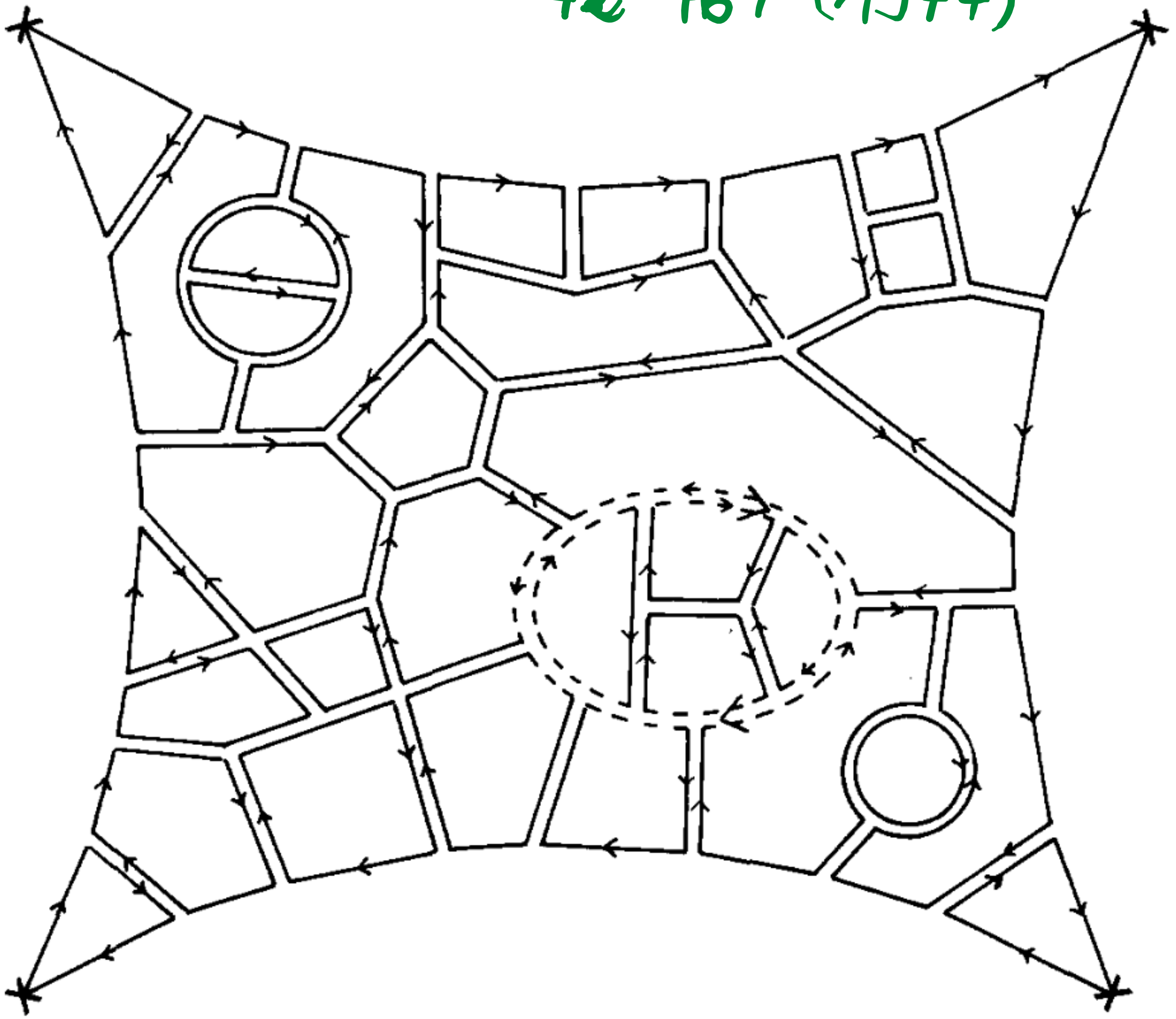
(b)

COMBINATORICS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

SCALING OF THE DIAGRAM

$g^{V_3 + 2V_4} N^{L-Q}$

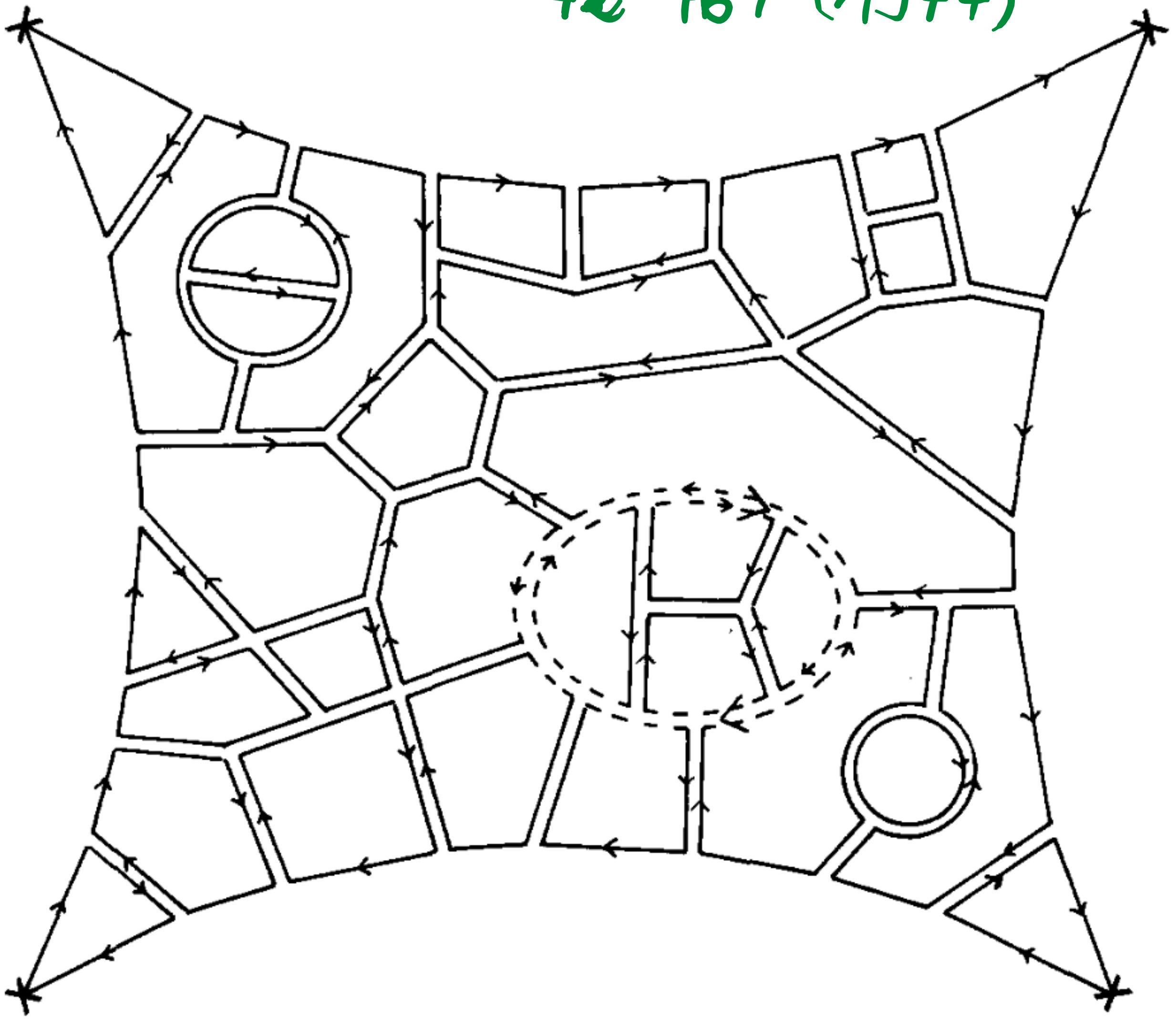


COMBINATORICS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

SCALING OF THE DIAGRAM

$g^{2P-2V} N^{L-Q}$



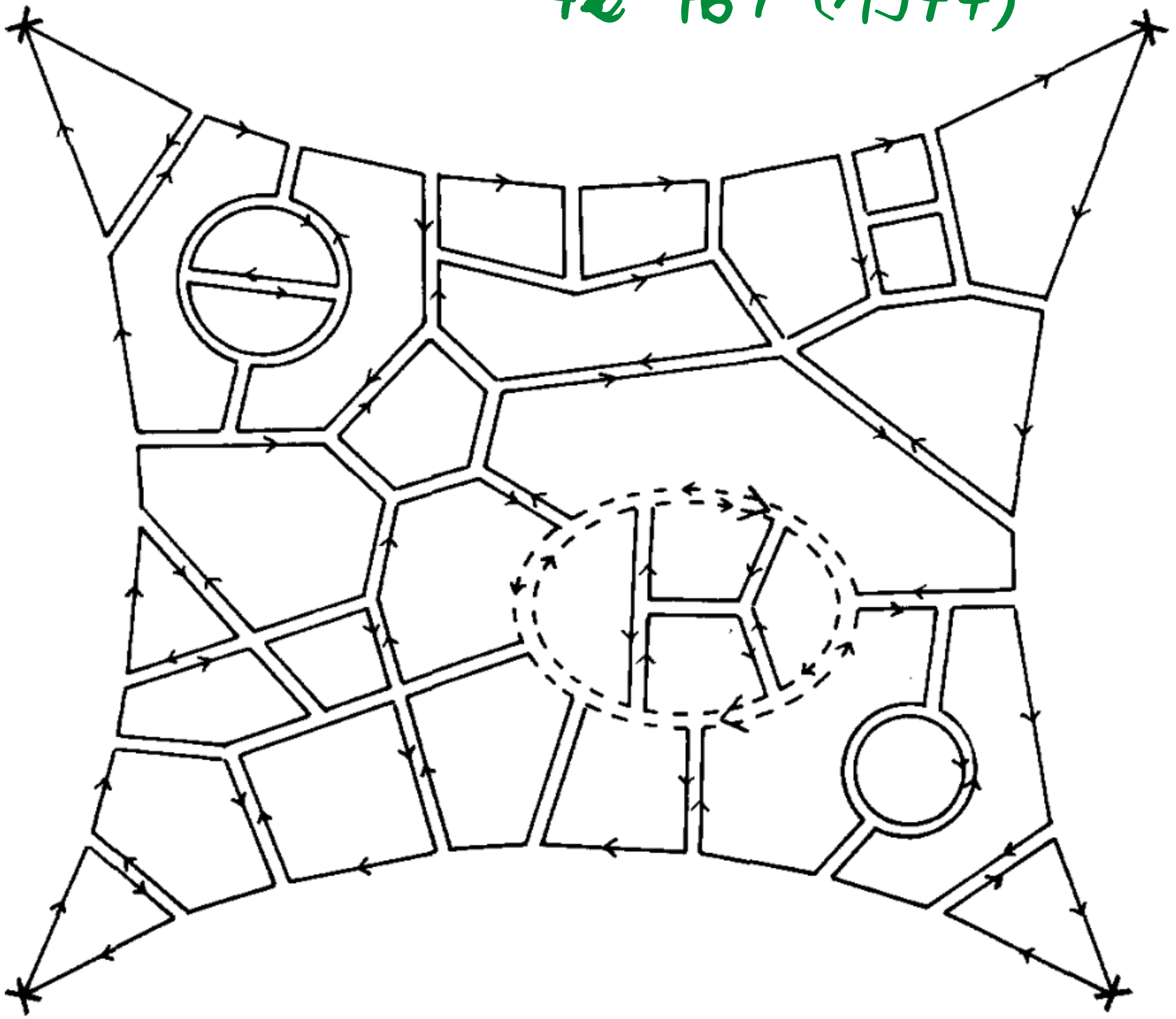
COMBINATORICS

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

SCALING OF THE DIAGRAM

$$g^{2P-2V} N^{L-Q}$$

$$= (g^2 N)^{P-V} N^{2-2H} \left(\frac{1}{N}\right)^{Q}$$



G. 't Hooft, Nucl. Phys. B 72, 461 (1974)

$\frac{1}{N}$-EXPANSION

$$\text{Diagram} \sim (g^2 N)^{P-V} N^{2-2H} \left(\frac{1}{N}\right)^{Q}$$

$$N \to \infty \,, \quad g \to 0 \,, \quad g^2 N = g_0^2 = \text{fixed}$$

$$\text{Diagram} \sim N^2 \left(\frac{1}{N^2}\right)^{H} \left(\frac{1}{N}\right)^{Q}$$

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

$\frac{1}{N}$-EXPANSION

$$\text{Diagram} \sim N^2 \left(\frac{1}{N^2}\right)^H \left(\frac{1}{N}\right)^Q$$

Similarity to string theory

$\frac{1}{N}$-EXPANSION

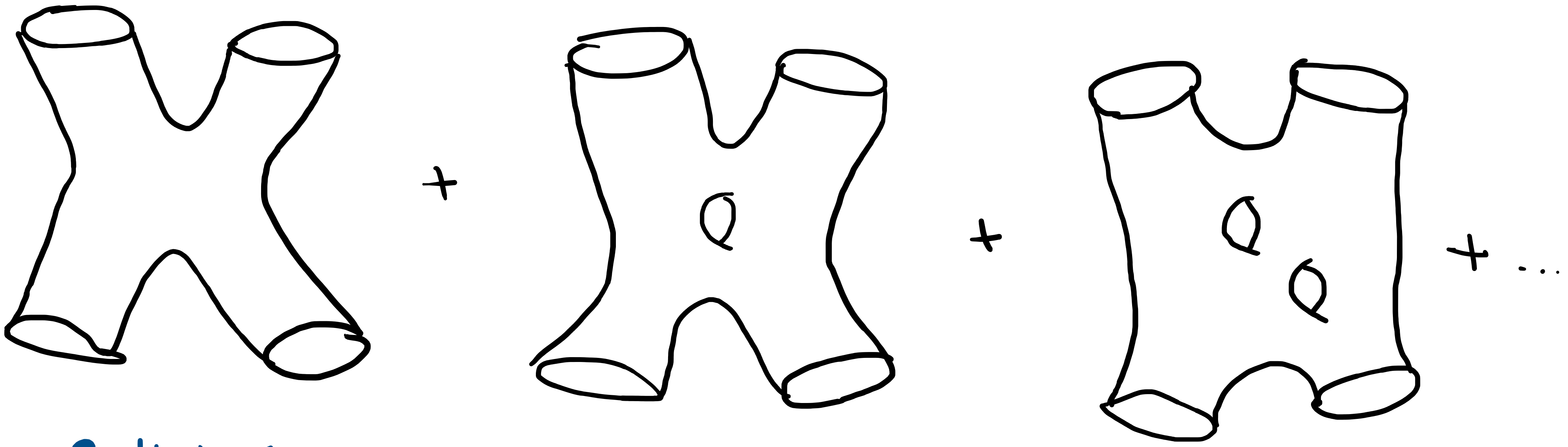
G. 't Hooft, Nucl. Phys. B 72 461 (1974)

$$\text{Diagram} \sim N^2 \left(\frac{1}{N^2}\right)^H \left(\frac{1}{N}\right)^Q$$

$\uparrow$ Suppression of quark loops ???

STRING THEORY?

G. 't Hooft, Nucl. Phys. B 72 461 (1974)
Stephen Weinberg The Quantum Theory of Fields, Vol. 3, (2013)



0 HOLES

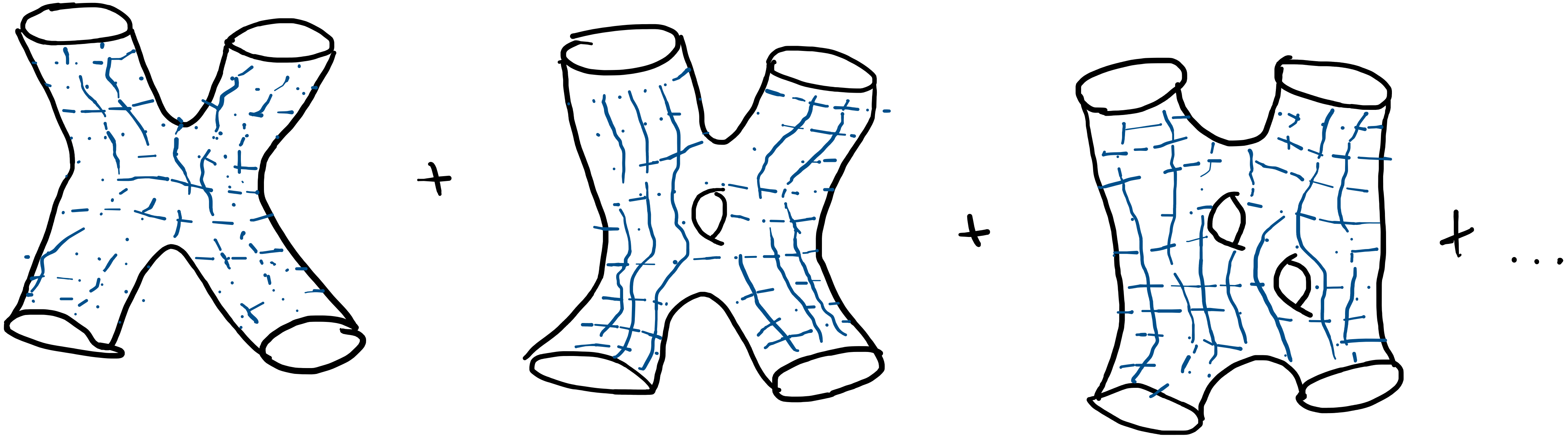
1 HOLES

2 HOLES

QUANTIZED STRING ACTION $S[\tilde{x}] = \int d\sigma \, d\tau \left[\left(\frac{\partial \tilde{x}}{\partial \sigma}\right)^2 + \left(\frac{\partial \tilde{x}}{\partial \tau}\right)^2 \right]$

STRING THEORY?

G. 't Hooft, Nucl. Phys. B 72 461 (1974)
Stephen Weinberg The Quantum Theory of Fields, Vol. 3, (2013)



$\frac{1}{N}$-EXPANSION: $\sum_{ij} \left| \frac{\Delta p^+_{ij}}{2(\tau_i - \tau_j)} \right| (\tilde{x}_i - \tilde{x}_j)^2$

TAKING THE LIMIT

For a large-N theory with fermions

G. 't Hooft, Nucl. Phys. B 72, 461 (1974)

IMPLICATIONS OF 'T HOOFT'S LIMIT

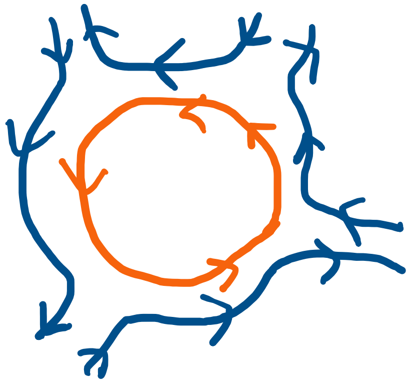
$$\text{Diagram} \sim N^2 \left(\frac{1}{N^2}\right)^H \left(\frac{1}{N}\right)^Q$$

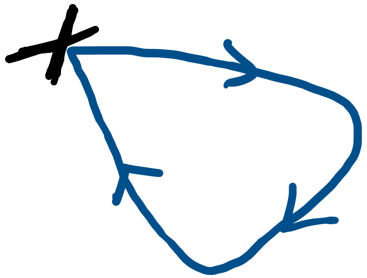
Suppression of quark loops ???

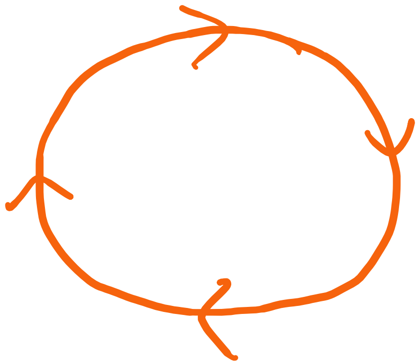
THE VENEZIANO LIMIT

G. Veneziano, Nucl. Phys. B 117 519 (1976)

QUARK LOOP







$\sim \sum_i \delta_i^i = N$

$\sim N_f$

CLOSED INDEX LOOP

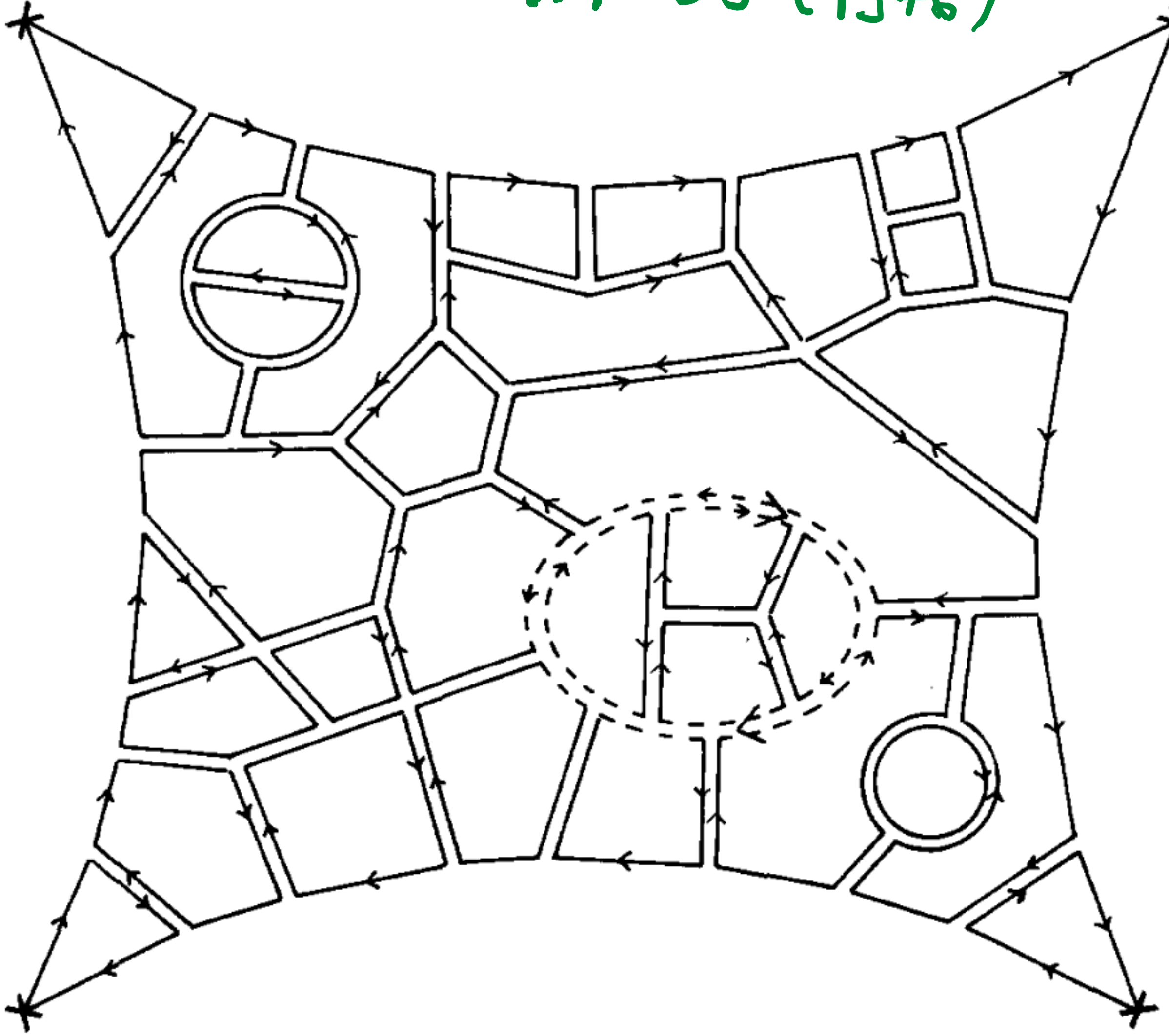
COLOR BLINDNESS

COMBINATORICS (VENEZIANO)

SCALING OF THE DIAGRAM

$$g^{V_3+2V_4} N^{I} N_f^{Q}$$

G. Veneziano, Nucl. Phys. B 117 519 (1976)



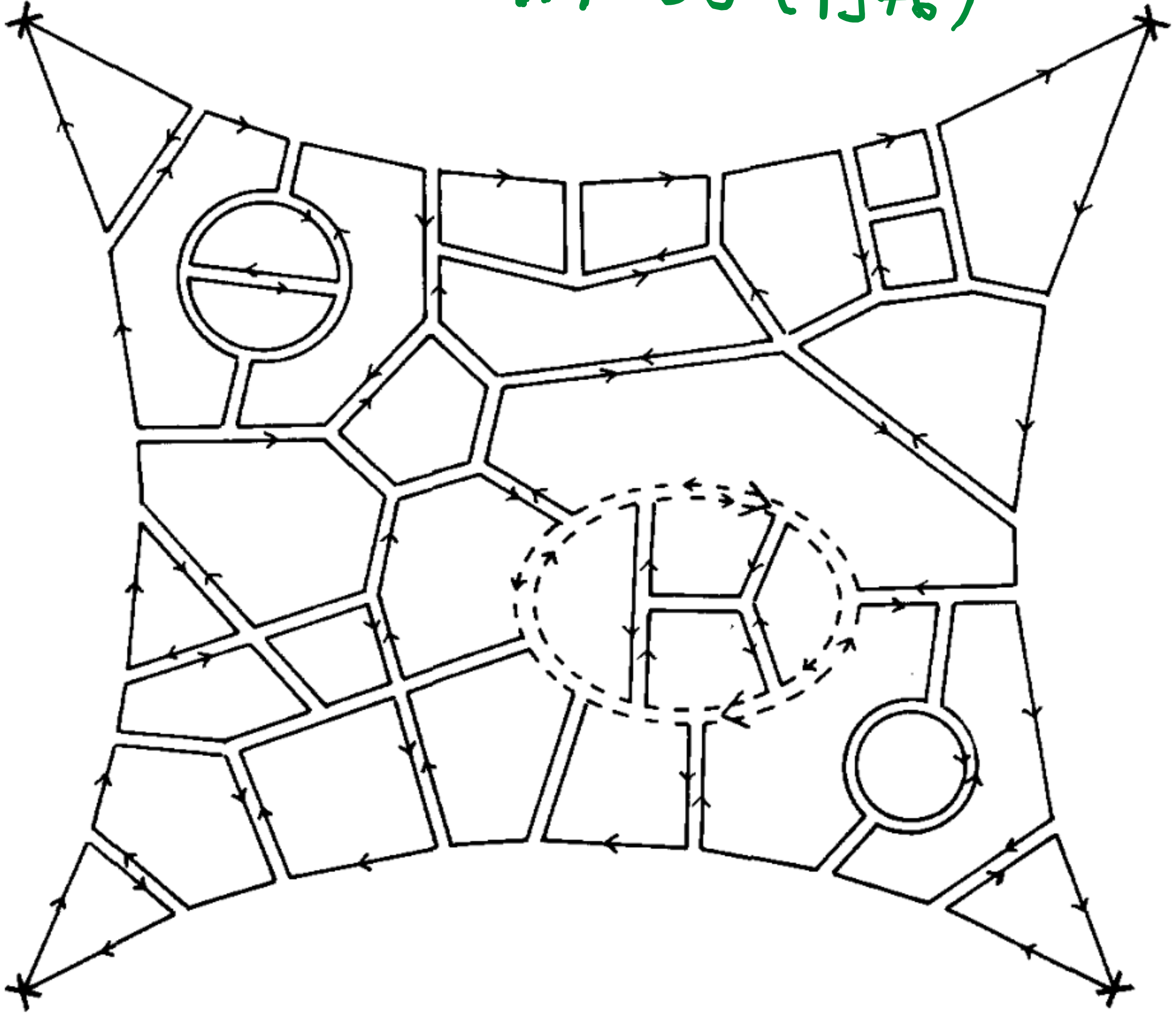
G. 't Hooft, Nucl. Phys. B 72 461 (1974)

COMBINATORICS (VENEZIANO)

Scaling of the diagram

$$g^{2P-2V} N^{I} N_f^{Q}$$

G. Veneziano, Nucl. Phys. B 117 519 (1976)



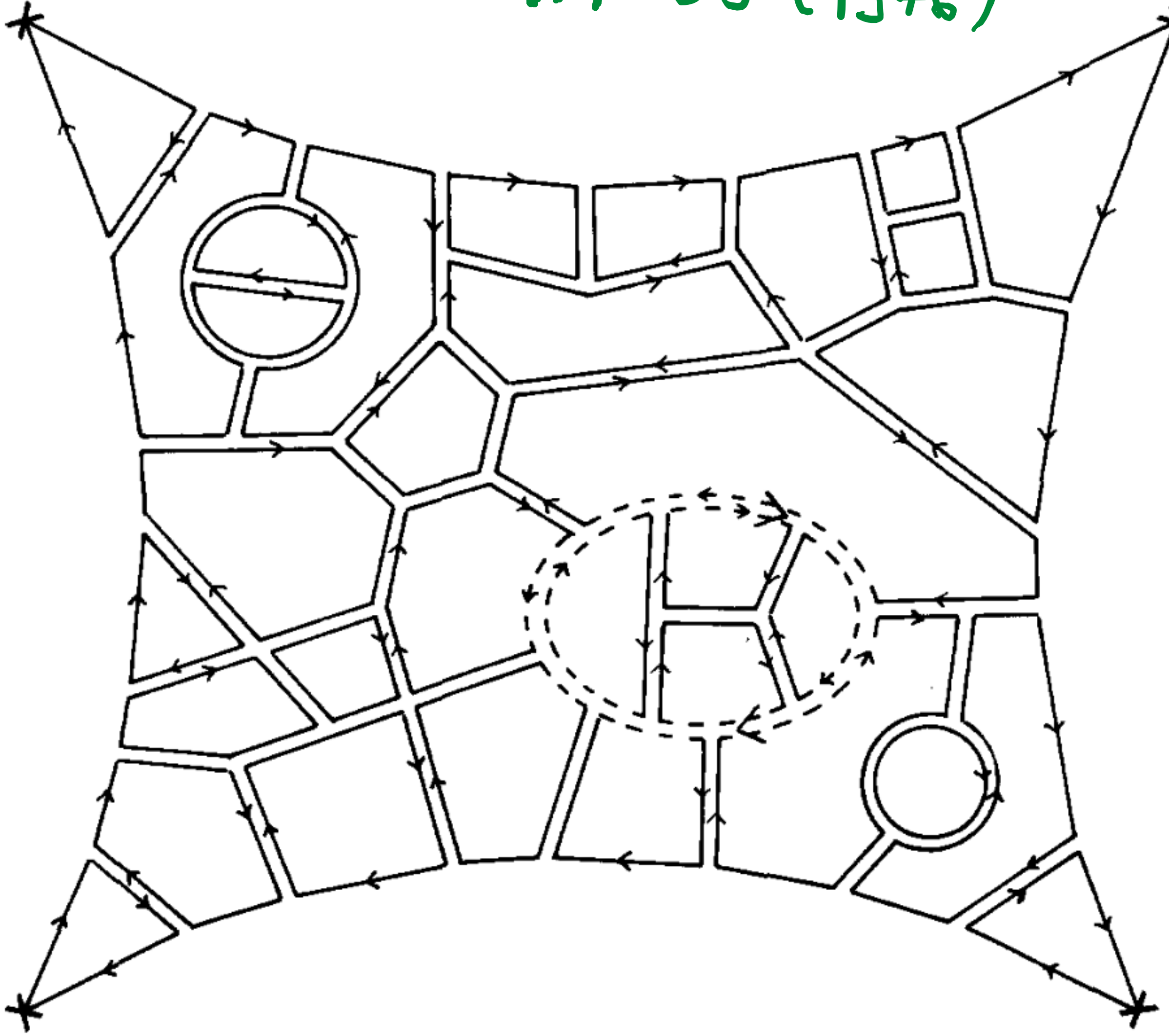
COMBINATORICS (VENEZIANO)

Scaling of the diagram

$$g^{2P-2V-2I-2Q}\,(g^2N)^I$$

$$\times\,(g^2N_f)^Q$$

G. Veneziano, Nucl. Phys. B 117 519 (1976)



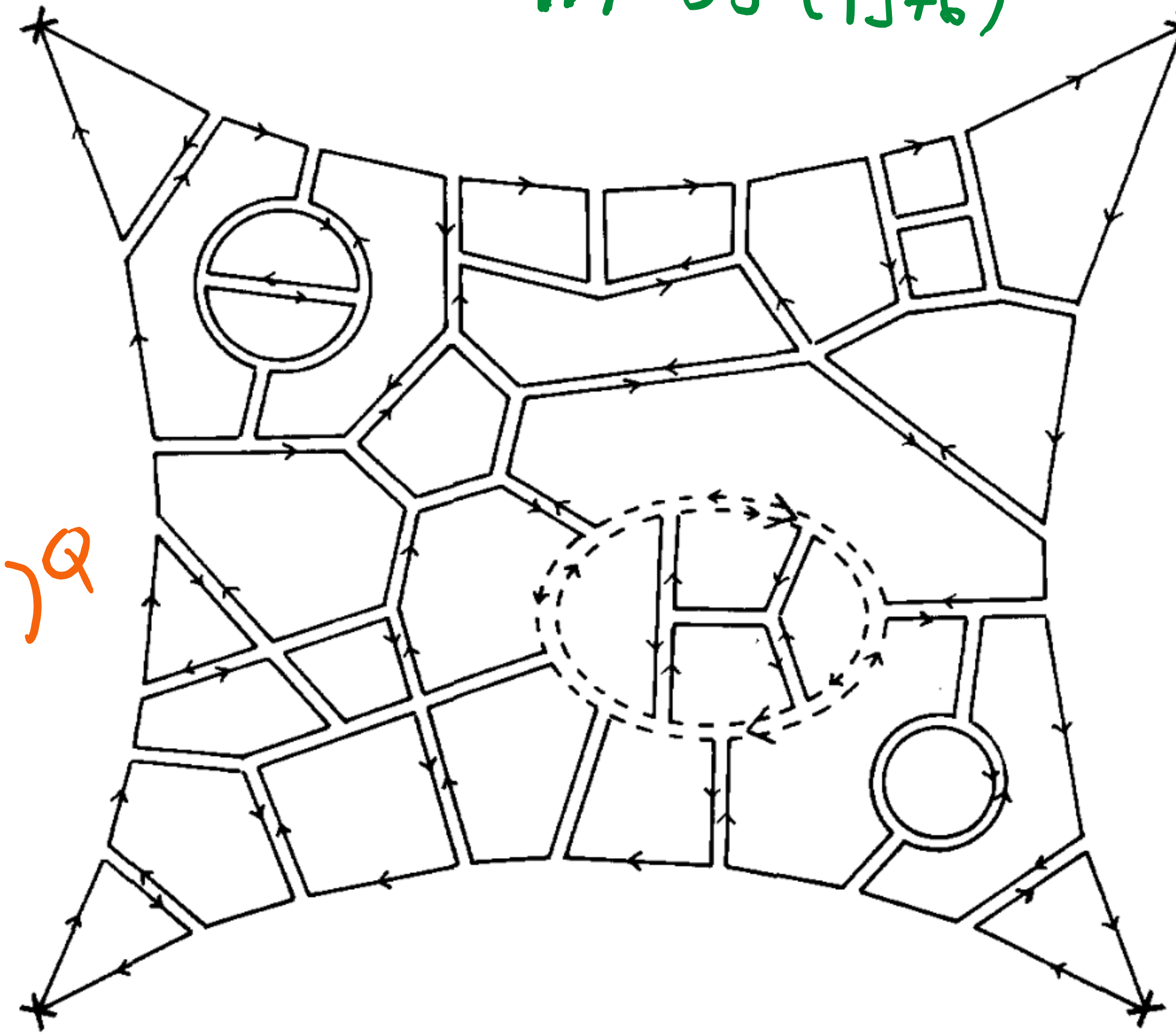
G. 't Hooft, Nucl. Phys. B 72 461 (1974)

COMBINATORICS (VENEZIANO)

SCALING OF THE DIAGRAM

$$g^{2(P-V-L)}(g^2N)^I(g^2N_f)^Q$$

G. Veneziano, Nucl. Phys. B 117 519 (1976)



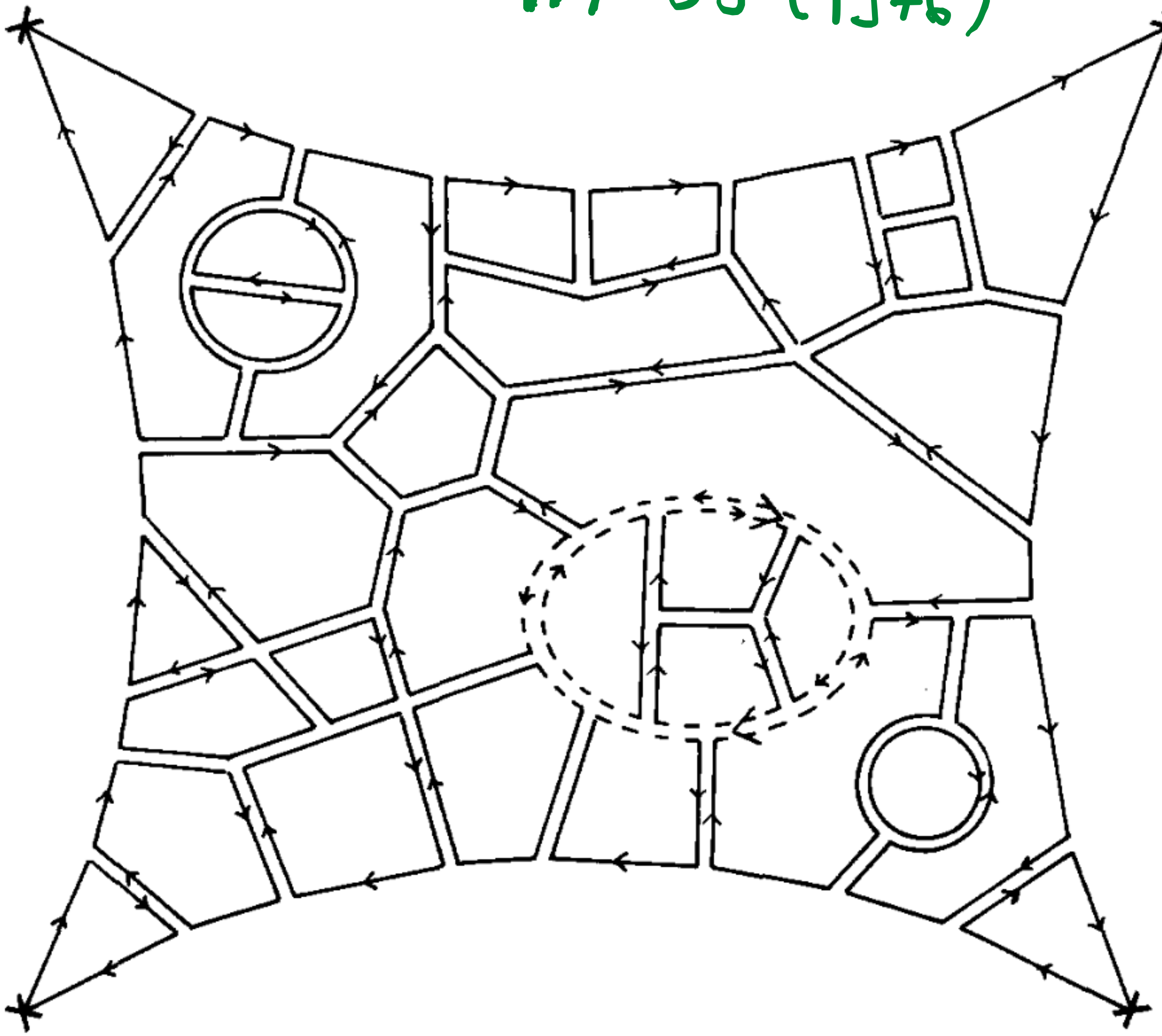
G. 't Hooft, Nucl. Phys. B 72 461 (1974)

COMBINATORICS (VENEZIANO)

SCALING OF THE DIAGRAM

$$(g^2)^{2H-2}(g^2N)^I(g^2N_f)^Q$$

G. Veneziano, Nucl. Phys. B 117 519 (1976)



G. 't Hooft, Nucl. Phys. B 72 461 (1974)

COMBINATORICS (VENEZIANO)

SCALING OF THE DIAGRAM

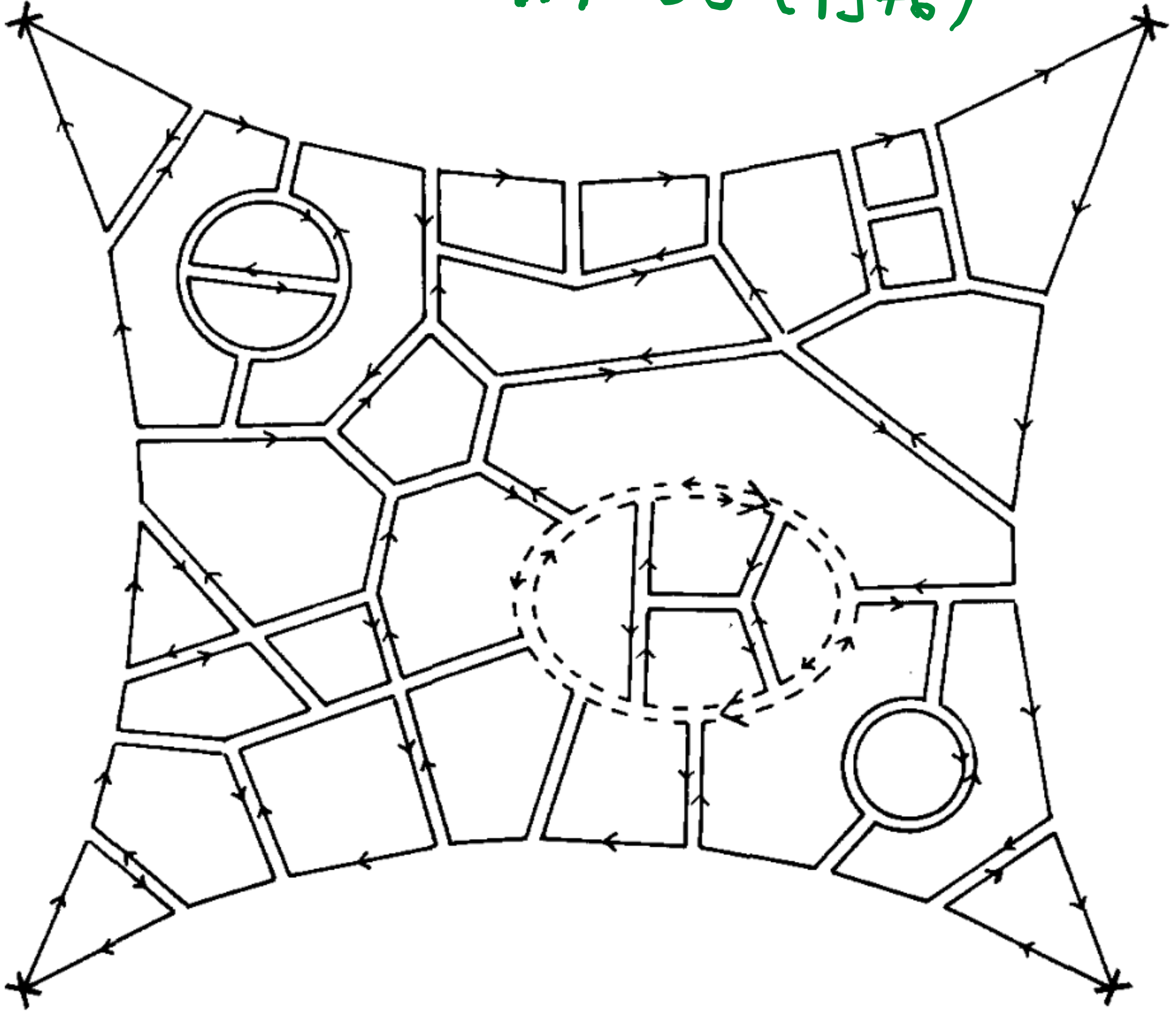
$$(g^2)^{2H-2}(g^2N)^I(g^2N_f)^Q$$

$g^2N \sim \text{const.}$

$g^2N_f \sim \text{const.}$

$\Rightarrow \quad N/N_f \sim \text{const.}$

G. Veneziano, Nucl. Phys. B 117 519 (1976)



G. 't Hooft, Nucl. Phys. B 72 461 (1974)

WHAT NOW?

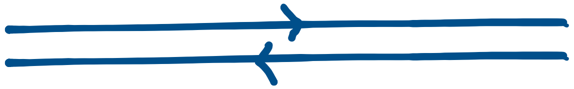
Lark theory

E. Corrigan & P. Ramond
Physics Letters B 87 73
1979

CORRIGAN & RAMOND

QUARKS

GAUGE
FIELD



+ GHOSTS

CORRIGAN & RAMOND

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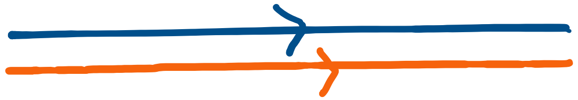
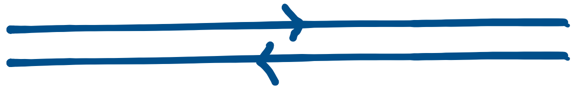
QUARKS



GAUGE
FIELD



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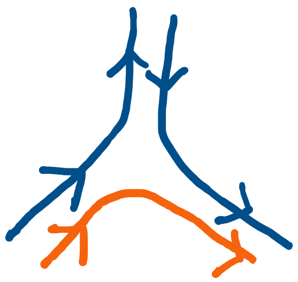
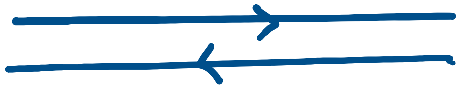
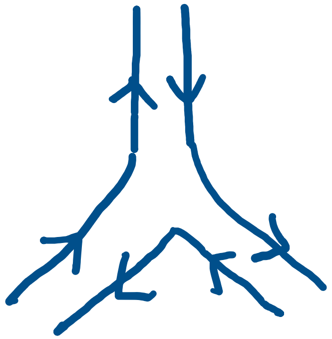


$\sim \mathcal{O}(g)$



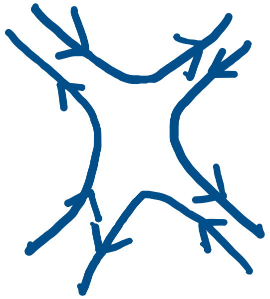
$\sim \mathcal{O}(1)$



$\sim \mathcal{O}(g)$

+ GHOSTS

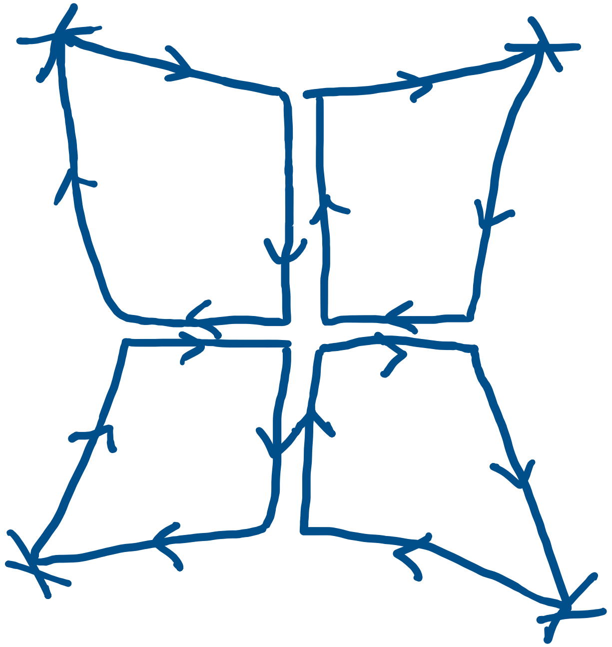
$\sim \mathcal{O}(1)$

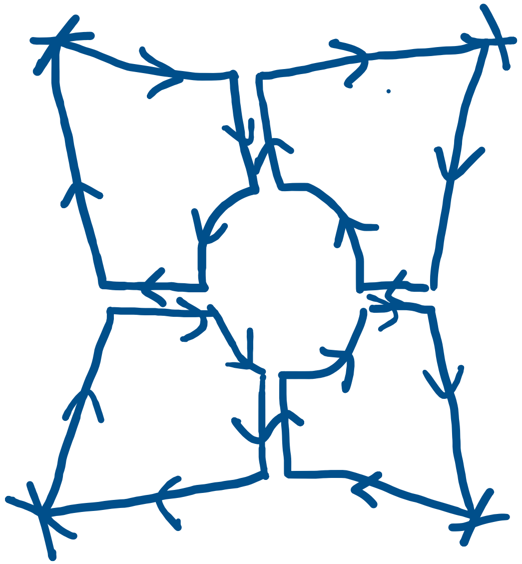


$\sim \mathcal{O}(g^2)$

G. 't Hooft, Nucl. Phys. B 72 461 (1974)

'T HOOFT





CORRIGAN & RAMOND

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Physics Letters B 87 73
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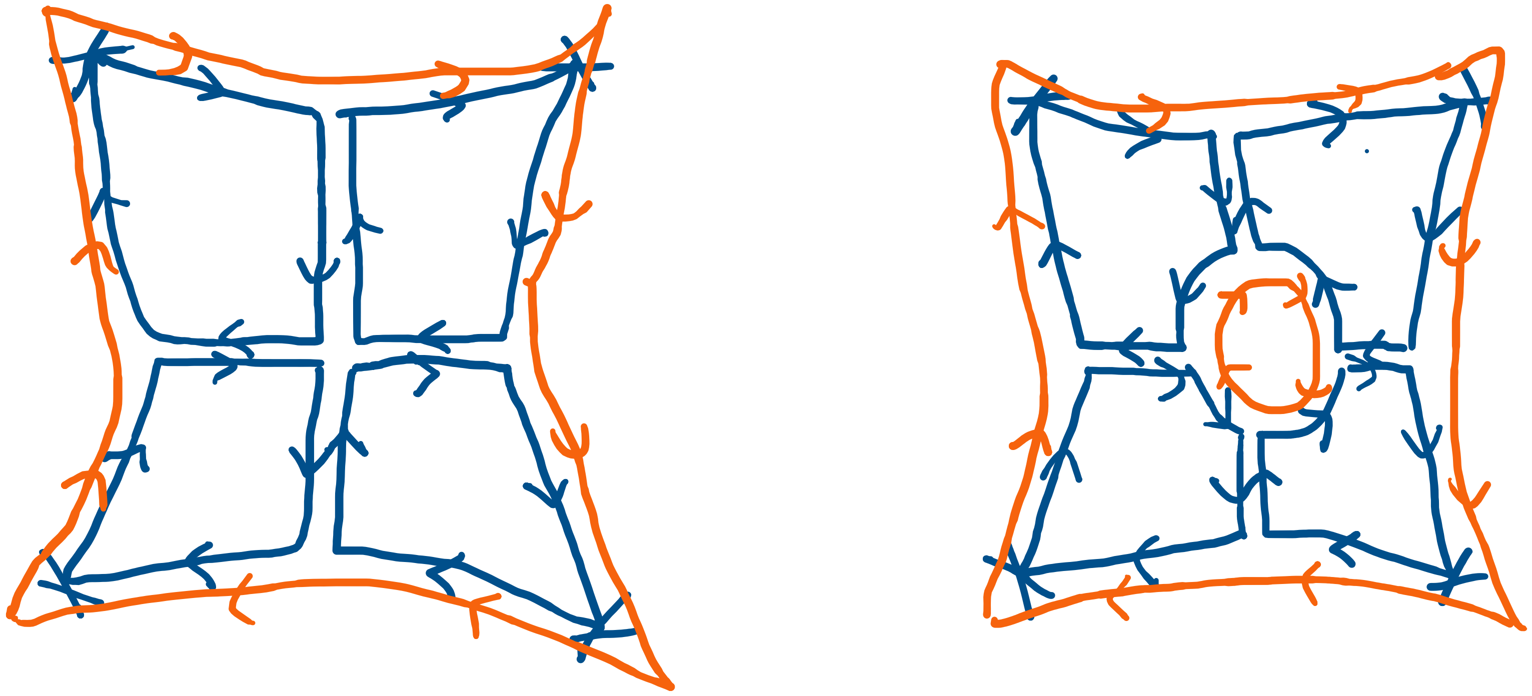

E. Corrigan & P. Ramond
Physics Letters B 87 73
1979

WHY 2AS?

Dimensions of representations of SU(N)

$N_{FUND} = N$ $\quad N_{ADJ} = N^2 - 1$ $\quad N_{2S} = \frac{N(N+1)}{2}$ $\quad N_{2AS} = \frac{N(N-1)}{2}$

E. Corrigan & P. Ramond
Physics Letters B 87 73
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WHY 2AS?

Dimensions of representations of SU(N)

$N_{FUND} = N$ $\quad N_{ADJ} = N^2 - 1$ $\quad N_{2S} = \frac{N(N+1)}{2}$ $\quad N_{2AS} = \frac{N(N-1)}{2}$

Fun fact for $N=3$

$N_{FUND} = 3 \quad , \quad N_{2AS} = \frac{3 \cdot 2}{2} = 3$

E. Corrigan & P. Ramond
Physics Letters B 87 73
1979

WHY 2AS?

Dimensions of representations of SU(N)

$$N_{FUND} = N \qquad N_{ADJ} = N^2 - 1 \qquad N_{2S} = \frac{N(N+1)}{2} \qquad N_{2AS} = \frac{N(N-1)}{2}$$

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they coincide

And 3 is already a very large number!

UNDERSTANDING TWO PREDICTIONS

From the lattice

UNDERSTANDING TWO PREDICTIONS

F. Sannino & M. Shifman
Phys. Rev. D 69 125004
2004
A. Armoni & E. Imeroni
Phys. Lett. B 631, 192
2005

Sannino & Shifman

Use a Veneziano-Yankielowicz effective theory and find (for N=3)

$$\frac{M_\eta}{M_\sigma} \lesssim 1 - \frac{22}{9\mathrm{N}} - \frac{4}{9}\beta\left(\frac{1}{\mathrm{N}}\right) \lesssim 0.185 + O(N^{-2})$$

$$\frac{M_\eta}{M_\sigma} \lesssim 0.290 \text{ (all-orders)}$$

(*) See the review by Francesco

Armoni & Imeroni

Use type 0' string theory

$$\frac{M_\eta}{M_\sigma} \sim \frac{N-2}{N} = 1 - \frac{18}{9\mathrm{N}} + O(N^{-2})$$

Lattice: Della Morte et. al.

$$\frac{M_\eta}{M_\sigma} = 0.356(54)$$

Della Morte et. al.
Phys. Rev. D 107, 11,
114506, 2023