AN INSIGHT ON:

EXACT EVALUATION OF LARGE-CHARGE CORRELATION FUNCTIONS IN NON-RELATIVISTIC CONFORMAL FIELD THEORY

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ABOUT THE PAPER

- arXiv:2403.18898v1 by Bean, Orlando and Reffert
- Evaluation of non-relativistic correlation function for both an ideal and near-unitarity Fermi gas in the large charge and conformal limit
- Evaluation of Schröedinger-breaking effects (.non conformality)

INTRODUCTION

EFFECTIVE THEORY APPROACH

The complexity of many physical system can be resolved expanding observables in terms of little parameters built of the typical scales involved

NEUTRON-NEUTRON SCATTERING

The low energy $(0.1 - 5\ MeV)$ neutron-neutron scattering has a large s-wave lenght scattering $(a \sim -19 fm)$ with a typical interaction lenght $r_0 \sim 2.75 fm$. $\frac{r_0}{a}$ is a little parameter to organize the perturbative expansion of physical observables

A NON-RELATIVISTIC CONFOMAL THEORY OF NEAR-UNITARITY GAS

At leading-order $a \to \infty$ and many-neutron systems can be treated as a unitary Fermi gas. Correlation functions are constrained by confomality. $a \to 0$ is the non-interacting conformal limit (ideal Fermi gas)

LARGE CHARGE OPERATORS

Correlation functions of large charge operator can be computed in the saddle-point approximation both in the ideal and unitary regime of the Fermi gas.

Useful for computations of nuclear low energy processes involving many neutrons

- Neutron-Neutron scattering at low energy can be studied using a NR CFT
- Schrödinger group: (4) Translations, (3) Rotations, (3) Galileian Boosts,
  (1) Scale transformations, (1) Special Transformations
- The goal is to compute the correlation functions, directly from the path integral using Large Charge Expansion
- Large Charge Expansion allows to use the semiclassical approximation (saddle-point approximation)

- Consider the Galileian invariant Low-Energy Lagrangian, with contact interaction

$$\mathcal{L} = \psi_\sigma^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi_\sigma - \frac{1}{2} C_0 (\psi_\sigma^\dagger \psi_\sigma)^2$$

- We want to study the fermion-fermion scattering

$$k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + O(k^4)$$

- a is the scattering length r is the effective range

$$C_0(\mu) = \frac{4\pi}{M} \frac{1}{1/a - \mu}$$

- Fixed points:

$C_0 = 0 \;\; (a = 0)$ $\qquad$ $C_0 = C_* \;\; (a \to \infty)$

Free particles
Non interacting fixed point

Unitarity:
Infinite Interaction Range

- At Unitarity we have a Non-Relativistic CFT describing a “Unitary Fermi gas”:

$$\mathcal{L}_{CFT} = \psi_\sigma^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi_\sigma + \frac{1}{C_\star} s^\dagger s + \psi_\downarrow^\dagger \psi_\uparrow^\dagger s + s^\dagger \psi_\uparrow \psi_\downarrow$$

- For an ideal Fermi gas: $E/N = \frac{3}{5}\frac{k_F^2}{2M}$
- At unitarity $E/N = \frac{3}{5}\frac{k_F^2}{2M}\xi$, ξ is the Bertsch parameter, determined experimentally
- At unitarity the Fermi gas is Superfluid. We write the Superfluid EFT in the IR
- In the Infrared the only physical degree of freedom is the phase of the condensate

$$\langle\psi\psi\rangle = |\langle\psi\psi\rangle|\, e^{-2i\theta} \longrightarrow X = D_t\theta - \frac{(\partial_i\theta)^2}{2M} , \quad D_t\phi = \dot{\theta} - A_0$$

- The Leading Order (LO) Lagrangian of fermions at unitarity is: $\mathcal{L}_{LO} = c_0\, M^{3/2} X^{5/2}$
- It can be seen that $\mathcal{L}_{LO}$ is invariant under the Schrödinger group

- The goal is to compute the two-point function using the semi-classical approximation

$$G(x_1,x_2) \;=\; -i\langle 0|T\left(\mathcal{O}_Q(x_1)\mathcal{O}_Q^\dagger(x_2)\right)|0\rangle \;=\; \int \mathcal{D}\phi\mathcal{D}\phi^\dagger\, \mathcal{O}_Q(x_1)\mathcal{O}_Q^\dagger(x_2)|e^{-\int \mathrm{d}^4x\mathcal{L}(x)}$$

- For the free theory we have $\mathcal{L} = \phi^\dagger\left[\partial_\tau - \frac{\vec{\nabla}^2}{2M} - \mu\right]\phi \qquad \mathcal{O}_Q = \mathcal{N}(\phi)^Q\ , \quad \mathcal{O}_Q^\dagger = \mathcal{N}(\phi^\dagger)^Q$

- Exponentiating we get the new effective action

$$S \;=\; \int \mathrm{d}^4x\,[\mathcal{L} \;-\; Q\log(\phi)\delta^4(x-x_1) \;-\; Q\log(\phi^\dagger)\delta^4(x-x_2)]$$

- We write the e.o.m and the solution

$$\phi^\dagger\left[\partial_\tau - \frac{\vec{\nabla}^2}{2M} - \mu\right]\phi \;=\; +Q\,\delta^4(x-x_2)\ \ ,\ \ \phi\left[\partial_\tau + \frac{\vec{\nabla}^2}{2M} + \mu\right]\phi^\dagger \;=\; -Q\,\delta^4(x-x_1)$$

$$\phi \;=\; i\,(Q)^{1/2}\,\frac{\mathcal{G}(x;x_2)}{\mathcal{G}(x_1;x_2)^{1/2}}\ \ ,\ \ \phi^\dagger \;=\; -i\,(Q)^{1/2}\,\frac{\mathcal{G}(x_1;x)}{\mathcal{G}(x_1;x_2)^{1/2}}$$

- We compute the action at the saddle point

$$S = Q - Q \log\left(-\frac{1}{2} Q\, \mathcal{G}(x_1; x_2)\right)$$

- Finally we find the two-point function

$$G(x_1, x_2) = \int \mathcal{D}\phi \mathcal{D}\phi^\dagger \, \mathcal{O}_Q(x_1) \mathcal{O}_Q^\dagger(x_2) \, e^{-\int \mathrm{d}^4x \mathcal{L}(x)} = \theta(\tau_{12}) \tau_{12}^{-3Q/2} \exp\left(-\frac{MQ\mathbf{x}_{12}^2}{2\tau_{12}}\right) \Psi_2 \, e^{\mu Q \tau_{12}}$$

- For $\mu = 0$ this is invariant under the Schrödinger group.

- The chemical potential gives symmetry breaking effects.

SCHRÖEDINGER SYMMETRY AND CONSTRAINS

Non-relativistic conformal transformations

$$\begin{cases} \boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{v}t, & t' = t \\ \boldsymbol{x}' = \boldsymbol{x} + s\boldsymbol{x}, & t' = t + 2st \\ \boldsymbol{x}' = \boldsymbol{x} - ct\boldsymbol{x}, & t' = t - ct^2 \end{cases}$$

Constrained two-point functions

$$-i\langle 0|O(x_1)O(x_2)|0\rangle = G(x_1; x_2)$$
$$= \theta(\tau_{12})\delta_{Q_1Q_2}\delta_{\Delta_1\Delta_2}\tau_{12}^{-\Delta_1}\exp[-MQ_1x_{12}^2/2\tau_{12}]\,\psi_1 + other\ time\ ordering$$

Constrained three-point functions

$-i\langle 0|O(x_1)O(x_2)O(x_3)|0\rangle$ depends on $v_{123} = \frac{1}{2}(\frac{x_{12}^2}{\tau_{12}} + \frac{x_{23}^2}{\tau_{23}} - \frac{x_{13}^2}{\tau_{13}})$

Many-point functions

Many-point functions becomes less and less constrained because one can build a lot of harmonic ratios

SADDLE POINT CONFIGURATION FOR THE UNITARITY FERMI GAS

$O = \mathrm{N}\,(\phi)^Q \qquad O^\dagger = N\left(\phi^\dagger\right)^Q$ large charge operators

To compute $\mathrm{G}(\mathrm{x}_1, \mathrm{x}_2) = \int D\phi\, D\phi^\dagger O_Q(x_1) O^\dagger(x_2) e^{-\int L_I\, d^4x}$ with $L_I = -\mathrm{c}_0 \mathrm{M}^{\frac{3}{2}} X^{\frac{5}{2}}$ we exponentiate, consider the effective action, recover the eom and substitute again in S_{eff}.

In the superfluid phase only the Goldstone boson is relevant and the operators can be written in terms of it

$$O = N\,\left(\frac{2}{\gamma}\right)^{\frac{\Delta}{2}} X^{\frac{\Delta}{2}} \exp[iQ\theta].$$

Taking A_0 we obtain the solution for the Goldstone field

$$\theta_s(\tau, \mathbf{x}) = \frac{i}{2}\gamma \log\left(\frac{\tau_1 - \tau}{\tau - \tau_2}\right) - \frac{i}{4} M \left[\frac{(\mathbf{x} - \mathbf{x}_2)^2}{(\tau - \tau_2)} - \frac{(\mathbf{x} - \mathbf{x}_1)^2}{(\tau_1 - \tau)}\right]$$

SADDLE POINT CONFIGURATION FOR THE SUPERFLUID PHASE

The solution can be expressed in term of the density of the system

$$\rho = \frac{5}{2} c_0 M^{3/2} X^{3/2}$$

where

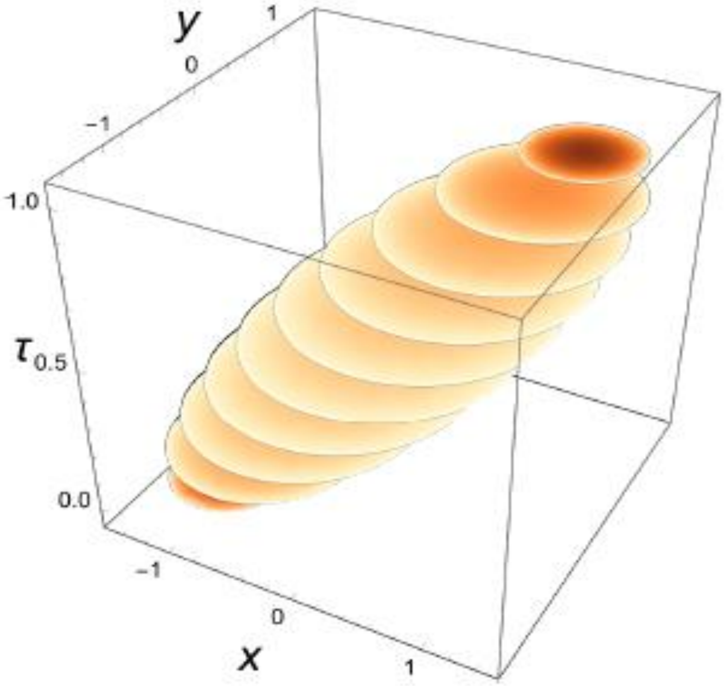
$$X = D_t\theta - \frac{(\partial_i\theta)^2}{2M}$$

We find that the density vanishes at

$$\bar{R}(\tau) = \sqrt{\frac{2\bar{\mu}}{M}}\frac{1}{\bar{\omega}} = \frac{2}{\sqrt{M}}\left(\gamma\frac{(\tau-\tau_2)(\tau_1-\tau)}{(\tau_1-\tau_2)}\right)^{1/2}$$

THE SUPERFLUID DROPLET

WE HAVE A PHYSICAL INTERPRETATION OF THE UNITARITY GAS IN THE PRESENCE OF LARGE CHARGE SOURCES



OVERVIEW

THE RESULTS ARE IN AGREEMENT WITH THE OPERATOR-STATE CORRESPONDENCE METHOD

NLO TERMS OF THE EXPANSION CAN BE INCLUDED

LARGE N AND LARGE CHARGE LIMIT