

Target group: PhD students and postdocs in mathematics

Course Title: **Microlocal Sheaf Theory**

Extend: 2 hours per week / 15 weeks / 5 ECTS

Aims The primary aim of this seminar is for students to learn the basics of microlocal sheaf theory. We will follow multiple texts, including “An Introduction to Symplectic Topology through Sheaf theory” by Claude Viterbo and “Quantitative Tamarkin Category” by Jun Zhang. Microlocal sheaf theory is a developing field in modern symplectic topology, so this seminar serves as preparation for students who wish to enter this area of study. It is likely that future seminars on topics in symplectic topology will apply techniques from this course. Another aim is for students to practice giving a math lecture. Students will learn how to manage time constraints during a talk and also how to field questions from the audience.

Synopsis We will begin the course with a review of classical sheaf theory, and then move on to the microlocal setting by defining the notion of microsupport. The seminar then splits into two threads.

The first of these is a study of the Tamarkin category. In this segment of the seminar, we will study propagation and prove the microlocal Morse lemma before defining the Tamarkin category. Then, we will investigate how many standard constructions in symplectic topology, such as generating functions and capacities, have analogues in the Tamarkin category. We will apply these techniques to prove the Gromov non-squeezing theorem. Having developed the basics of microlocal geometry, in the second parallel strand of this course, we explore symplectic resolutions and their quantizations.

We define (micro)differential operators, symplectic resolutions and their quantizations, and finish by covering the results of Braden, Proudfoot and Webster’s “Quantizations of Conical Symplectic Resolutions I: local and global structure.”

Evaluation method:

The evaluation will be based on an one hour presentation by each student in class (using the hours allocated for the course) on one of the topics covered in the course. The lecturer and an internal censor will be present during the talk to evaluate the presentation and ask the student questions. The evaluation will be pass/fail.

Teaching method: In person lectures

Responsible lecture(s) Ikshu Neithalath

Literature Tom Braden, Nicholas Proudfoot, and Ben Webster - *Quantizations of conical symplectic resolutions I: local and global structure*
 Pierre Schapira - *A short review on microlocal sheaf theory*
 Claude Viterbo - *An Introduction to Symplectic Topology through Sheaf theory*
 Jun Zhang- *Quantitative Tamarkin Category*