

Target group: PhD students and postdocs in mathematics

Course Title: **Advanced Topics in Geometry, Topology and Physics: Perverse sheaves, topological quantum field theory, and non-Abelian Hodge theory**

Extend: 2 hours per week / 15 weeks / 5 ECTS

Aims The primary aim of this seminar is for students to learn the basics of perverse sheaves, topological quantum field theory, and non-Abelian Hodge theory. We will follow multiple texts, including *Intersection homology and perverse sheaves* by B. Klingler, *Topological quantum field theory* by M. Aityah, and *Non-Abelian Hodge theory* by C. Simpson. This seminar serves as preparation for students who wish to learn more about each of these areas of study and will also serve to broaden their understanding of geometry, topology and physics. Another aim is for students to practice giving a math lecture. Students will learn how to manage time constraints during a talk and how to field questions from the audience.

Synopsis We will begin with a review of derived category and constructible sheaves, then move to perverse t-structure and the definition of perverse sheaves. As an example, we will describe the category of perverse sheaves over the punctured disk in terms of very explicit linear algebra data. Next, we move to intersection homology, which is a topological description of simple objects in the category of perverse sheaves. After explaining semi-simpleness of the Abelian category of perverse sheaves, we can have a well understanding of structure of perverse sheaves. Last, we will explain the Beilinson-Bernstein-Deligne-Gaber decomposition theorem that plays important roles in many fields of mathematics.

Then, we will introduce topological quantum field theory as a tool for computing topological invariants of manifolds. We will introduce geometric quantization, especially in the case of Chern-Simons theory, and discuss the volume conjecture which relates Chern-Simons theory with complex gauge group $SL(2, \mathbb{C})$ of a knot complement to the colored Jones polynomials of the knot.

Finally, we will understand the non-abelian Hodge correspondence. Its central statement is that two a priori very different algebraic varieties, namely the moduli space of representations of the fundamental group of a Riemann surface X in a group G and the moduli space of G -Higgs bundles over X , are homeomorphic.

Evaluation method:

The evaluation will be based on a one-hour presentation by each student in class (using the hours allocated for the course) on one of the topics covered in the course. The lecturer and an internal censor will be present during the talk to evaluate the presentation and ask the student questions. The evaluation will be pass/fail.

Teaching method: In person lectures

Responsible lecture(s) Guillermo Ayuso, Greyson Potter, Bingyu Zhang

Literature 1. Perverse Sheaves

- A. Dimca – *Sheaves in topology*
- P. N. Achar – *Perverse Sheaves and Applications to Representation Theory*
- G. Williamson – *An illustrated guide to perverse sheaves*
- F. Kirwan and J. Woolf – *An Introduction to Intersection Homology Theory*

J. Schurmann – *Topology of singular spaces and constructible sheaves*

B. Klingler – *Intersection homology and perverse sheaves*

2. Topological quantum field theory

K. Wernli – *Six Lectures on Geometric Quantization*

M. Atiyah – *Topological quantum field theory*

T. Dimofte and S. Gukov – *Quantum field theory and the volume conjecture*

3. Non-Abelian Hodge theory

C. Simpson – *Non-Abelian Hodge theory*

N. Hitchin – *The self-duality equations on a Riemann surface*