

Target group: PhD students and postdocs in mathematics & mathematical physics

Course Title: Mathematical aspects of 2d Mirror Symmetry

Extend: 2 hours per week / 15 weeks / 5 ECTS

Aims The aim of this seminar is for students to learn the basis of mathematical aspects of 2d Mirror Symmetry, especially the Homological Mirror Symmetry (HMS) conjecture. We will follow multiple texts, including “Mirror symmetry” by Kentaro Hori and “Dirichlet branes and mirror symmetry” by Paul S. Aspinwall, et al. 2d Mirror symmetry, which originally appears as the T-duality between IIA and IIB type superstring theory, is a main research topic in modern symplectic geometry and algebraic geometry, the seminar will serve as preparation for students who wish to enter this area of study. The content of the seminar fits into thesis topics of some of PhD students and will be useful for all PhD students in their further research. Another aim is for students to practice giving a math lecture. Students will learn how to manage time constraints during a talk and also how to field questions from the audience.

Synopsis After an overview for physical origin for related directions in mirror symmetry, we will begin the course with a review of basic symplectic geometry and algebraic geometry we need, including derived categories of coherent sheaves and Fukaya categories.

Next, we move to explain some of the known examples that the homological mirror symmetry conjecture is true. They will include Calabi-Yau hypersurfaces in P^n (Seidel, Sheridan), weighted projective plane (Auroux), some toric Fano manifold (Gammage-Shende). Meanwhile, we will also explain some structure results about the derived category of coherent sheaves (Bondal-Orlov) as ingredients for proving HMS.

Evaluation method:

The evaluation will be based on a one-hour presentation by each student in class (using the hours allocated for the course) on one of the topics covered in the course. The lecturer and an internal censor will be present during the talk to evaluate the presentation and ask the student questions. The evaluation will be pass/fail.

Teaching method: In person lectures

Responsible lecture(s) Bingyu Zhang

Literature -Overviews on mirror symmetry:

D. A. Cox, S. Katz, Mirror symmetry and algebraic geometry. 1999.

Kentaro Hori, Mirror symmetry. 2003.

Paul S. Aspinwall, et al., Dirichlet branes and mirror symmetry. 2009.

Anton Kapustin, Dmitri Orlov, Lectures on Mirror Symmetry, Derived Categories, and D-branes. 2003.

-Derived category of coherent sheaves on varieties:

R. P. Thomas, Derived categories for the working mathematician. 2001.

Alexei Bondal, Dmitri Orlov, Derived categories of coherent sheaves. [math.AG/0206295](#).

Dmitri Orlov, Derived categories of coherent sheaves and motives. [math.AG/0512620](#).

-Fukaya categories:

Paul Seidel, Fukaya Categories and Picard–Lefschetz Theory. 2008.

Denis Auroux, A beginner’s introduction to Fukaya categories. [math.SG/1301.7056](#).

-Mathematical proofs of homological mirror symmetry:

Paul Seidel, Homological mirror symmetry for the quartic surface. 2006.

Denis Auroux, Ludmil Katzarkov and Dmitri Orlov, Mirror symmetry for weighted projective planes and their noncommutative deformations. 2008.

Nick Sheridan, Homological mirror symmetry for Calabi–Yau hypersurfaces in projective space. 2015.

Benjamin Gammage, Vivek Shende, Mirror symmetry for very affine hypersurfaces. 2017.

Vivek Shende, Toric mirror symmetry revisited. 2021.