

Target group PhD students in Mathematics or/and theoretical Physics.

Course Title Introduction to the Integrable systems and examples

Extent 4 hours per week, 16 weeks, 8 ECTS credits

Aims The aim is for the students to get an understanding of integrable systems. The students are to understand various approaches to the subject and their interrelations. The contents of the course will be adjusted to the students' pre knowledge. We will rather have the students understand a few concepts really well than being introduced to many concepts but not understanding them. To make the students well familiar with the concepts, we will ask them to do many small calculations themselves during the lectures. Depending on the background of the students we can invest more time at the beginning to understand the basics or put more time into the further applications of physics or advanced mathematical topics at the end.

Synopsis The field of integrable systems is born together with Classical Mechanics, with a quest for exact solutions to Newton's equations of motion. It turned out that apart from the Kepler problem solved by Newton himself, after two centuries of hard investigations, only a handful of other cases were found. In the 19th century, Liouville finally provided a general framework characterizing the cases where the equations of motion are "solvable by quadratures". This course provides a thorough introduction to the theory of integrable systems, discussing the various approaches to the subject and explaining their interrelations. The course begins by introducing the central ideas of the theory of integrable systems, based on Lax representations, loop groups and Riemann surfaces. These ideas are then illustrated with detailed studies of model systems. The connection between isomonodromic deformation and integrability is discussed, and integrable field theories are covered. The KP, KdV and Toda hierarchies are explained using the notion of Grassmannian, vertex operators and pseudo-differential operators. We also explain about the inverse scattering method and we cover the necessary mathematical tools from symplectic geometry, Riemann surfaces and Lie algebras.

Evaluation method

The evaluation will be based on a 45 min. presentation by each student in class (using the hours allocated for the course) on a topic related to the course, for which minimum one of the lecturers as well as an internal censor will be present to evaluate the presentation and ask the student questions.

Teaching method

In person lectures including exercise classes.

Responsible lecturer(s)

Jørgen Ellegaard Andersen & Nezhla Aghaei.

Literature

- [1] Babelon, Bernard and Talon: Integrable systems and applications
- [2] Faddeev, Takhtajan: Hamiltonian methods in the Theory of Solitons
- [3] Gleb Arutyunov: Elements of Classical and Quantum Integrable Systems
- [4] Jens Hoppe: Lectures on integrable systems