

Target group: PhD students in Mathematics or/and theoretical Physics.

Course Title: Introduction to the theory of deformation quantization modules, Part II

Extend: 2 hours per week / 14 weeks / 5 ECTS

Aims The aim is for the students to get further understanding of deformation quantization modules after the first part of the course.

The contents of the course will be adjusted to the students' pre knowledge. We will rather have the students understand a few concepts well than being introduced too many concepts but not understanding them. Depending on the background of the students we can invest more time at the beginning to understand the basics or put more time into the further applications of advanced mathematical topics at the end.

Synopsis The theory of deformation modules can be seen as an introduction to non-commutative complex analytic geometry and to the study of micro differential systems. Indeed, on a complex manifold, we replace the structure sheaf $\mathcal{O}$ with a formal deformation of it, a DQ-algebroid, and study modules over this ring. This generalizes classical results of Cartan-Serre and Grauert. In another direction, this also generalizes the theory of D-modules.

Based on the basic materials covered last semester, we move on to more advanced topics. We begin our study with the theory of Hochschild homology in the setting of DQ-modules. Then we will cover two examples. In the commutative case, we will see that the theory of DQ-modules recovers the more familiar results for $\mathcal{O}$ -modules. The next example is the symplectic case which is one of the inspiring sources of the theory. An important class among these are the holonomic modules and we will see that they share a close relation with Lagrangian submanifolds.

We will follow the foundation notes, deformation quantization modules, by Kashiwara and Schapira.

Evaluation method:

The evaluation will be based on an one hour presentation by each student in class (using the hours allocated for the course) on one of the topics covered in the course. The lecturer and an internal censor will be present during the talk to evaluate the presentation and ask the student questions.

Teaching method: In person lectures

Responsible lecture(s) Christopher Kuo & Vivek Shende

Literature [1] Kashiwara and Schapira: Deformation quantization modules