

Target group: PhD students in Mathematics or/and theoretical Physics.

Course Title: Floer homology and the Fukaya category

Extend: 2 hours per week / 14 weeks / 5 ECTS

Aims The primary aim of this seminar is for students to learn the basics of Floer homology and the Fukaya category.

We will follow multiple texts, and mainly use “A beginner’s introduction to Fukaya categories” by Denis Auroux. The Fukaya category is an essential ingredient in modern symplectic topology, so this seminar serves as preparation for students who wish to enter the field. It is likely that future seminars on topics in symplectic topology will apply techniques from this course. Another aim is for students to practice giving a math lecture. Students will learn how to manage time constraints during a talk and further how to field questions from the audience.

Synopsis We will begin the course with a review of finite dimensional Morse theory including the construction of the Morse complex. We will then turn to the infinite dimensional setting of Floer theory and use analogous methods to construct the Floer complex. We will prove that the differential on the Floer complex squares to zero under certain hypotheses. We will then describe the Fukaya category of a symplectic manifold and define the A^∞ structure on this category. We will show how one can identify sets of generating objects for the Fukaya category and use these methods to explicitly compute the category in some simple examples. We will then define the wrapped Fukaya category. The final part of the course will consist of special topics depending on the students’ interests. Examples of possible topics include mirror symmetry, monopole Floer homology, Heegaard-Floer homology, and embedded contact homology.

Evaluation method:

The evaluation will be based on an one hour presentation by each student in class (using the hours allocated for the course) on one of the topics covered in the course. The lecturer and an internal censor will be present during the talk to evaluate the presentation and ask the student questions.

Teaching method: In person lectures

Responsible lecture(s) Ikshu Neithalath

Literature

- [1] Auroux, Denis - A beginner’s introduction to Fukaya categories
- [2] Hutchings, Michael - Lecture notes on Morse homology
- [3] Smith, Ivan - A symplectic prolegomenon
- [4] Seidel, Paul - Fukaya Categories and Picard-Lefschetz Theory