

Target group: PhD students and postdocs in Mathematics

Course Title:

Introduction to the theory of J-holomorphic curves, Part I

Extent: 2 hours per week, 14 weeks, 5 ECT credit

Aims: The primary aim of this seminar is for students to learn the analytical foundations of the modern theory of J-holomorphic curves. We will closely follow Chapters 2 and 3 of the text "Lectures on Holomorphic Curves in Symplectic and Contact Geometry" by Chris Wendl. This material is a prerequisite for the second part of this seminar, which will cover Chapters 4 and 5 of the same text. Furthermore, the subject of J-holomorphic curves is an essential ingredient in modern symplectic topology, so this seminar serves as preparation for students who wish to enter the field. Future seminars on topics in symplectic topology will rely upon the results of this course. Another aim is for students to practice giving a math lecture. Students will learn how to manage time constraints during a talk and also how to field questions from the audience.

Synopsis: We begin with a short introduction of the basic elements of symplectic geometry and almost complex structures. The majority of this course is focused on the analysis of solutions to the non-linear Cauchy-Riemann equation with respect to an arbitrary almost complex structure. We will prove elliptic regularity and use it to demonstrate that solutions of this equation are smooth. This requires that we introduce Sobolev spaces and the theory of elliptic partial differential equations. Next, we apply the regularity result to establish various properties of J-holomorphic curves. For example, we prove the similarity principle, unique continuation, local existence, and positivity of intersections for J-holomorphic curves. After these local considerations, we turn to the global theory of J-holomorphic curves. We show that the linearization of the Cauchy-Riemann operator at a J-holomorphic curve is Fredholm and compute its index, thereby establishing the Riemann-Roch formula.

Evaluation method:

The evaluation will be based on an one hour presentation by each student in class (using the hours allocated for the course) on a topic related to the course. The lecturer and an internal censor will be present to evaluate the presentation and ask the student questions.

Teaching method:

In person lectures, where the students are encouraged to give at least one talk presenting material from the literature.

Responsible lecturer:

Ikshu Neithalath

Literature:

Wendl, Chris - *Lectures on Holomorphic Curves in Symplectic and Contact Geometry*