

Target group: PhD students in Mathematics or/and theoretical Physics.

Course Title:

Deformation quantization modules

Extend: 2 hours per week, 15 weeks, 5 ECTS credits

Aims The aim is for the students to get an understanding of deformation quantization modules.
The contents of the course will be adjusted to the students' pre knowledge. We will rather have the students understand a few concepts well than being introduced too many concepts but not understanding them. Depending on the background of the students we can invest more time at the beginning to understand the basics or put more time into the further applications of advanced mathematical topics at the end.

Synopsis The theory of deformation modules can be seen as an introduction to non-commutative complex analytic geometry and to the study of micro differential systems. Indeed, on a complex manifold, we replace the structure sheaf $\mathcal{O}$ with a formal deformation of it, a DQ-algebroid, and study modules over this ring. This generalizes classical results of Cartan-Serre and Grauert. In another direction, this also generalizes the theory of D-modules.
We begin this course with the theory of (formal) deformation of a sheaf of rings and the modules over it. Then we globalize the results to modules over algebroids. In particular, we will be focus on DQ-algebroids, those locally come from a deformation of the structure sheaf $\mathcal{O}$ by star-products. Finally, we mention the theory of kernel in this setting, which provides the natural framework for functors between modules. We will follow the foundation notes, deformation quantization modules, by Kashiwara and Schapira.

Evaluation method:

The evaluation will be based on an one hour presentation by each student in class (using the hours allocated for the course) on a topic related to the course. Minimum one of the lecturers as well as an internal censor will be present to evaluate the presentation and ask the student questions.

Teaching method: In person lectures

Responsible lecture(s) Vivek Shende and Christopher Kuo

Literature [1] Kashiwara and Schapira: Deformation quantization modules