## Abstract

This thesis comprises of four works done during the PhD studies of the author. In the first part, we extend Chiral Perturbation Theory ( $\chi$ PT), a successful method to describe low-energy quantum chromodynamics (QCD), to include a light isosinglet scalar state. Such a framework is needed in cases where one is interested in describing the dynamics involving energy scales near or around the isosinglet state, or if the pseudo-Goldstone boson masses are comparable to the scalar mass. The framework is applied to various physical realizations of the isosinglet, such as the dilaton, the (pseudo) Goldstone boson, the  $\sigma$  state in QCD, and the linear sigma model.

Secondly, we investigate the phenomenological viability of a recently proposed class of composite Dark Matter (DM) models where the relic density is determined by  $3 \rightarrow 2$  number-changing processes in the dark sector. Here the dark pions of the strongly interacting field theory constitute the DM particles. This newly proposed DM paradigm is called Strongly Interacting Massive Particle (SIMP). This part discusses the results of two separate works. In the first, we perform a consistent next-to-leading and next-to-next-to-leading order investigation of the SIMP using  $\chi$  PT. Our work demonstrates that a leading order analysis cannot be used to draw conclusions about the feasibility of the model. We further show that higher order corrections substantially increase the tension with phenomenological constraints challenging the viability of the simplest realization of the SIMP paradigm. In the second work, we introduce the hidden sector SIMP, where the strongly interacting dark sector has, at most, feeble interactions with the Standard Model particles and evolves with its own separate thermal history. This model provides an attractive model building framework for self-interacting DM, and allows, contrary to the standard SIMP, for a controlled perturbative treatment within  $\chi$ PT.

In the last part of the thesis, we determine the phase diagram of completely asymptotically free  $SU(N_c)$  gauge theories featuring  $N_s$  complex scalars and  $N_f$  Dirac quarks transforming according to the fundamental representation of the gauge group.