Information index augmented eRG to model vaccination behaviour: A case study of COVID-19 in the US

Based on *arXiv:2407.20711v1* , in collaboration with B. Buonomo, R. Della Marca and F. Sannino

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Overview

1. Introduction to the framework
2. eRG: a review
3. BeRG
4. Application to the COVID-19 case in the US
5. Part I: The US vaccination campaign as BeRG test-bed
6. Part II: Testing the validity of the BeRG

Introduction to the framework

Framework

renormalization
group approach
in high energy physics

Framework

renormalization
group approach
in high energy physics

the beta function β governs
the time (inverse energy)
variation
of the interaction strength
among fundamental particles

Framework

renormalization
group approach
in epidemics

Framework

β is an epidemic strength function

renormalization group approach in epidemics

Framework

renormalization group approach in epidemics

β is an epidemic strength function

whose derivative with respect to time
provides a new quantity: the beta-function of an underlying microscopic model.

Framework

renormalization
group approach
in epidemics

β is an epidemic strength function

whose derivative with respect to time
provides a new quantity: the beta-function of an underlying microscopic model.

it governs the time (inverse energy) dependence of the social interactions.

Toy model

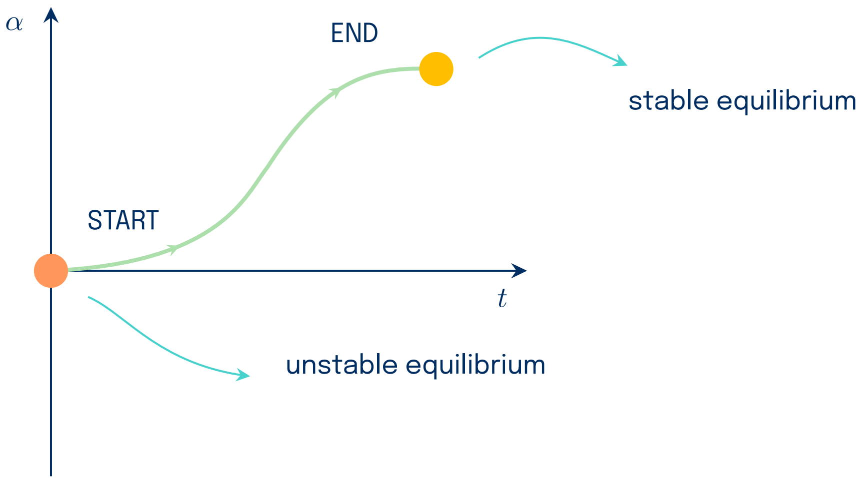
END

stable equilibrium

START

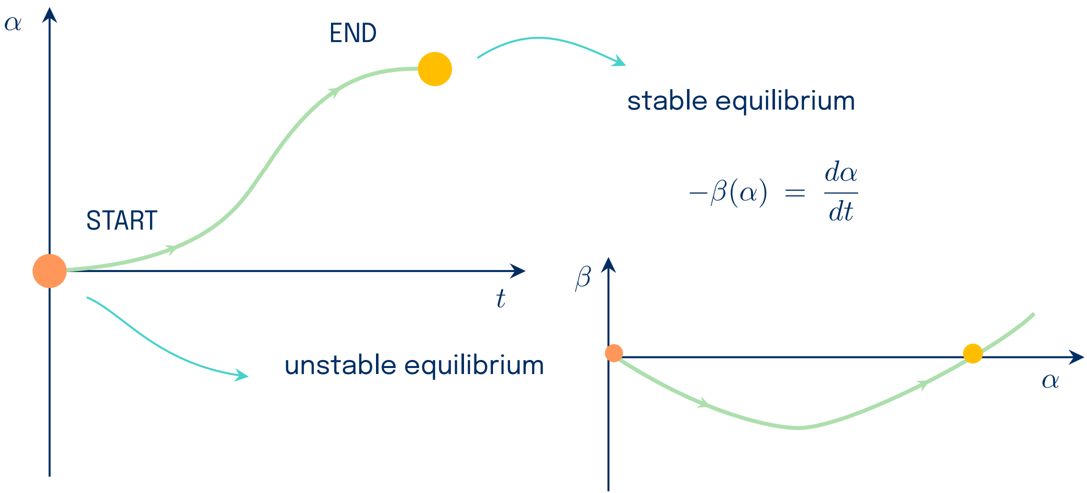
unstable equilibrium

Toy model



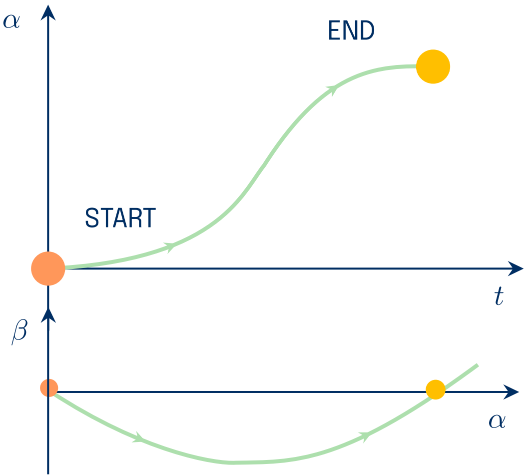
α
END
START
t
stable equilibrium
unstable equilibrium

Toy model



$$-\beta(\alpha) = \frac{d\alpha}{dt}$$

eRG: a review

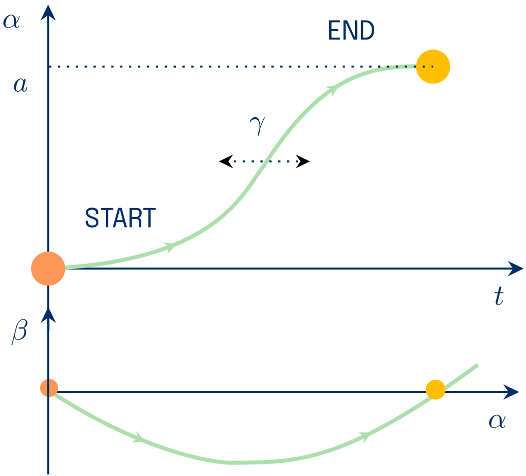


β may be interpreted as the time variation of the *epidemic strength* $\alpha(t)$

$$-\beta(\alpha) = \frac{d\alpha}{dt} \tag{1}$$

where $\alpha(t)$ is a measure of the interactions among individuals leading to infections from the beginning of the epidemic wave up to the time t

eRG



The quantity α is typically assumed to be the slowly varying function:

$$\alpha(t) = \ln \mathcal{I}(t), \tag{2}$$

where $\mathcal{I}(t)$ is the cumulative number of infectious individuals

Two parameters to describe a single wave:

- a : the total number of cases at the end of the wave.
- γ : the growth rate of the wave.

eRG: dictionary

A single wave is seen as a flow between two fixed points representing:

- the beginning of the epidemic wave, where $\alpha = 0$

and

- the end of the epidemic wave, when the final size of epidemic strength is reached, i.e. $\alpha = a$

the (approximate) temporal symmetries of the problem yields

$$-\beta(\alpha) \equiv \frac{d\alpha}{dt} = \gamma\, \alpha \left(1 - \frac{\alpha}{a}\right) \tag{3}$$

that captures the *initial* and *final* temporal scale invariance of a single epidemic outbreak

eRG: vaccine implementation

vaccination campaign

eRG: vaccine implementation

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vaccination
campaign

aims to

reduce the total number
of infected individuals: a

aims to

reduce the growth rate
of the epidemic wave: γ

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the parameters a and γ evolve in time

eRG: vaccine implementation

vaccination campaign

aims to

reduce the total number of infected individuals: a

aims to

reduce the growth rate of the epidemic wave: γ

the parameters a and γ evolve in time

they should decrease in time

eRG: vaccine implementation

$$\frac{d\gamma}{dt} = -\boxed{c}\;\gamma(t_v)\,, \qquad \frac{dA}{dt} = -\boxed{c}\;(A - e^{\alpha})\,. \tag{4}$$

with t_v the initial time of the vaccination campaign

- $A = e^a$
- c the fraction of vaccinated individuals in the unit of time

$$\begin{aligned} \frac{d\alpha}{dt} &= \gamma\alpha\left(1 - \frac{\alpha}{\log A}\right) \\ \frac{d\gamma}{dt} &= -c\,\gamma(t_v) \\ \frac{dA}{dt} &= -c\,(A - e^{\alpha}) \end{aligned} \tag{5}$$

eRG: travellers

in any realistic description of a pandemics

exchange across different regions

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exchange across different regions

the corresponding
waves
are influenced

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introduction of an interaction term affecting the epidemic strength

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introduction of an interaction term affecting the epidemic strength

coupling between the different regions

eRG: interaction term

W regions that interact

$$\sum_{j \neq i}^{W} \frac{K_{ij}}{n_{mi}} \left(e^{\alpha_j - \alpha_i} - 1\right) \tag{6}$$

$K_{ij} \in \mathbb{R}$ is a measure for the number of travellers between the populations i and j

system of W coupled differential equations:

$$\begin{aligned} \frac{d\alpha_i}{dt} &= \gamma_i \alpha_i \left(1 - \frac{\alpha_i}{\log A_i}\right) + \sum_{j \neq i}^{W} \frac{K_{ij}}{n_{mi}} \left(e^{\alpha_j - \alpha_i} - 1\right) \\ \frac{d\gamma_i}{dt} &= -c\,\gamma_i(t_v) \\ \frac{dA_i}{dt} &= -c\,\left(A_i - e^{\alpha_i}\right) \end{aligned} \tag{7}$$

BeRG

Social alarm and human behaviour

Goal

Social alarm

some people are
afraid
and get vaccinated

does it affect
the dynamics of
epidemics?

Information index

we introduce the function $information\ index$ taking in consideration:

1. the $information - induced\ perception$ regarding the status of the disease
2. the $memory$ of past spread

$$M(t) = \int_{-\infty}^{t} k\ \mathcal{I}(\tau)\ H(\tau)\theta e^{-\theta(t-\tau)} d\tau \tag{8}$$

H is the Heaviside step function

Information index II

$$M(t) = \int_{-\infty}^{t} \boxed{k}\ \mathcal{I}(\tau) H(\tau) \theta e^{-\boxed{\theta}(t-\tau)} d\tau \tag{9}$$

1. $k > 0$ is the $\boxed{information\ coverage}$:
   - $0 < k < 1$ disease under-reporting and leads to underestimate the number of ill cases in the community
   - $k > 1$ disease over-reporting and leads to overestimate the number of ill cases in the community

2. $\boxed{1/\theta}$ is the characteristic memory length: average time delay in the collection of the information on the disease

Information index III

$$M(t) = \int_{-\infty}^{t} k\,\mathcal{I}(\tau)H(\tau)\theta e^{-\theta(t-\tau)}d\tau \tag{10}$$

results in a differential equation for M:

$$\frac{M(t)}{dt} = \theta\left(ke^{\alpha} - M\right) \tag{11}$$

impact of the human
behaviour
on the vaccination campaign

the time variations of γ and
a are affected by M

upgrade c
to be $c(M)$

Impact on vaccinated individuals

upgrade of the original constant c to be a function of the information index $c(M)$

$$c(M) = c_0 + c_1(M) \tag{12}$$

Impact on vaccinated individuals

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- fraction of individuals who get vaccinated independently of the information

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- fraction of individuals who get vaccinated independently of the information
- fraction of individuals who get vaccinated because of the social alarm caused by the spreading information on the disease

Impact on vaccinated individuals

upgrade of the original constant c to be a function of the information index $c(M)$

$$c(M) = \boxed{c_0} + c_{1,\max} \frac{\boxed{D}\, M}{1 + \boxed{D}\, M} \tag{12}$$

- fraction of individuals who get vaccinated independently of the information
- fraction of individuals who get vaccinated because of the social alarm caused by the spreading information on the disease
- $D \geq 0$, the *reactivity*, measuring how quickly the individuals react to information

BeRG equations

We therefore arrive at the following system of equations:

$$
\begin{aligned}
\frac{d\alpha_i}{dt} &= \gamma_i \alpha_i \left(1 - \frac{\alpha_i}{\log A_i}\right) + \sum_{j \neq i}^{W} \frac{K_{ij}}{n_{mi}} \left(e^{\alpha_j - \alpha_i} - 1\right) \\
\frac{d\gamma_i}{dt} &= -c(M_i)\gamma_i(t_v) \\
\frac{dA_i}{dt} &= -c(M_i)\left(A_i - e^{\alpha_i}\right) \\
\frac{dM_i}{dt} &= \theta_i(k_i e^{\alpha_i} - M_i).
\end{aligned}
\tag{13}
$$

The above defines the Behavioural epidemiological Renormalisation Group (BeRG) including the vaccination impact that can be used for any pandemic across the world

BeRG parameters

already included in eRG:

- a : the total number of cases at the end of the wave.
- γ : the growth rate of the wave.

new parameters:

- $k\ (information\ coverage)$: balance between the disease under-reporting and the media and rumours amplification of the social alarm
- θ : rate at which memory of past is retained in individuals
- $D\ (reactivity\ parameter)$: how quickly the individuals react to information

Application to the COVID-19 case in the US

History

Part I: The US vaccination campaign as BeRG test-bed

History and Plan

we have official data
of infected people

History and Plan

STEP 1
eRG fit

we have official data
of infected people

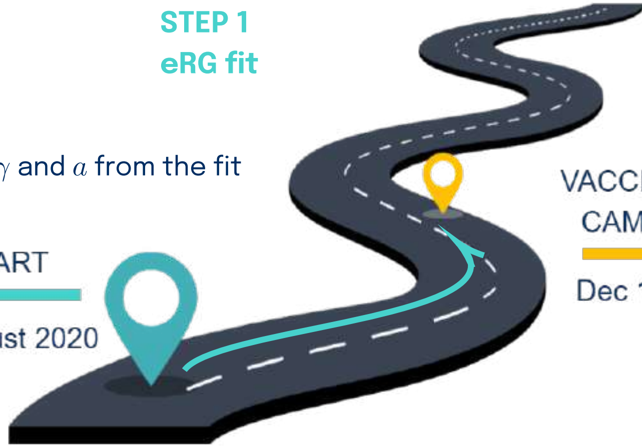
History and Plan

STEP 1
eRG fit

γ and a from the fit

START

August 2020



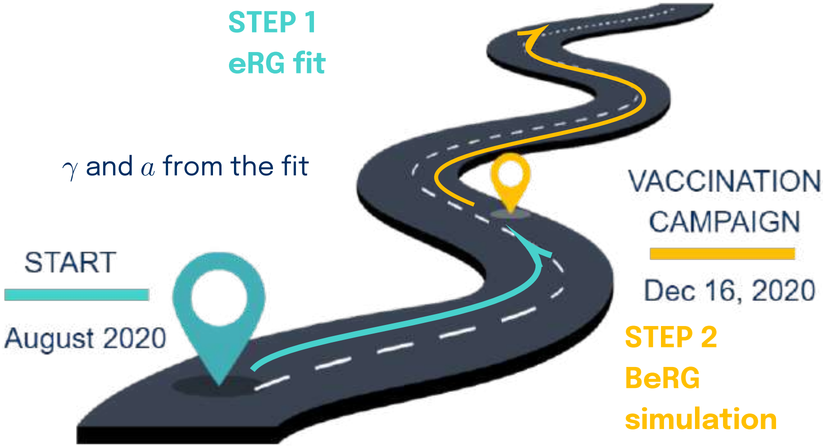
VACCINATION CAMPAIGN

Dec 16, 2020

History and Plan

STEP 1
eRG fit

γ and a from the fit



STEP 2
BeRG
simulation

Step 1: eRG to obtain a and γ

COVID-19 in the US

Step 1: eRG to obtain a and γ

COVID-19 in the US

study of the second wave
started in August 2020

Step 1: eRG to obtain a and γ

COVID-19 in the US

study of the second wave started in August 2020

eRG until vaccination campaign starts on December 16, 2020

Step 1: eRG to obtain a and γ

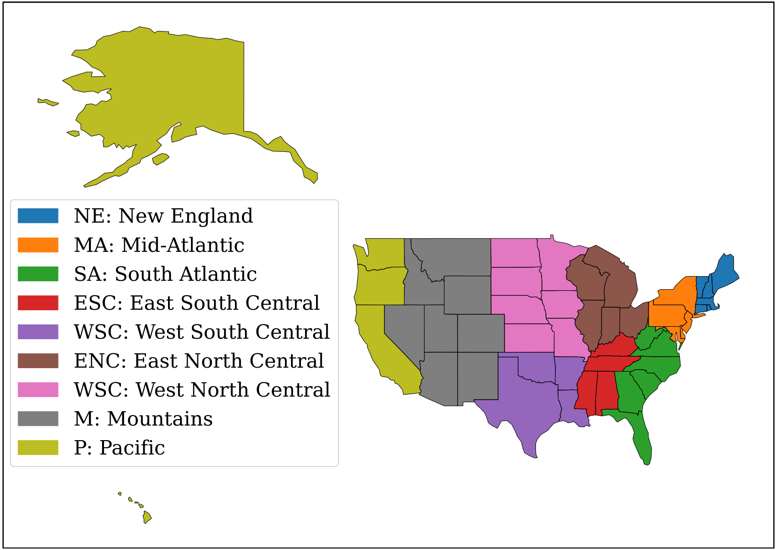
COVID-19 in the US

study of the second wave started in August 2020

eRG until vaccination campaign starts on December 16, 2020

Fit a and γ using the cumulative infected data

The US census divisions



The US are separated into 9 different regions with similar cultural area and populations

Study of the second wave

COVID-19 in the US

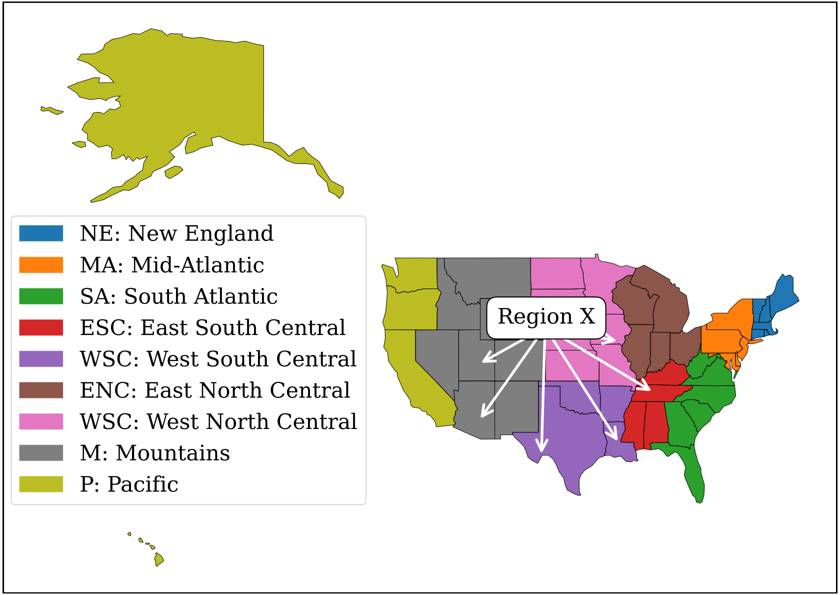
study of the second wave
started in August 2020

as a source-start

Divisions that experienced a surge
in COVID-19 cases during
July and August 2020

define Region-X as
the their average sum

Region X


NE: New England
MA: Mid-Atlantic
SA: South Atlantic
ESC: East South Central
WSC: West South Central
ENC: East North Central
WSC: West North Central
M: Mountains
P: Pacific
Region X

Region X

Region-X

made of

an average sum of all the divisions with a pandemic peak occurring in the July–August period

SA, ESC, WSC, ENC, WNC, M, P

Methodology

optimization process to find a and γ comparing with the official data

$$\frac{d\alpha_i}{dt} = \gamma_i \alpha_i \left(1 - \frac{\alpha_i}{a_i}\right) + \sum_{j \neq i}^{9} \frac{K_{ij}}{n_{mi}} \left(e^{\alpha_j - \alpha_i} - 1\right) + \sum_{i}^{9} \frac{K_{R_X i}}{n_{R_X i}} \left(e^{\alpha_{R_X} - \alpha_i} - 1\right) \tag{14}$$

Methodology

optimization process to find a and γ comparing with the official data

$$\frac{d\alpha_i}{dt} = \gamma_i \alpha_i \left(1 - \frac{\alpha_i}{a_i}\right) + \boxed{\textstyle\sum_{j \neq i}^{9} \frac{K_{ij}}{n_{mi}} \left(e^{\alpha_j - \alpha_i} - 1\right)} + \textstyle\sum_{i}^{9} \frac{K_{R_X i}}{n_{R_X i}} \left(e^{\alpha_{R_X} - \alpha_i} - 1\right) \qquad (14)$$

- interactions among the 9 divisions using flight data for K_{ij}

Methodology

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$$\frac{d\alpha_i}{dt} = \gamma_i \alpha_i \left(1 - \frac{\alpha_i}{a_i}\right) + \boxed{\sum_{j \neq i}^{9} \frac{K_{ij}}{n_{mi}} \left(e^{\alpha_j - \alpha_i} - 1\right)} + \boxed{\sum_{i}^{9} \frac{K_{R_X i}}{n_{R_X i}} \left(e^{\alpha_{R_X} - \alpha_i} - 1\right)} \tag{14}$$

- interactions among the 9 divisions using flight data for K_{ij}
- interactions between the 9 divisions and the Region-X to model the source of the start of the second wave

eRG fit to obtain a and γ

$$\frac{d\alpha_i}{dt} = \gamma_i\,\alpha\left(1-\frac{\alpha_i}{a_i}\right)$$

$$+\sum_{j\neq i}^{9}\frac{K_{ij}}{n_{ij}}\left(e^{\alpha_j-\alpha_i}-1\right)$$

$$+\sum_{i}^{9}\frac{K_{R_X i}}{n_{R_X i}}\left(e^{\alpha_{R_X}-\alpha_i}-1\right)$$

a and γ using the cumulative infected data

Second wave parameters			
Division	**Code**	a	γ
New England	NE	11.006	0.214
Mid-Atlantic	MA	10.882	0.206
South Atlantic	SA	10.885	0.185
East South Central	ESC	11.201	0.207
West South Central	WSC	10.713	0.213
East North Central	ENC	11.074	0.250
West North Central	WNC	11.060	0.263
Mountains	M	11.089	0.213
Pacific	P	11.535	0.171

Table: Parameters of the eRG model for the second wave in the 9 divisions and their baseline values, chosen to reproduce the data until December 16, 2020

STEP 2: BeRG implementation

study vaccination campaign to model the dynamics from December 16, 2020 on

STEP 2: we move to BeRG

Find the BeRG parameters: D, θ, k

D and θ parameters

For each census division we establish that

$$c(M) = c_0 + c_{1,\max} \frac{D\,M}{1 + D\,M}, \tag{15}$$

- $c_0 = 0.0064$ weeks^{-1} since 0.64% of the total population was vaccinated as of December 28, 2020
- $c_{1,max}$ is obtained assuming that $c_0 + c_{1,max}$ corresponds to the maximum vaccination rate reached during the COVID-19 pandemic in US
- $D = 5 \times 10^{-6}$ so that $c(M)$ has a steep growth at the beginning of the vaccination campaign, and then it smooths over time until it reaches a plateau by the end of July 2021

and the characteristic memory length $1/\theta$ is of four days hence $\theta = 4$ weeks^{-1}

The information coverage parameter

estimate the information coverage parameter k

compare the model solutions with the data stemming from the first dose vaccinated individuals

a first glance on the response of human behaviour from official data

The information coverage parameter II

partially vaccinated are those individuals who have received the first dose of Pfizer or Moderna vaccines

we demand

the number of partially vaccinated individuals per millions (PVM) from the model during the vaccination campaign,
is as much as possible close to the official number of partially vaccinated people

The information coverage parameter III

The estimate is obtained minimising

$$\chi_i^2 = \frac{1}{t_f - t_1 - 1} \sum_{l=1}^{f} \left(V_{BeRG,i}^{l} - V_{0,i}^{l} \right)^2 \quad \text{with} \quad i = 1, \ldots 9 \qquad (16)$$

The information coverage parameter III

The estimate is obtained minimising

$$\chi_i^2 = \frac{1}{t_f - t_1 - 1} \sum_{l=1}^{f} \left(\boxed{V^l_{BeRG,i}} - V^l_{0,i} \right)^2 \quad \text{with} \quad i = 1, \dots 9 \qquad (16)$$

- number of PVM
  from the BeRG model,
  N_i is the population
  corresponding to the ith-division

The information coverage parameter III

The estimate is obtained minimising

$$\chi_i^2 = \frac{1}{t_f - t_1 - 1} \sum_{l=1}^{f} \left(\boxed{V^l_{BeRG,i}} - \boxed{V^l_{0,i}} \right)^2 \quad \text{with} \quad i = 1, \ldots 9 \qquad (16)$$

- number of PVM
  from the BeRG model,
  N_i is the population
  corresponding to the ith-division

- official number of PVM

The information coverage parameter III

The estimate is obtained minimising

$$\chi_i^2 = \frac{1}{t_f - t_1 - 1} \sum_{l=1}^{f} \left(\boxed{\int_{t_0}^{t_l} c(M_i) N_i dt} - \boxed{V_{0,i}^l} \right)^2 \quad \text{with} \quad i = 1, \ldots 9 \qquad (16)$$

- number of PVM
  from the BeRG model,
  N_i is the population
  corresponding to the ith-division
- official number of PVM
- t_0 marks the official start of the US vaccination campaign (i.e., December 16, 2020), l increments the number of weeks at which we evaluate the χ_i^2

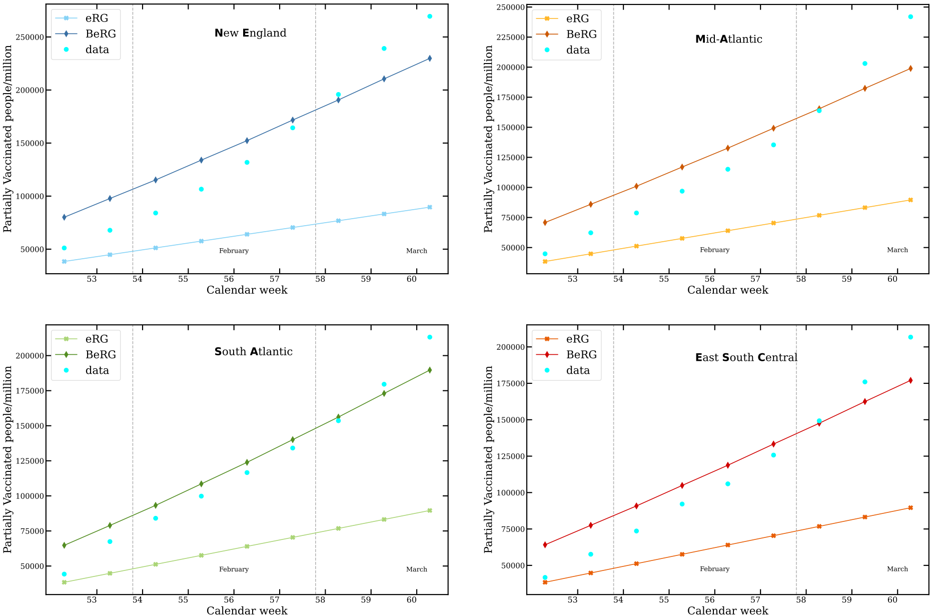
The information coverage parameter III

The estimate is obtained minimising

$$\chi_i^2 = \frac{1}{t_f - t_1 - 1} \sum_{l=1}^{f} \left(\boxed{\int_{t_0}^{t_l} c(M_i) N_i dt} - \boxed{V_{0,i}^l} \right)^2 \quad \text{with} \quad i = 1, \ldots 9 \qquad (16)$$

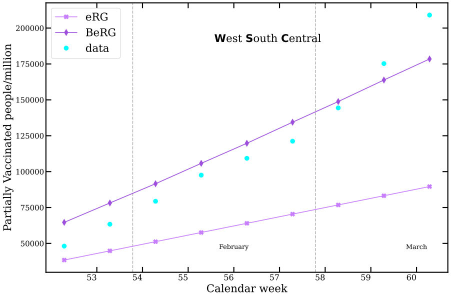
- number of PVM from the BeRG model, N_i is the population corresponding to the ith-division
- official number of PVM
- t_0 marks the official start of the US vaccination campaign (i.e., December 16, 2020), l increments the number of weeks at which we evaluate the χ_i^2
- computations from week $t_1 = 26$ (January 21, 2021) to week $t_9 = 34$ (March 18, 2021)

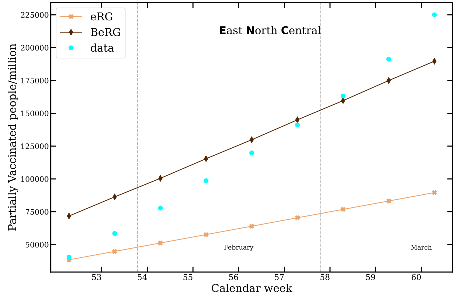
Fit on PVM

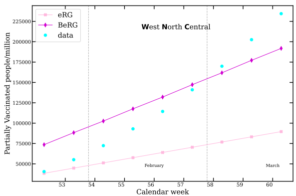


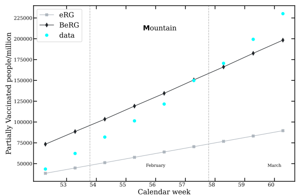
New England
Mid-Atlantic
South Atlantic
East South Central
eRG
BeRG
data
Partially Vaccinated people/million
Calendar week
February
March

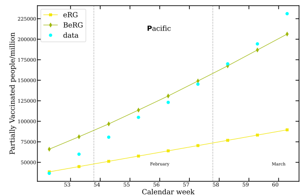
Fit on PVM II


West South Central
eRG
BeRG
data
Partially Vaccinated people/million
Calendar week
February
March

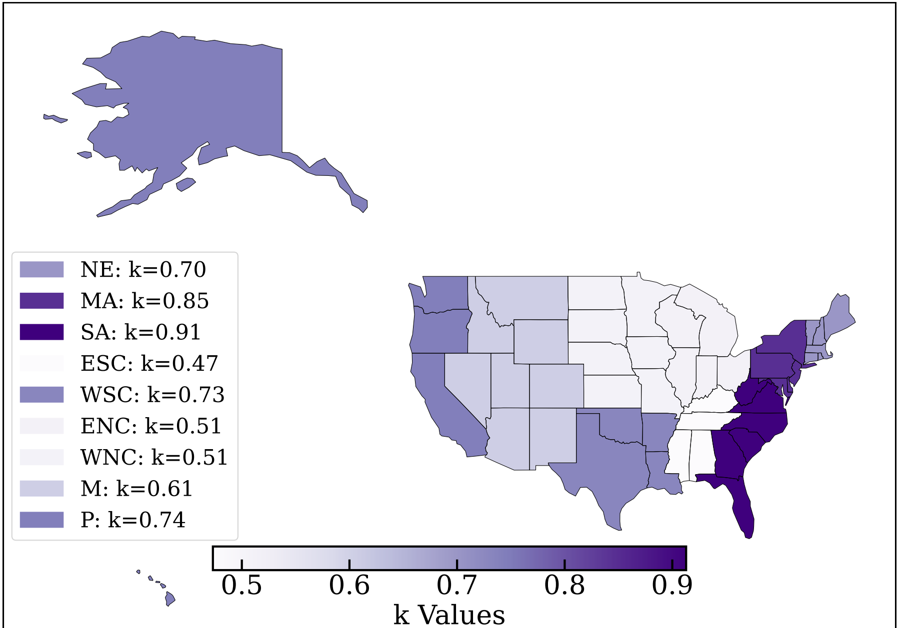

East North Central
eRG
BeRG
data
Partially Vaccinated people/million
Calendar week
February
March


West North Central
eRG
BeRG
data
Partially Vaccinated people/million
Calendar week
February
March


Mountain
eRG
BeRG
data
Partially Vaccinated people/million
Calendar week
February
March


Pacific
eRG
BeRG
data
Partially Vaccinated people/million
Calendar week
February
March

Results: the k parameter



Summary of Part I

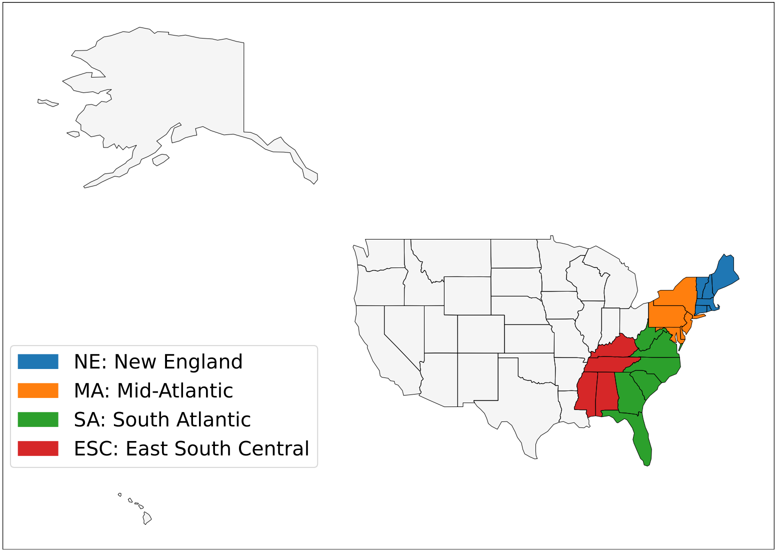
To assess the performance of the the BeRG model, we perform the best fits on PVM and we compared with eRG curves and official data:

- the eRG curve is reproduced by taking $c = c_0$ while the BeRG curve is obtained by assuming $c = c_0 + c_1(M)$, with $c_1(M)$ that further depends on the parameter k
- the BeRG model is able to reproduce the official data better than eRG, as expected
- the variability in the information coverage parameter depends on several factors influencing:
  1. how the population experiences the pandemics in different US divisions
  2. personal believes to social-economical status

Part II: Testing the validity of the BeRG

History and Plan

The US census divisions



we only focus on divisions where the vaccination campaign starts when the epidemic peak has not yet been achieved

BeRG fit to obtain a, γ, D and k

$$\frac{d\alpha_i}{dt} = \gamma_i\, \alpha_i \left(1 - \frac{\alpha_i}{a_i}\right)$$

$$+ \sum_{j\neq i}^{4} \frac{K_{ij}}{n_{ij}} \left(e^{\alpha_j - \alpha_i} - 1\right)$$

$$+ \sum_{i}^{4} \frac{K_{R_X i}}{n_{R_X i}} \left(e^{\alpha_{R_X} - \alpha_i} - 1\right)$$

$$\frac{d\gamma_i}{dt} = -c(M_i)\, \gamma_i(t_v)$$

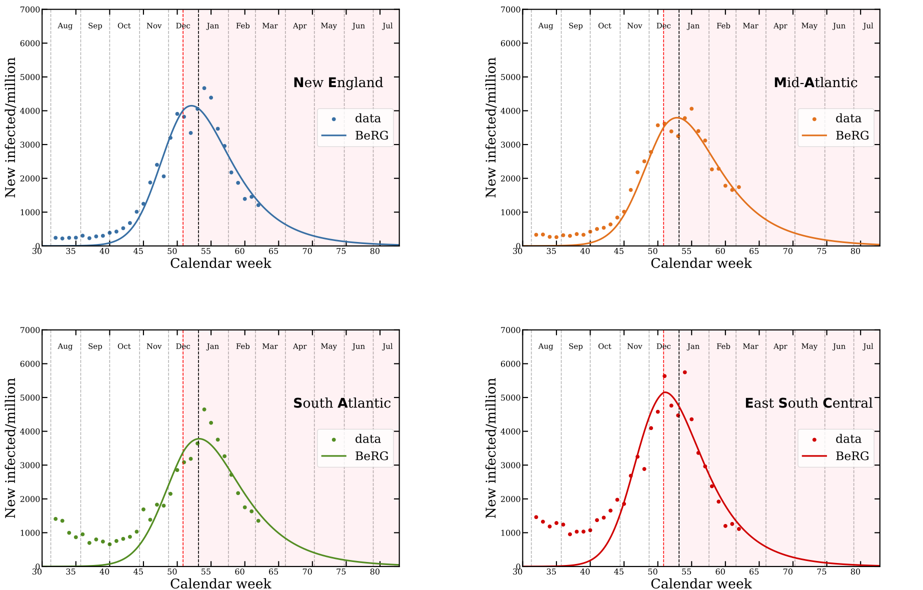
$$\frac{da_i}{dt} = -c(M_i)\, \left(e^{a_i} - e^{\alpha_i}\right)$$

a, γ, D and k using
the new infected data

Second wave parameters				
Code	a	γ	k	D
NE	11.036	0.204	0.52	5×10^{-6}
MA	11.075	0.185	0.75	8×10^{-6}
SA	11.075	0.185	0.60	10^{-5}
ESC	11.221	0.207	0.64	5×10^{-6}

Table: Parameters of the BeRG model for the second wave in 4 divisions: NE, MA, SA and ESC. The values are chosen to reproduce the data until March 4, 2021.

Results

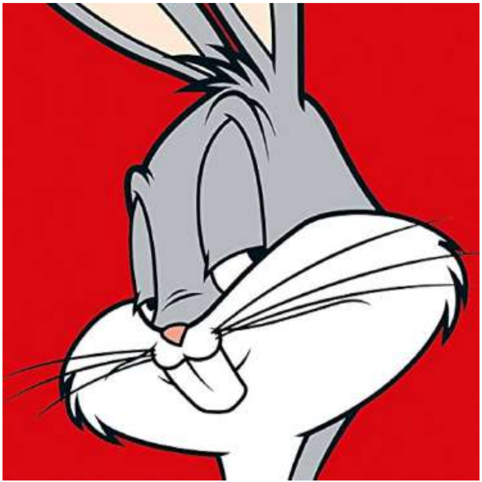

New England
data
BeRG
Mid-Atlantic
data
BeRG
South Atlantic
data
BeRG
East South Central
data
BeRG
Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul
New infected/million
Calendar week

Conclusions

We introduced the information index augmented eRG model to describe vaccination behaviours during an epidemic outbreak performing multiple tests of the model by comparing it to the data corresponding to nine US divisions:

1. Part I
   - we observe a strong behavioural impact on the increase of the number of vaccinated for all divisions

2. Part II
   - the model provides a better representation of the data for the divisions where the peak of the number of new infected is reached after the start of the vaccination campaign
   - the result is consistent with the expectation that the vaccination campaign and associated human behaviour has an impact when the epidemic is still rising



That's all Folks!

Thank you!