

# COMPOSITE HIGGS FROM $SU(2)$ WITH TWO FUNDAMENTAL FLAVORS

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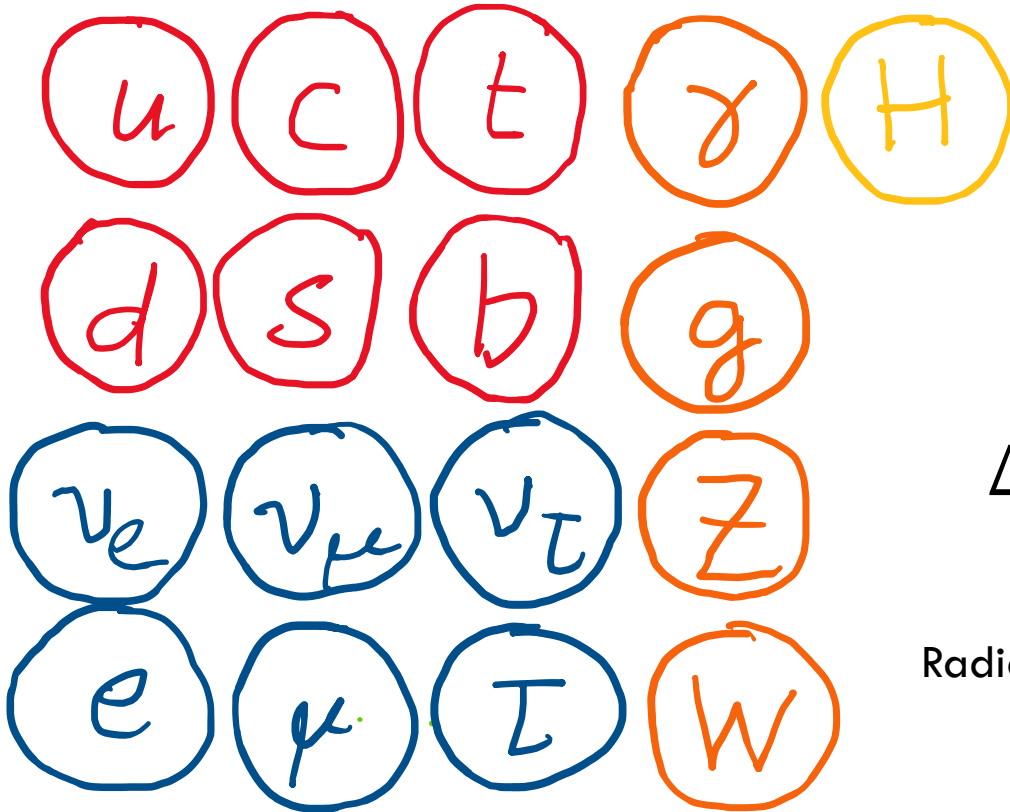
# OUTLINE

1. Introduction
  1. Composite Higgs
  2. Lattice Gauge Theory
2. Standard Model Quark Mass Generation
  1. Walking Technicolor
  2. Partial Compositeness
3. Lattice Studies of CH
  1. Software
  2.  $SU(2)$  with two fundamental flavors
  3. The pseudoscalar decay constant
  4. The mass of the new Higgs
4. Outlook

# INTRODUCTION

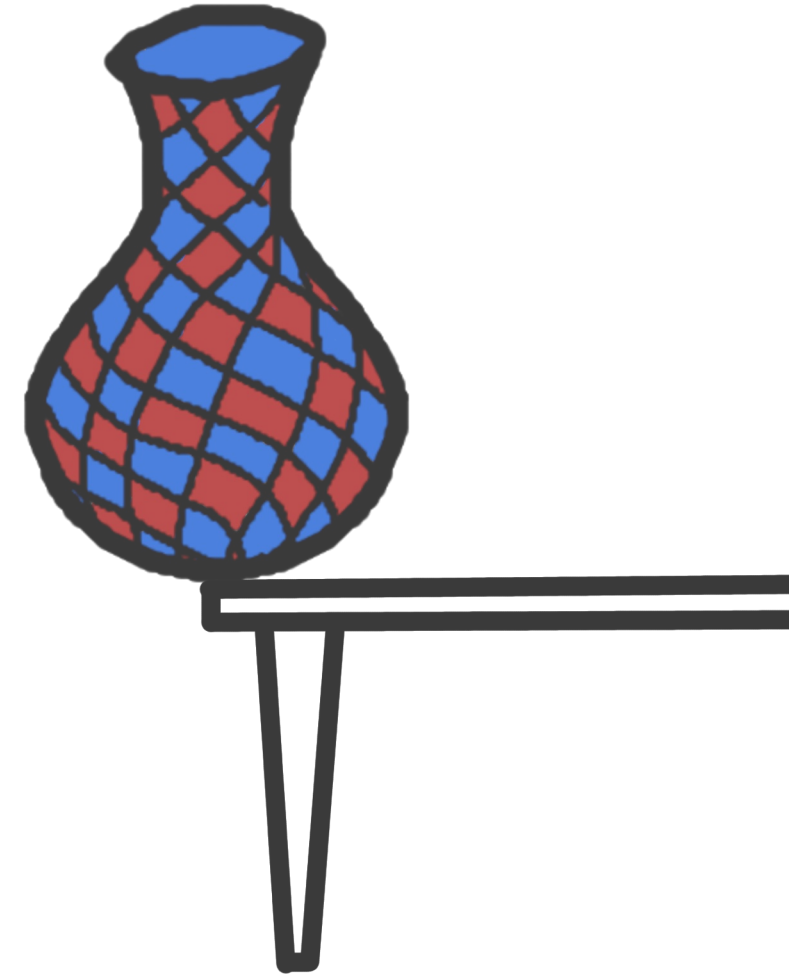
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# WHY COMPOSITE HIGGS?



$$\Delta m_H^2 = \frac{|\lambda_f|^2}{8\pi} \Lambda_{UV}^2$$

Radiative corrections to the Higgs-mass



# WHY COMPOSITE HIGGS?

Standard Model symmetries

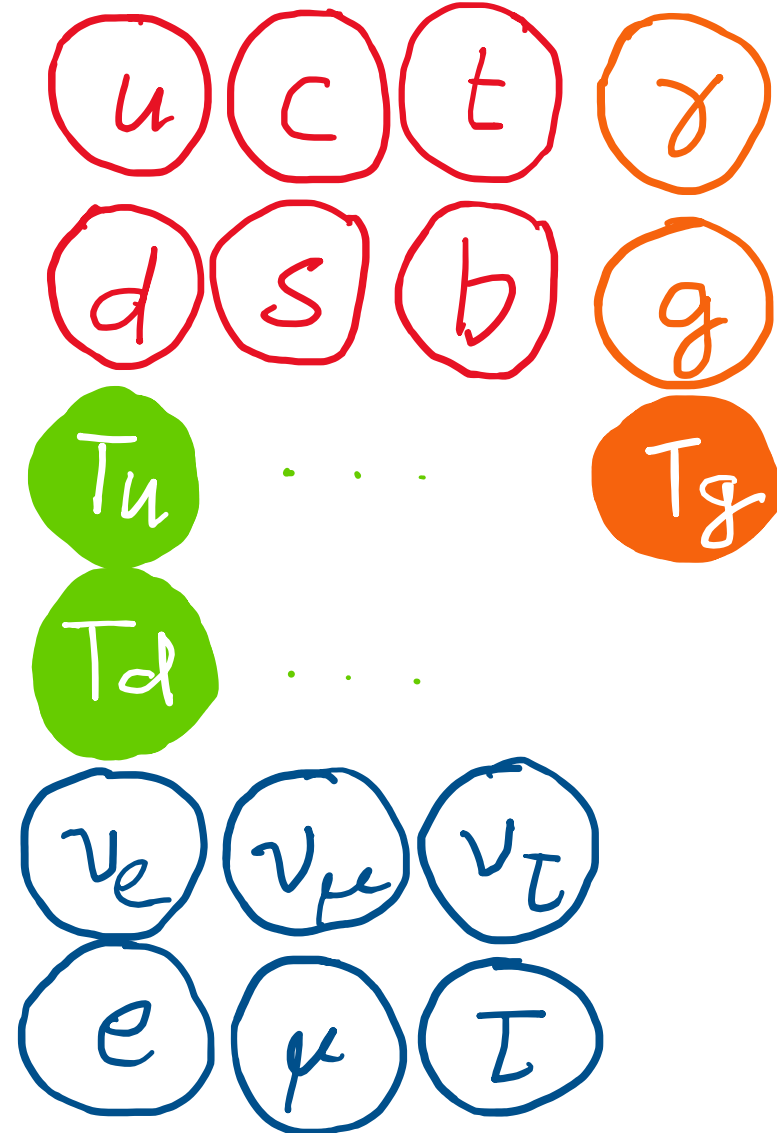
$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Protect the  $\rho$ -parameter with additional custodial symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{EM}$$

Replace the weak sector with a **fundamental color** theory

$$SU(3)_c \times \mathbf{SU(2)}_L \times \mathbf{SU(2)}_R \times U(1)_Y \rightarrow SU(3)_c \times U(1)_Y \times \mathbf{SU(N_{FC})}$$





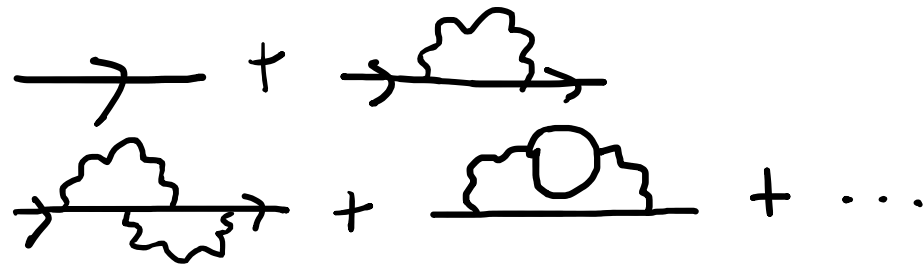
# THE ROLE OF LATTICE GAUGE THEORY

Minkowskian 4D continuum physics

$$Z = \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{iS[U, \Psi, \bar{\Psi}]}$$

Generating Functional with conjugate variables

$$Z = \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{iS[U, \Psi, \bar{\Psi}] - (J, U) - (\pi, \Psi) - \overline{(\pi, \Psi)}}$$

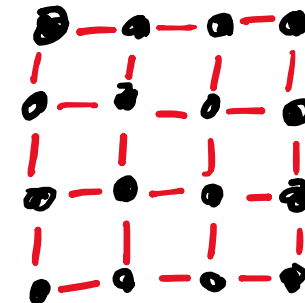


QFT on a Euclidean 4D lattice

$$Z = \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{-S[U, \Psi, \bar{\Psi}]}$$

Generate sample of gauge fields  $U_i, i = 1, \dots, N$

$$\langle \widehat{O} \rangle = \frac{1}{N} \sum_{i=1}^N O(U_i) \pm \sqrt{\frac{\text{var}(O(U_i))}{N}}$$

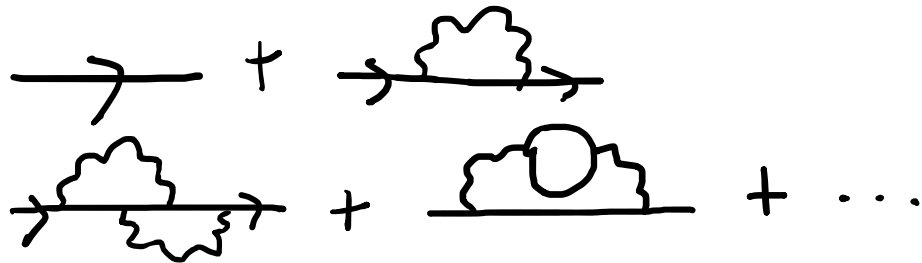


# PROBLEMS WITH BOTH APPROACHES

## Minkowskian 4D continuum physics

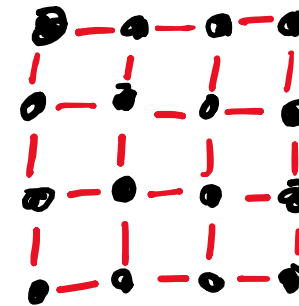
$$Z = \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{iS[U, \Psi, \bar{\Psi}]}$$

Breaks down at large couplings



## QFT on a Euclidean 4D lattice

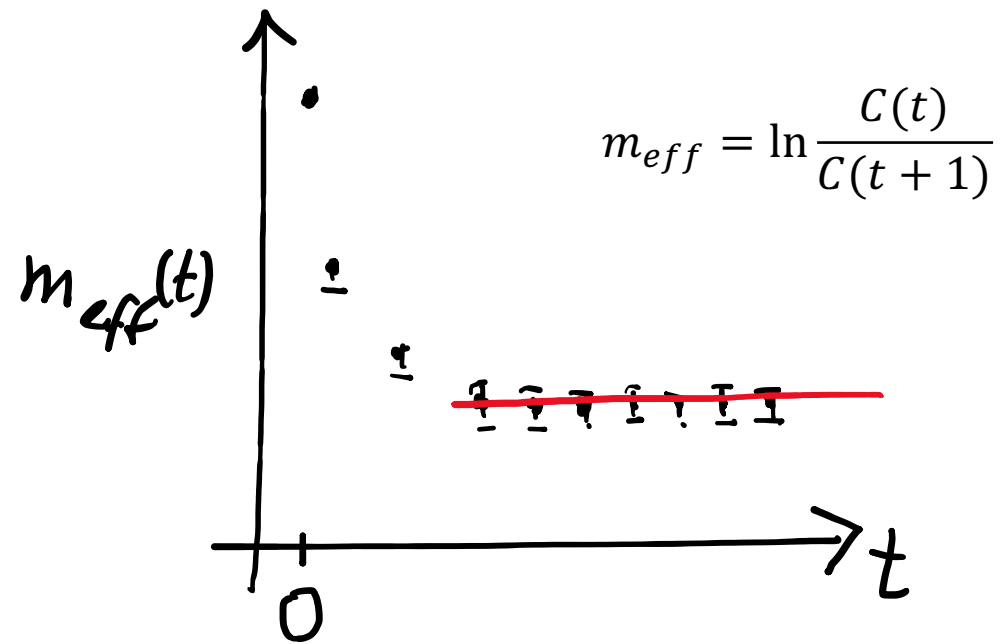
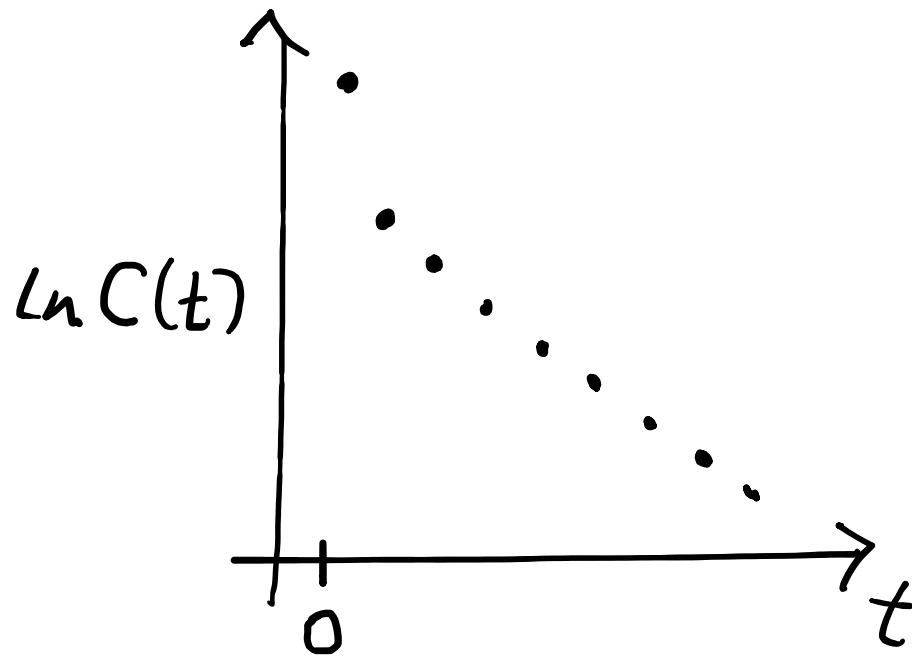
- Systematic uncertainties: Finite lattice spacing, volume and quark mass
- Statistical uncertainties: Sample size



# THE ROLE OF LATTICE GAUGE THEORY

Lattice gauge theory is good at **extracting masses**

$$C(t_1, t_0) = \langle O(t_1) O(t_0) \rangle = \sum_n |\langle 0 | O(0) | n \rangle|^2 e^{-E_n(t_1 - t_0)} \rightarrow |\langle 0 | O(0) | n \rangle|^2 e^{-m(t_1 - t_0)}$$



# STANDARD MODEL QUARK MASS GENERATION

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# WALKING TECHNICOLOR

This is some necessary context for lattice studies on **Technicolor**

# EXAMPLES OF RG TRANSFORMATIONS

\* RG = RENORMALIZATION GROUP

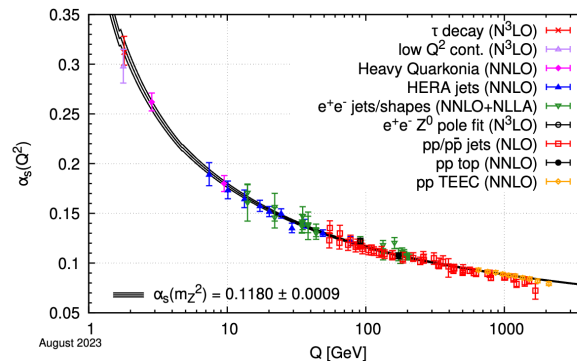
Infectious disease spread



Predicting the number of infections at a future time from the current time

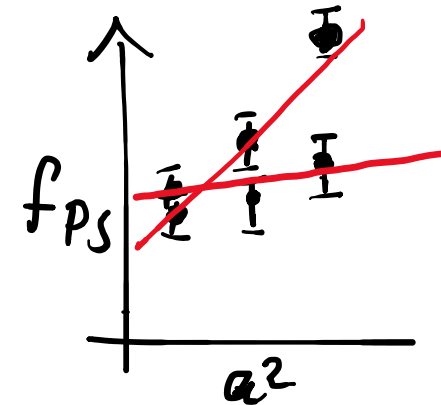
[de Hoffer, A., Vatani, S., Cot, C. et al., 2022, Nature, 0.1038/s41598-022-12442-8]

SU(N) theories



Increasing the center of mass energy<sup>1</sup>

Lattice Field Theory



Performing a continuum extrapolation

# EXAMPLES OF CONFORMAL BEHAVIOR

\* RG = RENORMALIZATION GROUP

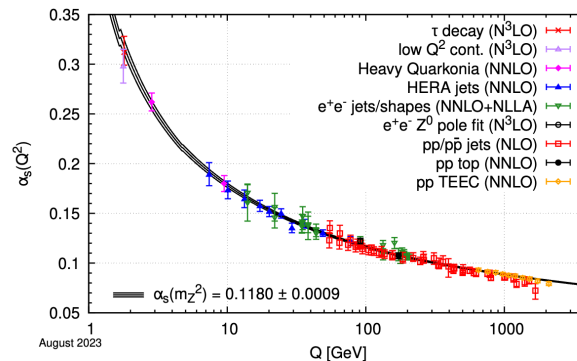
Infectious disease spread



The disease is endemic

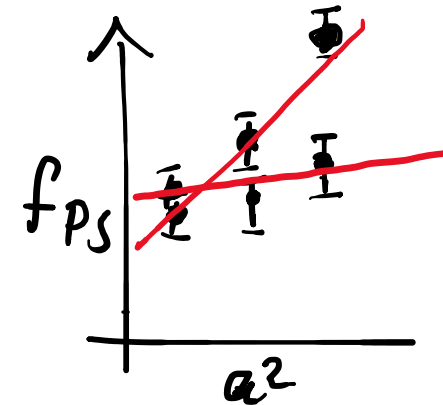
[de Hoffer, A., Vatani, S., Cot, C. et al., 2022, Nature, 0.1038/s41598-022-12442-8]

SU(N) theories



The coupling stays constant<sup>1</sup>

Lattice Field Theory



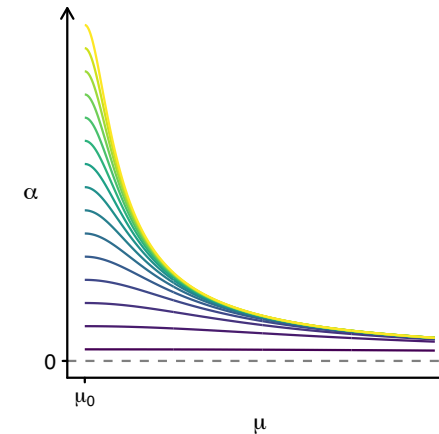
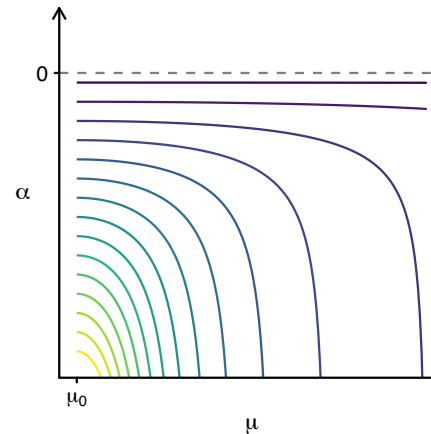
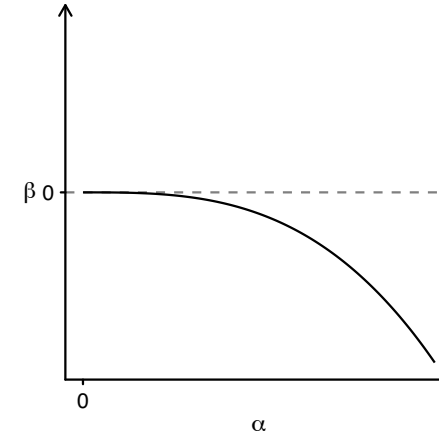
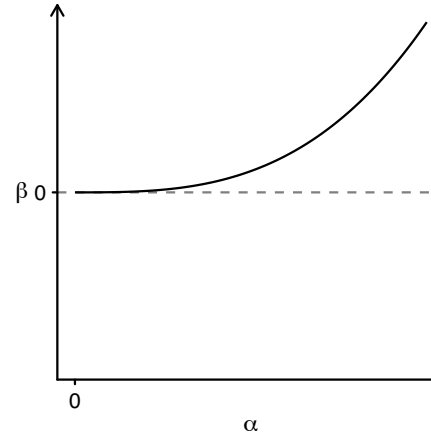
"Perfect actions"

[Hasenfratz, Niedermayer, 1993, Phys. Rev. D, hep-lat/9308004]

# ANALYZING THE BETA FUNCTION — QED & QCD

$$\mu \frac{d\alpha}{d\mu} = -\beta(\alpha)$$

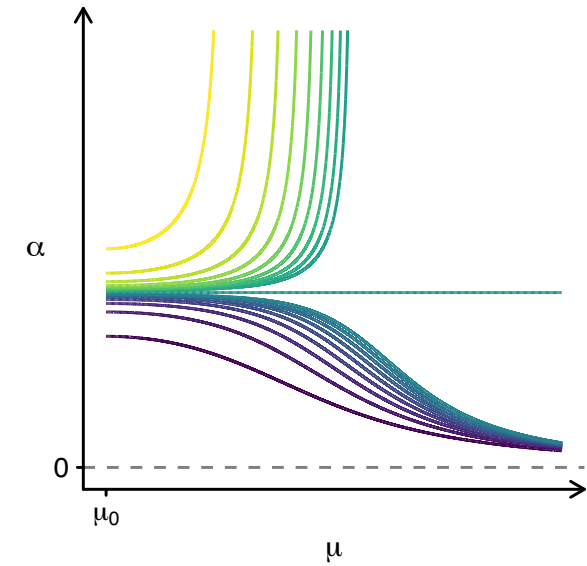
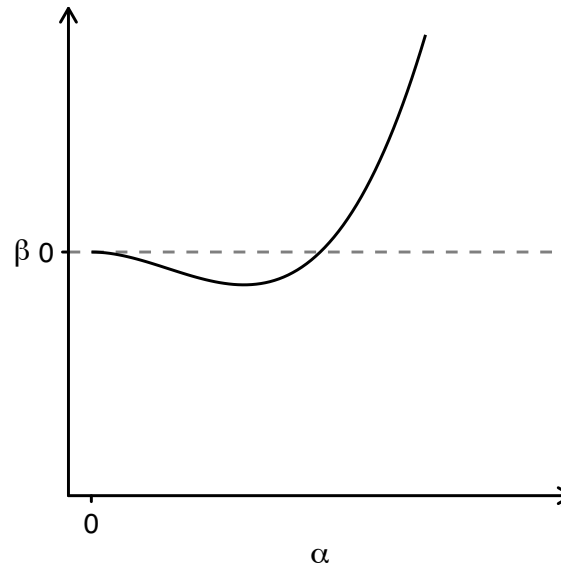
1. Trivial fixed point
2. Sign of beta function
3. Divergence of coupling



# ANALYZING THE BETA FUNCTION - CFT

$$\mu \frac{d\alpha}{d\mu} = -\beta(\alpha)$$

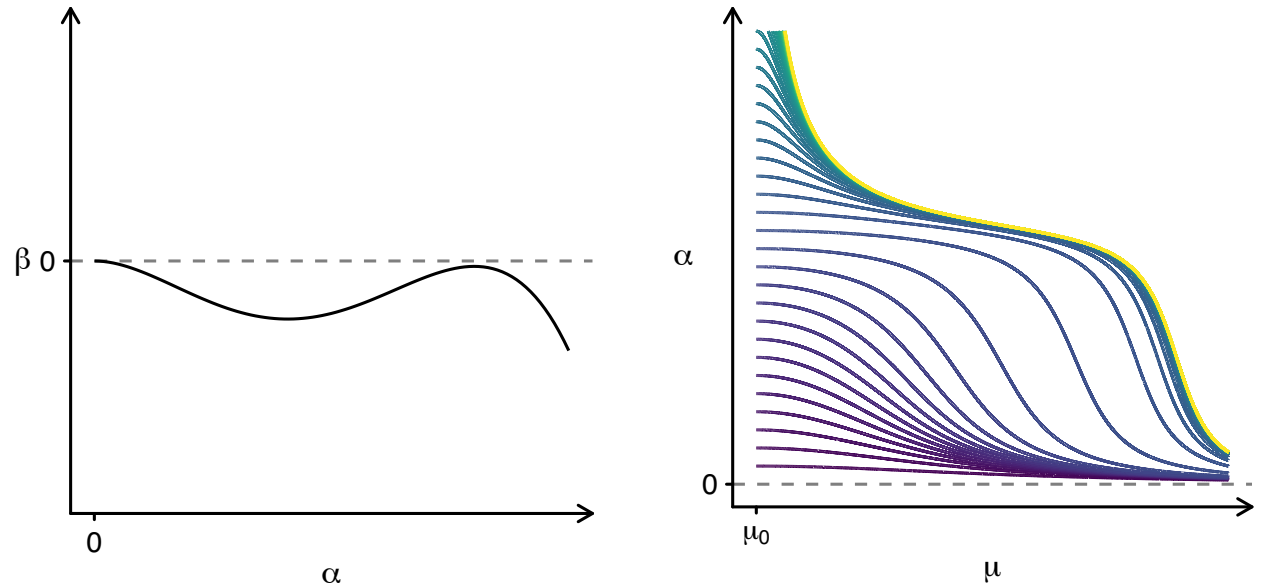
1. **Attractive or repulsive** fixed points
2. **Conformal phases** with IR fixed point
3. Phase structure defined by Beta-function



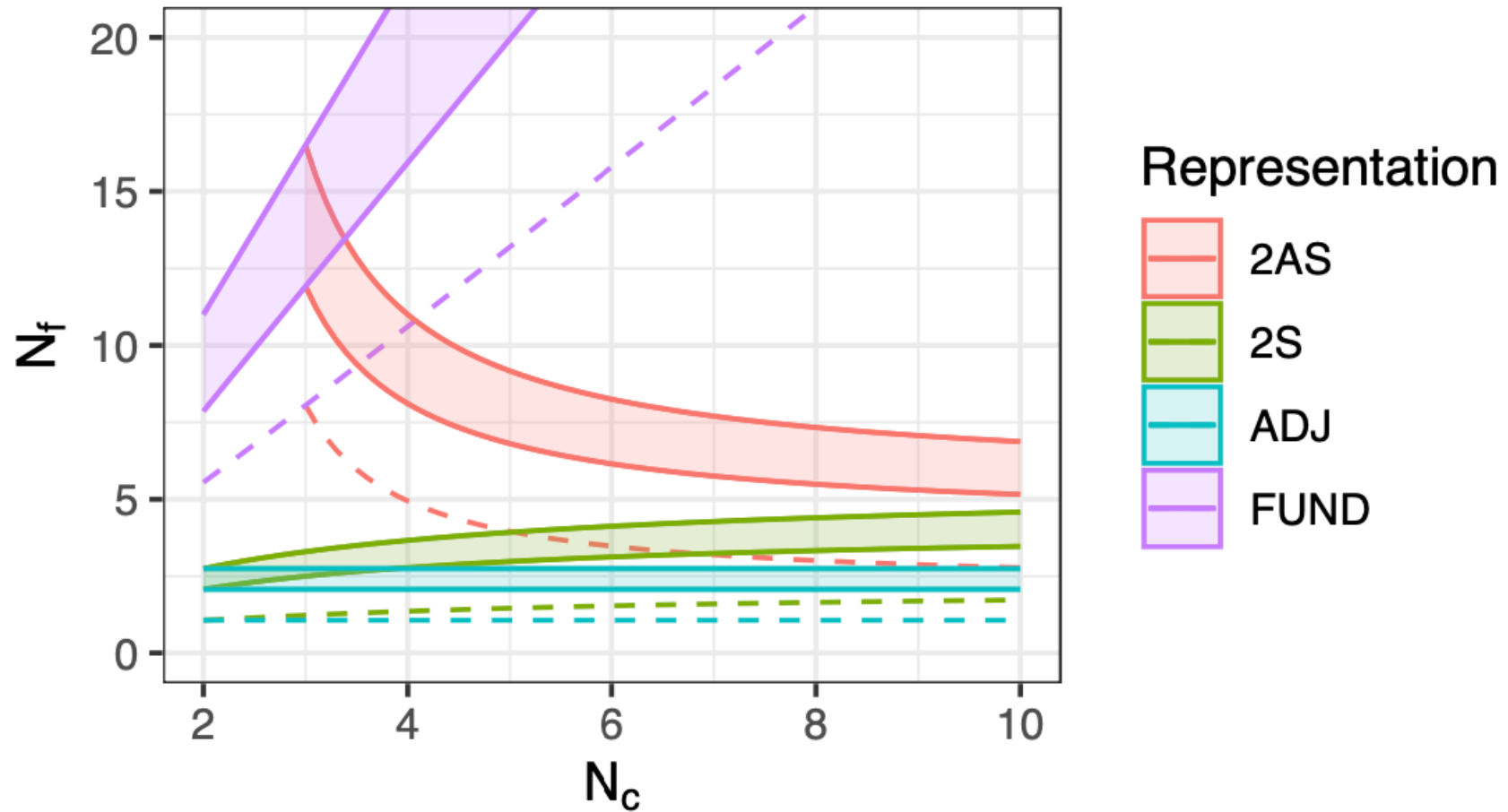
# ANALYZING THE BETA FUNCTION — WALKING TC

$$\mu \frac{d\alpha}{d\mu} = -\beta(\alpha)$$

1. Near conformality = Walking TC
2. AF & ChiSB are still there, like regular QCD



# CONFORMAL WINDOW (PERTURBATIVE)

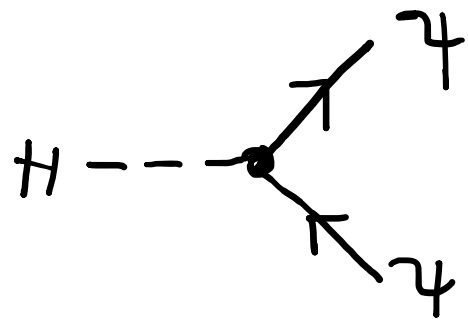


# STANDARD MODEL MASS GENERATION

This aims to generate the masses of the Standard Model quarks.

**Why is the top so heavy??**

Higgs interaction with  
Standard Model quarks



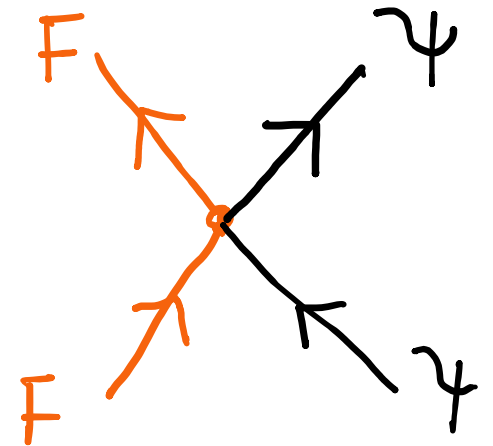
$$= gH\psi\bar{\psi}$$

*becomes*

$\rightarrow$

$$\alpha_{ab} \frac{\bar{F}T^a F \bar{\psi}T^b \psi}{\Lambda^2}$$

New sector quarks F interact with Standard  
Model quarks



# STANDARD MODEL MASS GENERATION

This aims to generate the masses of the Standard Model quarks.

**Why is the top so heavy??**

Electroweak  
symmetry  
breaking

*becomes*

Chiral  
Symmetry  
Breaking

$\langle H \rangle$

$\rightarrow$

$\langle \bar{F} F \rangle$

# STANDARD MODEL MASS GENERATION

This aims to generate the masses of the Standard Model quarks.

**Why is the top so heavy??**

$$m_q \sim \frac{\alpha}{\Lambda^2} \langle \bar{F} F \rangle \approx \frac{\alpha}{\Lambda^2} \langle \bar{F} F \rangle_{SU(N)} = \frac{\alpha}{\Lambda^2} \langle H \rangle$$

# STANDARD MODEL MASS GENERATION

This aims to generate the masses of the Standard Model quarks.

**Why is the top so heavy??**

$$m_q \sim \frac{\alpha}{\Lambda^2} \langle \bar{F} F \rangle \approx \frac{\alpha}{\Lambda^2} \left( \frac{\Lambda^2}{\Lambda_{SU(N)}^2} \right)^{\gamma(\alpha^*)} \langle \bar{F} F \rangle_{SU(N)}$$

# QUICK SUMMARY

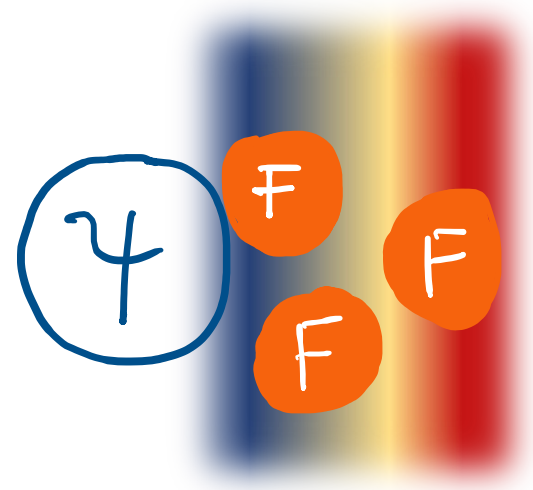
1.  $SU(N)$  gauge theories can exhibit chiral symmetry breaking or a conformal phase in the IR
2. Theories that are close to conformal have chSB & AF but also a region where the coupling stays close to constant
3. All of these phenomena are non-perturbative and need lattice gauge theory for proof

# PARTIAL COMPOSITENESS

This is some necessary context for lattice studies on **Composite Higgs**

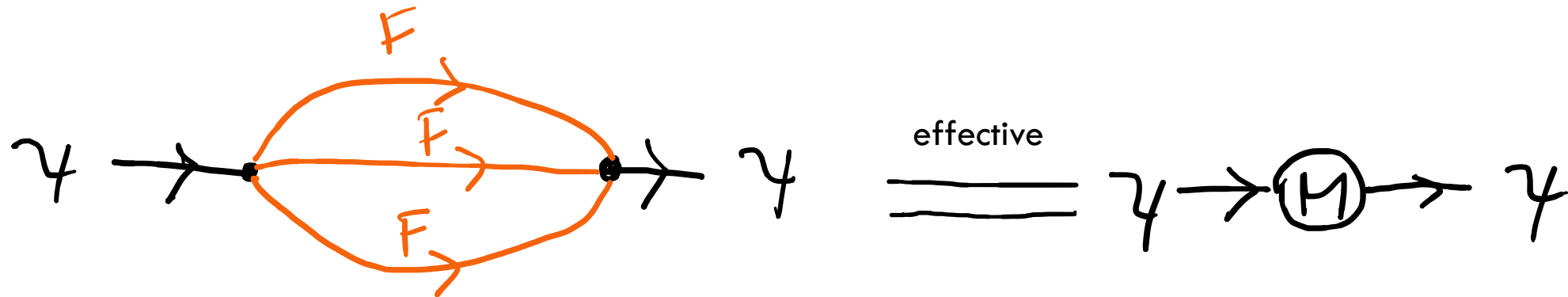
# SM QUARKS MIX WITH QUARKS FROM THE NEW SECTOR

$$\frac{4\pi}{M^2} (\kappa \Psi FFF + \kappa^* \bar{\Psi} \bar{F} \bar{F} \bar{F}) = \Psi \rightarrow \begin{array}{c} \nearrow \bar{F} \\ \nearrow F \\ \searrow F \end{array}$$



$FFF$  must have the same quantum numbers as  $\Psi$ .

# SM QUARKS MIX WITH QUARKS FROM THE NEW SECTOR



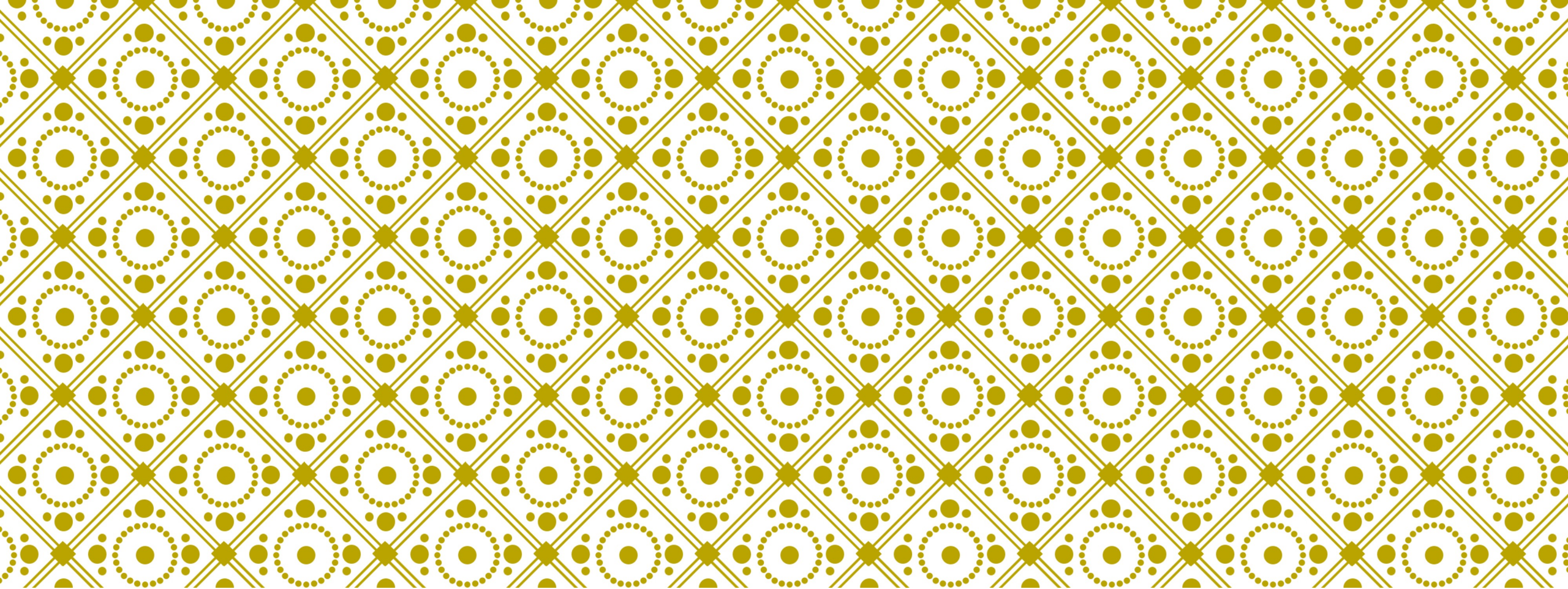
Mass generation is linked to new sector quark mixing and not the condensate

# QUICK SUMMARY

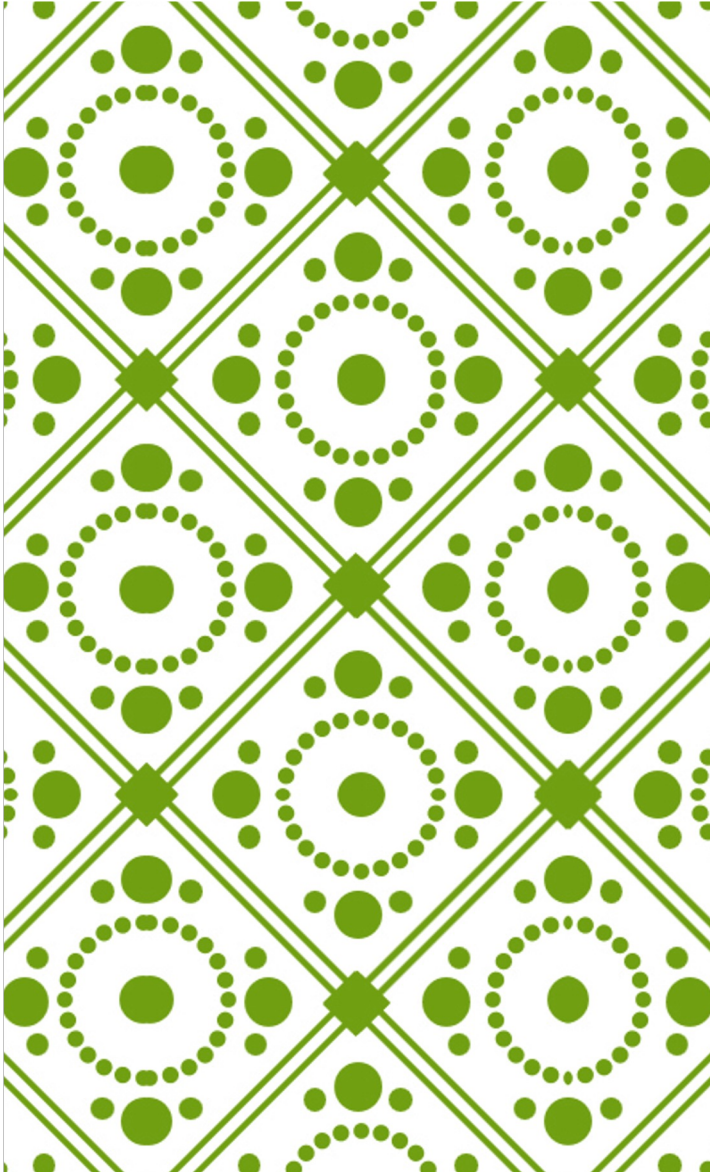
1. Another mass generation mechanism is partial compositeness, mixing of standard model quarks with fundamental color quarks
2. In this case one avoids coupling the Standard Model masses to the fundamental color quark condensate

# SUMMARY

1. Walking behavior due to near conformality can explain large/different quark masses
2. Partial compositeness can explain large/different quark masses
3. To understand the workings of technicolor/Goldstone Higgs theory we need to have an estimate of  $f_{PS}$  and  $m_H$  from the new sector
4. This is very feasible on the lattice



# LATTICE STUDIES OF CH



# SOFTWARE

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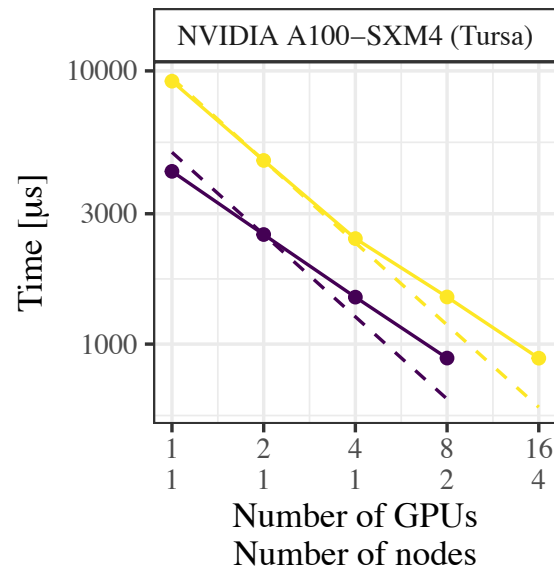
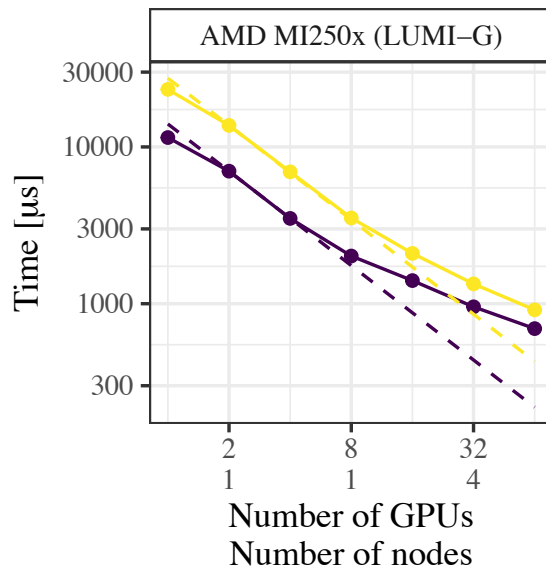
HiRep



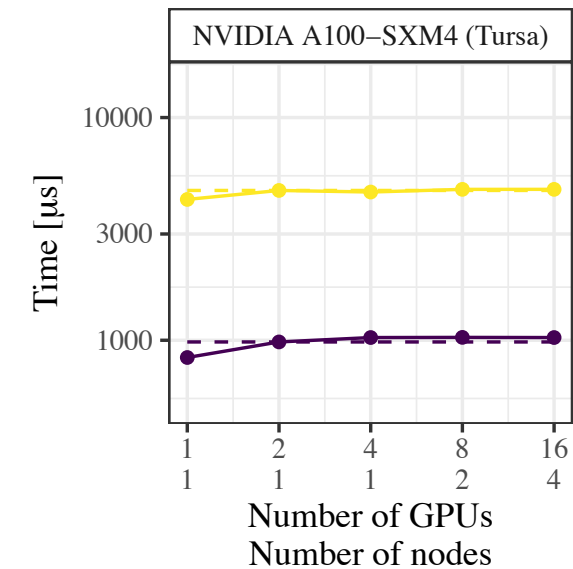
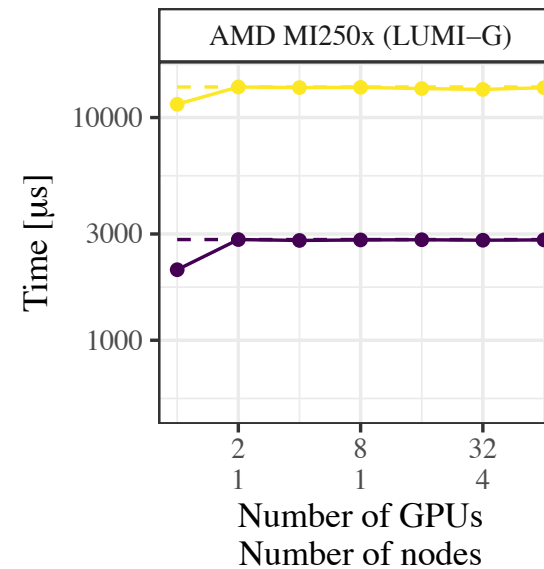
# A HIGH PERFORMANCE WILSON DIRAC OPERATOR

1. The software runs on **multiple architectures**
2. It achieves **peak bandwidth** on NVIDIA GPUs

Global lattice —●—  $48^4$  —●—  $96 \times 48^3$



Local lattice —●—  $32^4$  —●—  $48^4$



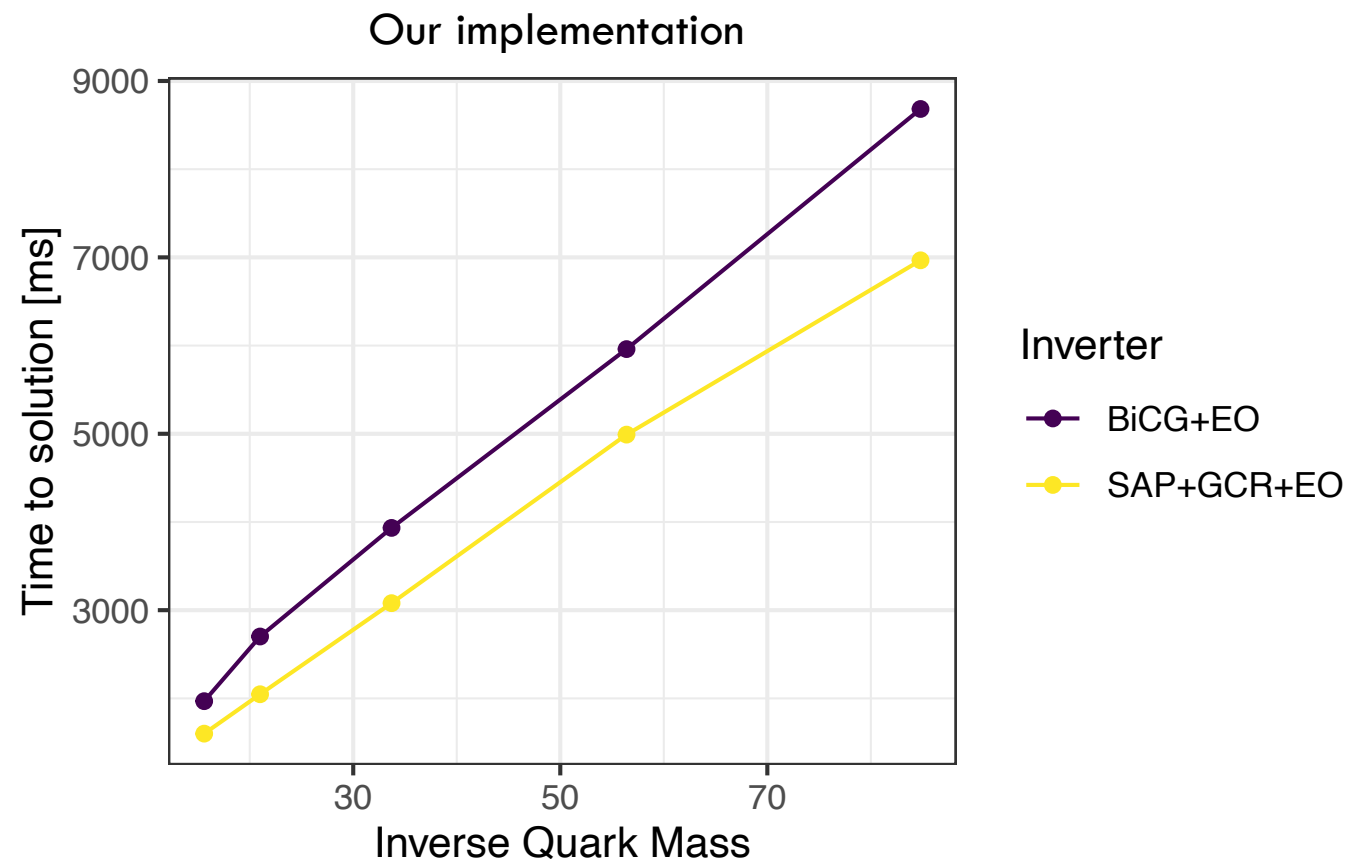
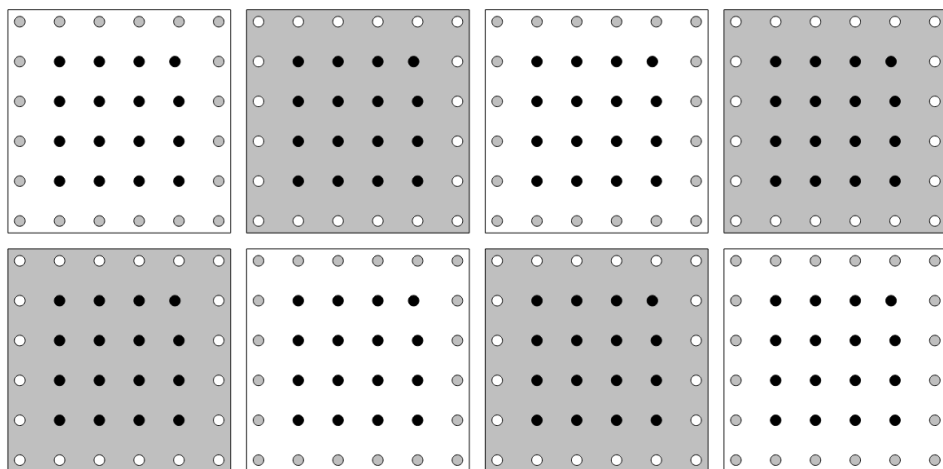
1. We can reduce execution time by using more processors for the same lattice (**strong scaling**)
2. If we increase the volume together with the processors, the execution time stays the same (**weak scaling**)



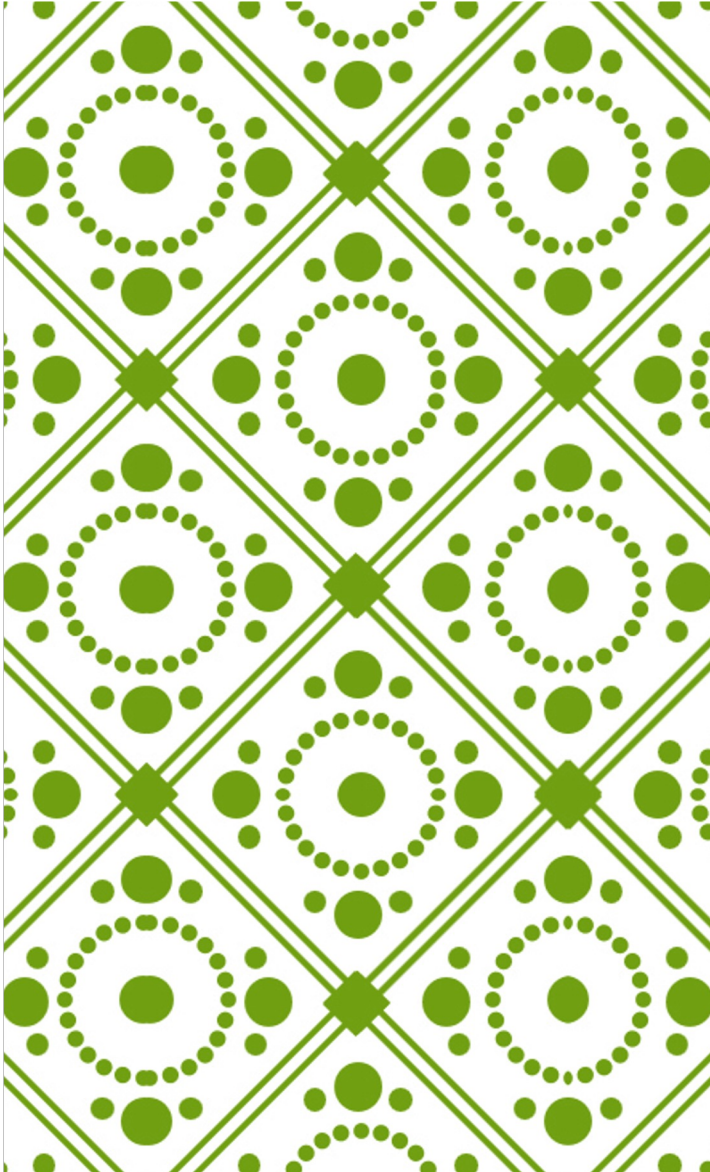
# DOMAIN DECOMPOSITION FOR HIGHER REPRESENTATIONS

WORK IN PROGRESS

1. GPU kernels are executed **block by block**
2. We should perform inversions on the lattice block by block



Picture and idea: [Luscher, 2003, Comput. Phys. Commun., hep-lat/0310048]



# SU(2) WITH FUNDAMENTAL FLAVORS

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The minimal composite higgs scenario

# THE MINIMAL SCENARIO

Two massless fermions and a two-color gauge boson, solves a few problems

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{u}(i\gamma^\mu D_\mu - m)u + \bar{d}(i\gamma^\mu D_\mu - m)d$$



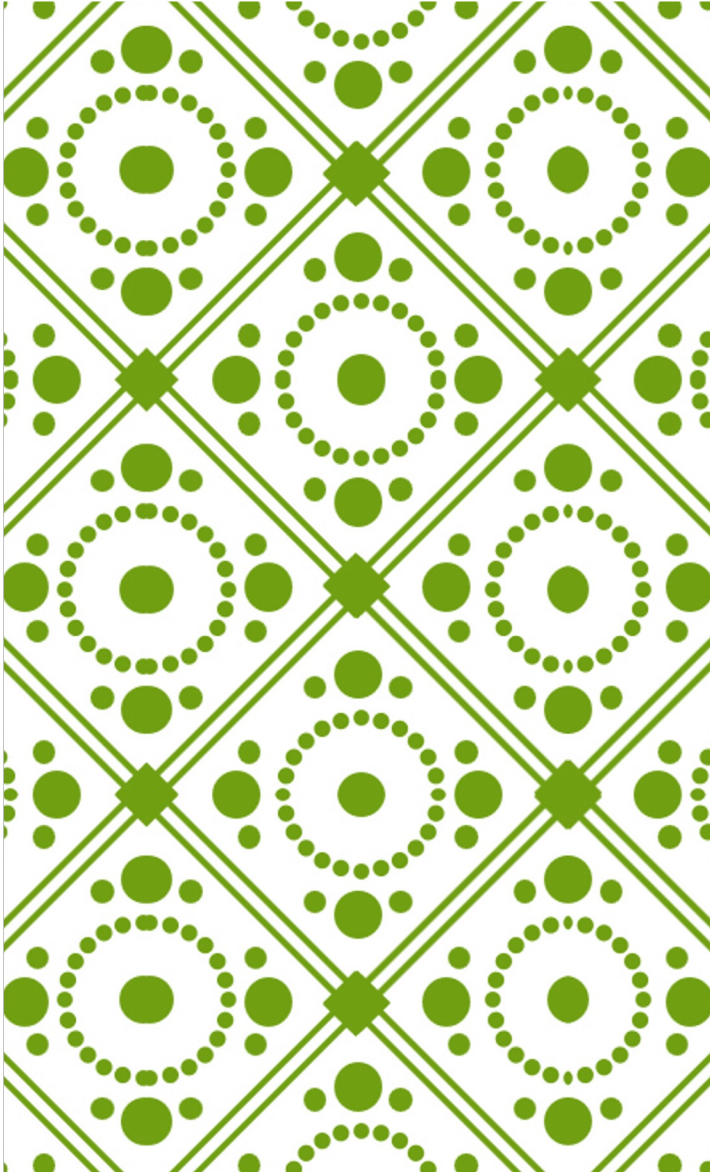
Preservation of  
custodial symmetry



Minimal = cheap  
on the lattice



No hierarchy  
problem



# DETERMINATION OF THE PSEUDOSCALAR DECAY CONSTANT

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This is needed for deriving a valid low-energy theory and that has to correspond with the current weak sector

# RECIPE: FROM THE LATTICE TO A PHENO PREDICTION

1. **Extrapolate to a line of constant physics**
2. **Continuum extrapolation = extrapolating to vanishing lattice spacing**
3. **Chiral extrapolation = extrapolation to vanishing quark mass**



# PRECISION SCALE SETTING THROUGH $f_{PS}$



## Controlling statistical uncertainties

Large lattices + High statistics on GPUs

Hasenbusch Acceleration



## Controlling systematic uncertainties

Non-perturbative Exponential-Clover Improvement

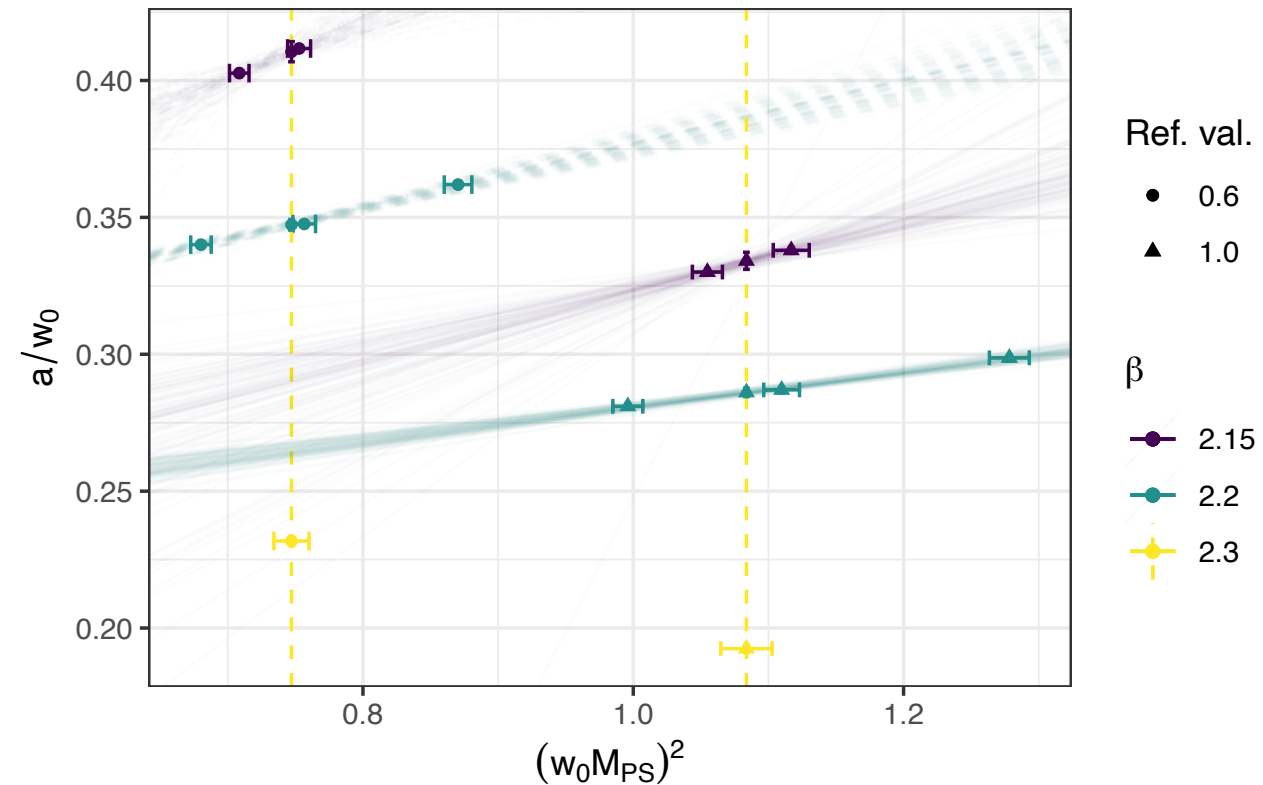
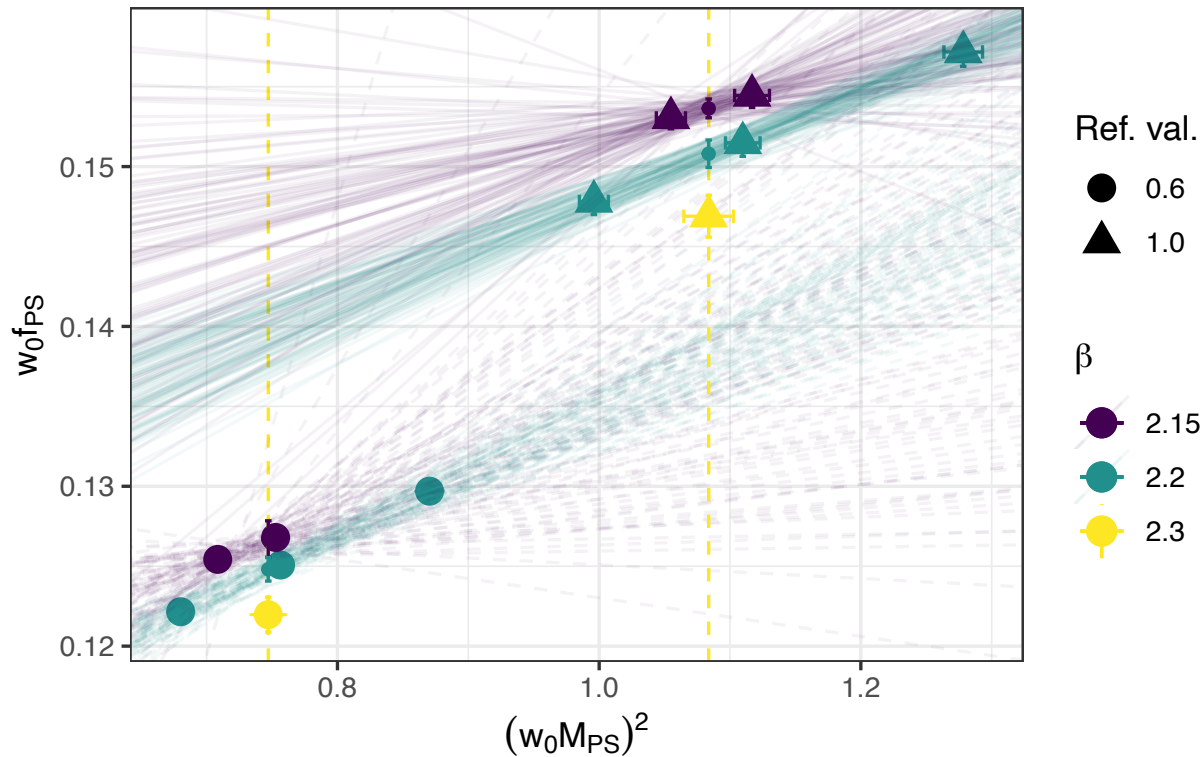
Twisted-Mass Mixed Action Measurements

Continuum extrapolation scale setting, compare scales  $w_1/w_0$

# LINE OF CONSTANT PHYSICS

## 1. Simulate at different quark masses, then extrapolate to a line of constant physics:

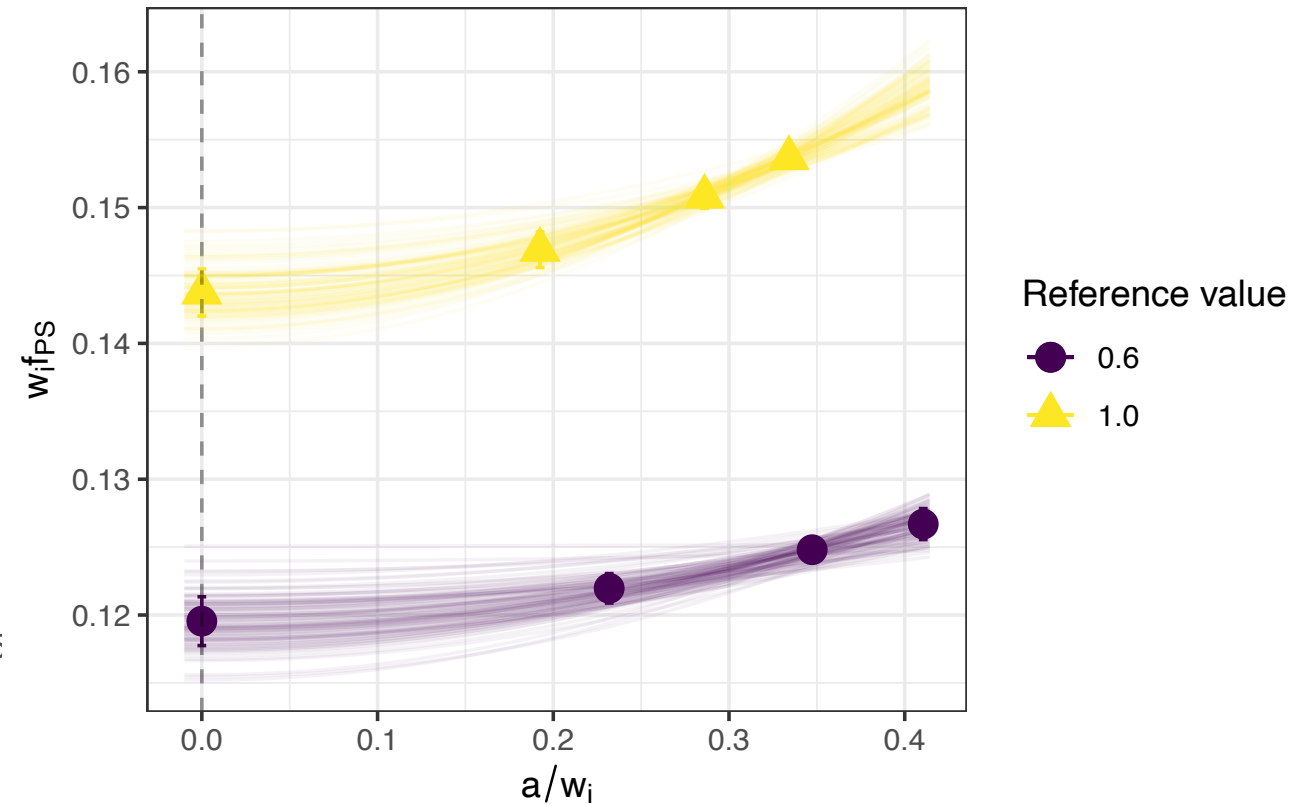
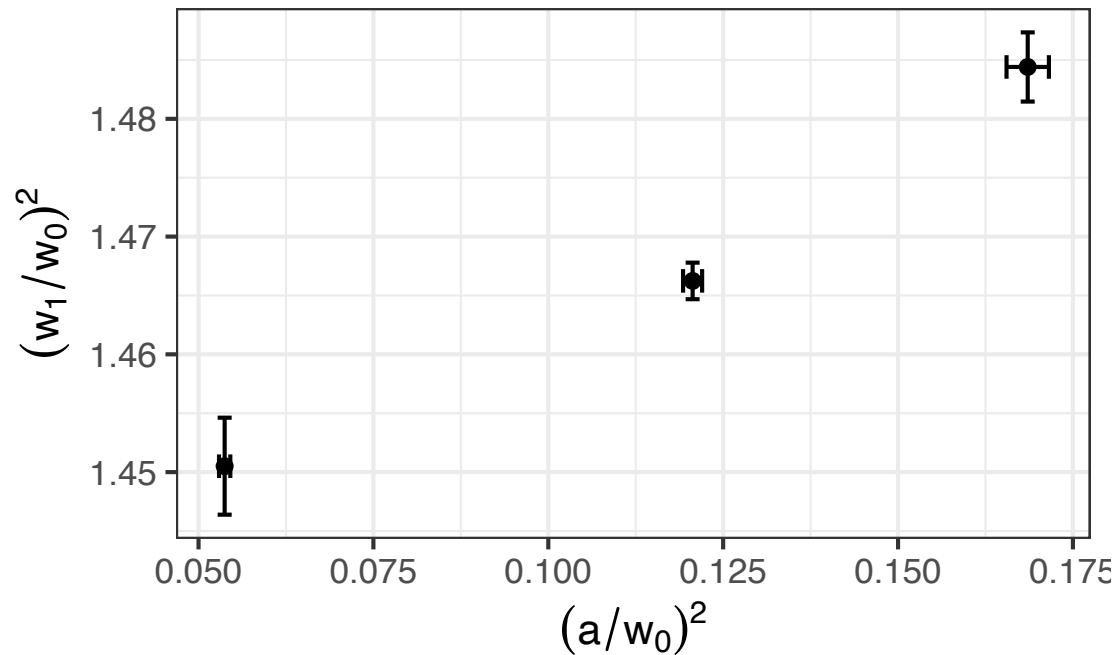
1. Find the prediction, for example  $f_{PS}$ , for a fixed mass of the pseudoscalar
2. Find the mass of the pseudoscalar  $m_{PS}$
3. Extrapolate the  $f_{PS}$  to a common fixed  $m_{PS}$ , the line of constant physics



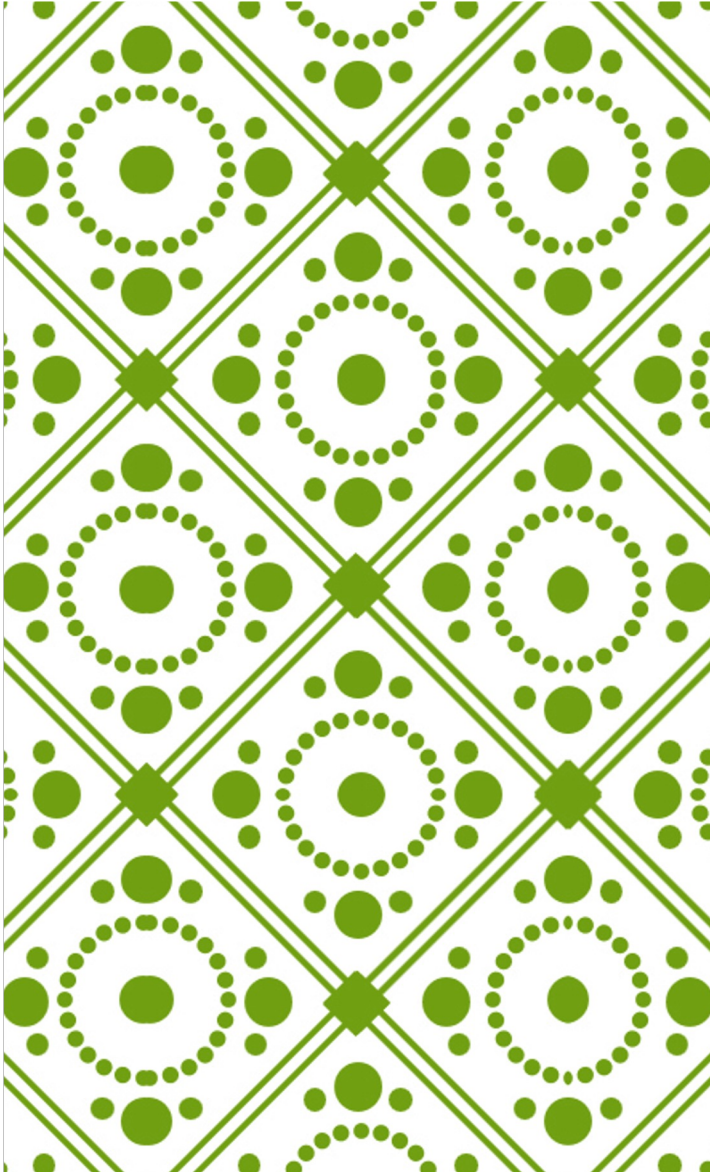
# CONTINUUM EXTRAPOLATION

## 1. Continuum extrapolation

1. Determine the lattice spacing of the ensemble
2. Extrapolate to the continuum



We have at most 10% discretization effects. This is quite precise!



# THE MASS OF THE NEW HIGGS

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Spectroscopy

# DISCONNECTED CONTRIBUTIONS TO THE SINGLET

Calculate propagator

$$\langle \bar{\Psi}(x_0) \Psi(x_1) \rangle = D^{-1}(x_0|x_1) = S(x_0, x_1)$$

Consider all Wick-contractions:

$$\langle O(t_0) O(t_1) \rangle = -\frac{1}{2} \text{tr}[\Gamma S(t_0|t_1) \Gamma S(t_0|t_1)] + \frac{1}{2} \text{tr}[\Gamma S(t_0|t_0)] \text{tr}[\Gamma S(t_1|t_1)]$$



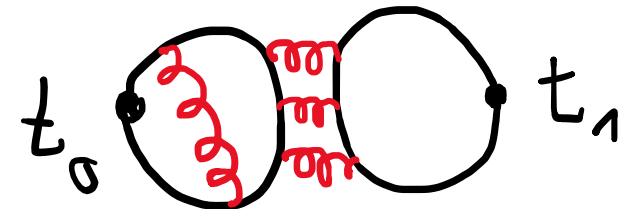
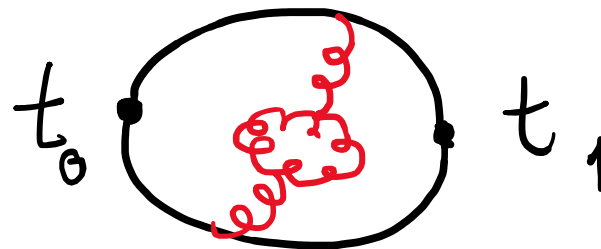
# THIS IS NOT PERTURBATION THEORY

Calculate propagator

$$\langle \bar{\Psi}(x_0) \Psi(x_1) \rangle = D^{-1}(x_0|x_1) = S(x_0, x_1)$$

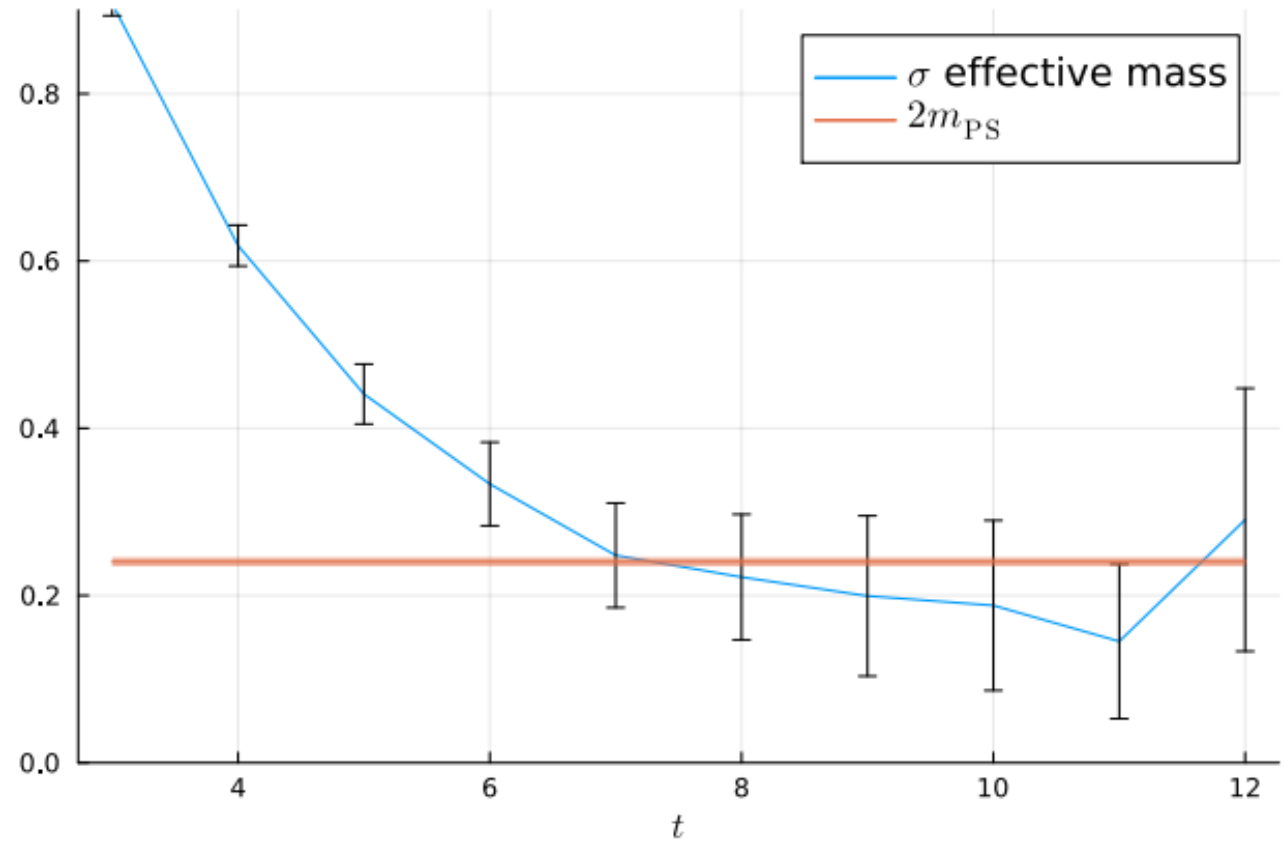
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# SINGLET & TWO-PION STATE

1. We see a plateau
2. Is this the singlet or the two-pion state?





# CONCLUSION & OUTLOOK

# SUMMARY

1. Composite Higgs theories are a promising candidate for BSM physics because of good phenomenological properties
2. We are examining  $SU(2)$  with two fundamental-color quarks on the lattice
3. We have managed to use state-of-the-art techniques to evaluate the pseudoscalar decay constant and Higgs mass on the lattice with very few lattice artifacts
4. A chiral extrapolation remains to be done

# PRECISION PREDICTIONS FOR BSM PHYSICS

1. The GPU porting of HiRep allows the generation of high statistics on modern supercomputers
2. We were able to reach exceptionally high precisions for BSM physics

**The lattice is becoming able to produce high-precision results  
for BSM theories**